Heat Flux in a Non-Maxwellian Solar Coronal Plasma.

N.N.Ljepojevic[1] and P.MacNeice[2]

[1]Department of Applied Mathematics and Theoretical Physics
University of Cambridge, Cambridge CB3 9EW, UK

[2]NASA Goddard Space Flight Center, Code 682,
Greenbelt, MD 20771, USA

Abstract.

We determine the electron distribution function within a hot coronal loop using a hybrid numerical scheme which couples the Spitzer-Härm method at low velocities with the solution to the high velocity form of the Landau-Fokker-Planck equation. From this we calculate the heat flux throughout the loop and compare it with the classical fourier law of Spitzer and Härm(1953).

1. Introduction.

Studies of the energy balance within solar coronal structures have long been hampered by poor estimates of the heat flux carried by ionized plasmas. The heat flux is only known accurately in the highly collisional regime studied by Spitzer and Härm(1953) – hereafter referred to as SH – and Braginskii(1965). In this paper we consider the failure of their solutions to properly account for the contribution of collisionless high velocity electrons, which lend a global character to the energy balance.

In many cases, although recognising its inappropriateness, authors have had to adopt the SH fourier law heat flux for lack of anything better (ie. Kopp et al 1986). SH calculated the time-independent electron distribution function which results from the presence of a weak electric field and temperature gradient. Their solution was based on a perturbation analysis, effectively expanding the distribution function as a power series in the Knudsen parameter $K = \lambda/L$, where λ is the electron mean free path and L is the scale height associated with inhomogeneity in the plasma. Since K is an increasing function of electron velocity the SH solution always breaks down at high electron velocity. For weak electric fields and temperature gradients this breakdown does not significantly compromise the ability of the SH solution to describe the transport properties of the plasma. However, as Gray and Kilkenny(1980) have shown, this is no longer the case when K reaches values of ≥ 0.02. Then the heat flux receives a substantial contribution from electrons which are collisionless, and whose behaviour is poorly described by the SH theory.

A more accurate description of the high velocity particles is provided by the high velocity form of the Landau equation (HVL). We have constructed an approximate solution to the Fokker-Planck equation by solving the HVL equation subject to the condition that it match the SH solution at the lower velocity boundary. This technique has been applied to determine the distribution function within a model coronal loop which is representative of active region and flaring loops. From this we are able to calculate the heat flux carried by the plasma and compare it with the standard heat flux estimates used in coronal modelling work.

2. Model

Our calculation, described in more detail in Ljepojevic and MacNeice(1989), can be summarised as follows :

(1) The plasma is assumed to be fully ionized hydrogen.

(2) We assume the electron distribution is in steady-state.

(3) We adopt the Spitzer and Härm(1953) solution

$$f = f_0\Big(1 - \lambda_0\Big[\frac{ZD_E}{A}\big(\frac{eE}{kT_e} + \frac{1}{p_e}\frac{\partial p_e}{\partial z}\big) - 2\frac{ZD_T}{B}\frac{1}{T_e}\frac{\partial T_e}{\partial z}\Big]\mu\Big)$$

for $\xi = v/v_{th} \leq 2$, where ZD_E/A and ZD_T/B are tabulated in SH, p_e is the electron pressure, T_e is the electron temperature, E the electric field, A and B represent a normalised electric field and temperature gradient respectively,λ_0 is the mean free path of a thermal electron, v_{th} is the electron thermal velocity, and $\mu = \mathbf{v}\cdot\hat{\mathbf{z}}/v$ is the cosine of the electron pitch angle.

(4) For $\xi > 2$ we solve the high velocity form of the Landau equation (cf. Gurevich and Istomin 1979, Shoub 1983, Ljepojevic 1988)

$$v\mu\frac{\partial f}{\partial z} + \frac{e\mathrm{E}}{m_e}\cdot\frac{\partial f}{\partial \mathbf{v}} = \frac{1}{v^2}\frac{\partial}{\partial v}\Big[v^2\nu(v)\big(\frac{kT}{m_e}\frac{\partial f}{\partial v} + vf\big)\Big] - \nu(v)\frac{\partial}{\partial\mu}\Big[(1-\mu^2)\frac{\partial f}{\partial\mu}\Big]$$

where

$$\nu(v) = \frac{4\pi e^4 n\ \ln\Lambda}{m_e^2 v^3}$$

with n being the density characterising the bulk of the electron distribution at height z and $\ln\Lambda$ the Coulomb logarithm. We require this solution to match the SH solution at $\xi = 2$.

(5) We adopt the temperature and density (T, n) characterising the bulk of the electron distribution from the loop model of MacNeice(1986), shown in figure 1, which is symmetric about the apex.

(6) The magnetic and gravitational forces are considered only in so far as they have already influenced the bulk distribution parameters $T(z)$ and $n(z)$.

(7) The only electric field which is considered is the self-consistent polarisation electric field (Spitzer and Härm 1953)

$$E_z = -.703\frac{k}{e}\frac{\partial T}{\partial z}.$$

required to maintain zero electric current in the presence of a temperature gradient.

3. Results

Typical results for $s(\mu, \xi)$ are displayed in figure 2 where

$$f/f_0 = exp(-s\xi^2)/exp(-\xi^2)$$

with f_0 a Maxwellian distribution. Note that $s = 1$ indicates a Maxwellian value, $s > 1$ indicates underpopulation in comparison with a Maxwellian and $s < 1$ overpopulation.

The SH solution shows that at low velocities the distribution is overpopulated for particles moving up the temperature gradient ($\mu > 0$) and underpopulated for particles moving down the gradient. This is the return current established by the polarisation electric field to give zero nett current.

It is significant that, despite the fact that the matching of the solutions involved only continuity in f and not in it's higher derivatives the slope of s across $\xi = 2$ also appears fairly continuous.

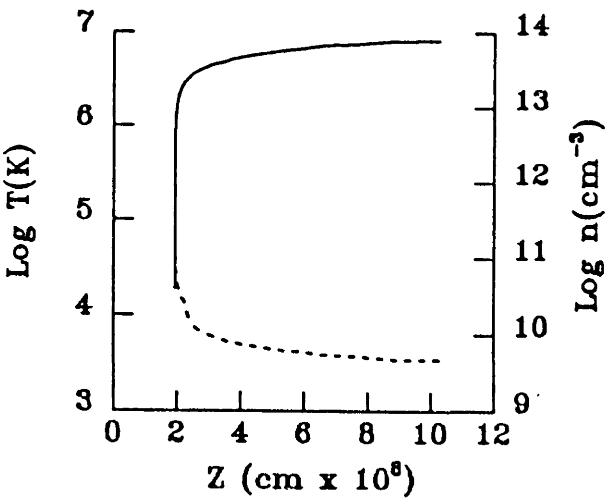

Figure 1 The electron temperature(solid line) and density profiles in the adopted loop model. The loop is assumed symmetric about its apex at $z = 1.2 \times 10^9$cm and so only one half is shown.

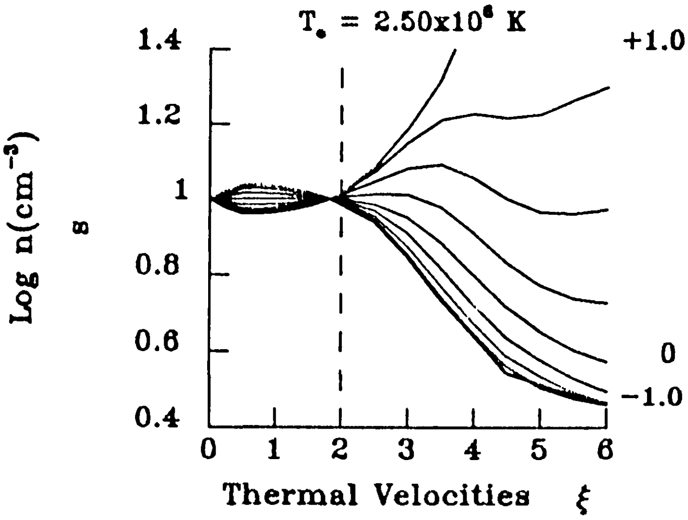

Figure 2 Variation of the non-Maxwellian function s with velocity and pitch angle at one location within the loop model. The curves represent $\mu = +1.0, .924, .707, .383, 0, -.383, -.707, -.924$ and -1.0. The vertical dashed line shows the velocity boundary at which the Spitzer-Härm and HVL solutions are matched.

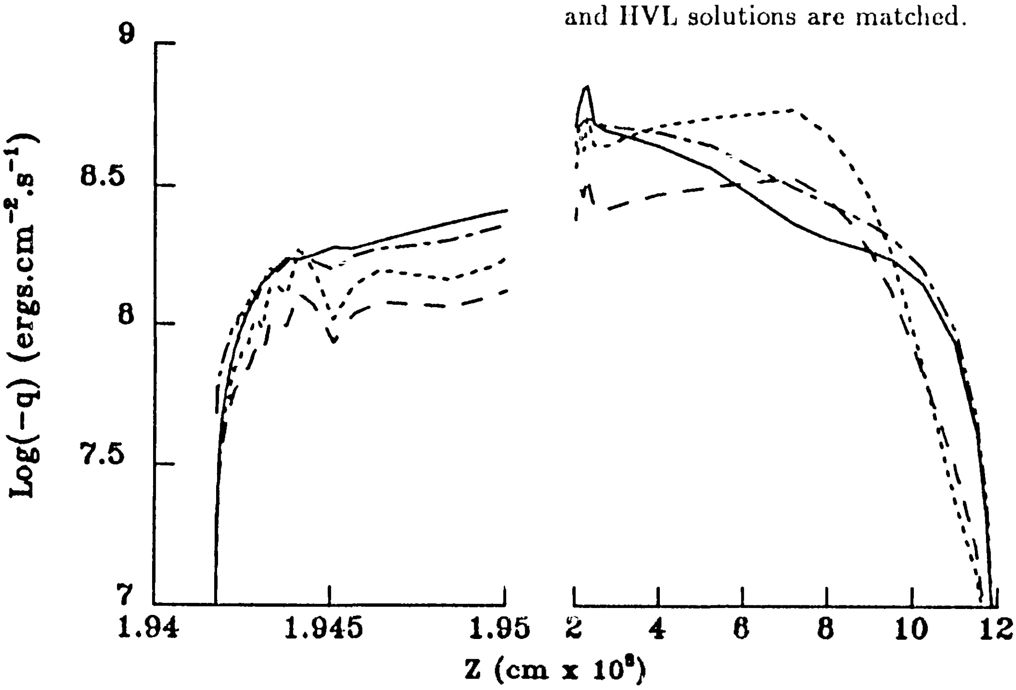

Figure 3 Heat flux as a function of position along the loop axis. The left frame illustrates the variation through the transition region while the right frame (which has the z axis drawn to much coarser scale) illustrates the upper transition region and coronal variation. The solid line represents the heat flux q_{pr} from the present calculation, the dotted line shows the Spitzer and Härm(1953) heat flux q_{sh}, the dashed line shows q_c following Campbell(1984), and the dot-dash line is q_L from the approximation of Luciani et al(1985) with $\alpha = 31$.

https://doi.org/10.1017/S0252921100154363 Published online by Cambridge University Press

This suggest that adopting an iterative scheme which refines the SH and HVL solutions to achieve complete consistency is not necessary in this case.

The heat fluxes from the present calculation (hereafter designated q_{pr}) are compared in figure 3 with those calculated by the methods of Spitzer-Härm(1953), Campbell(1984) and Luciani et al(1985) -- hereafter referred to as q_{sh}, q_c and q_L respectively.

In the high density lower transition region ($T \leq 2 \times 10^5$K) q_{pr} and q_{sh} are in excellent agreement. However at higher temperatures they differ considerably both in magnitude and in spatial variation. In the lower corona ($3 \times 10^8 \leq z \leq 7 \times 10^8$cm) they have different spatial gradients indicating that at these heights in our loop model the present calculation predicts conductive cooling while the Spitzer-Härm distribution predicts conductive heating.

Campbell(1984) assumes an essentially isotropic high energy tail thus allowing separation of μ and ξ dependences in f. His results should be expected to be appropriate only in regions of large K, which in this case means the coronal section of the loop. As shown in figure 3, this approximation gives very poor agreement with the present calculation.

Luciani et al(1983, 1985) suggested the following form for heat flux :

$$q_L(x) = \int_{-\infty}^{\infty} q_{sh}(z)\ w(x,z)\ dz$$

with

$$w(x,z) = \frac{1}{2\lambda(z)} exp\Big(-\Big|\int_z^x \frac{n(y)}{n(z)\lambda(z)}\ dy\Big|\Big)$$

and

$$\lambda = \alpha\lambda_0.$$

This heat flux (with $\alpha = 31$) is also plotted in figure 3. Although it has a consistently higher value than q_{pr}, it does have essentially the same spatial variation for $T \geq 3 \times 10^5$K. However at lower temperatures ($T < 2 \times 10^5$K) it gives a considerably larger heat flux than does our hybrid solution. The adjustable parameter α controls the effective mean free path. Reducing α makes the system more collisional and results in closer agreement between q_L and q_{sh} and therefore between q_L and q_{pr} at lower temperatures. Experimentation shows that smaller α leads to a better agreement between q_L and q_{pr} in the lower transition region. However this improvement occurs at the expense of the agreement at higher temperatures.

References

Braginskii,S.I.: 1965, "Reviews of Plasma Physics", Vol.1, p.205

Campbell,P.M.: 1984, Phys. Rev. A, **30**, 365

Gray,D.R., and Kilkenny,J.D. : 1980, Plasma Phys., **22**, 81

Gurevich, A.V. and Istomin, Ya.N. : 1979, JETP, **50(3)**, 470

Kopp,R.A., Fisher,G.H., MacNeice,P., McWhirter,R.W.P., and Peres,G.: 1986, Chapter 7 in "Energetic Phenomena on the Sun", eds. Kundu,M. and Woodgate,B., NASA Conf. Publ. 2439.

Ljepojevic, N.N. : 1988, submitted to Phil. Trans. Royal. Soc.

Ljepojevic,N.N. and MacNeice,P.: 1989, in preparation

Luciani,J.F., Mora,P. and Pellat,R.: 1985, Phys. Fluids, **28**, 835

MacNeice, P. : 1986, Solar Phys., **103**, 47

Shoub, E.C. : 1983, Astrophys. J., **266**, 339

Spitzer, L. and Härm, R.: 1953, Phys. Rev., **89**, 977

https://doi.org/10.1017/S0252921100154363 Published online by Cambridge University Press