*Bull. London Math. Soc.* 36 (2004) 271–280 
DOI: 10.1112/S0024609303002765

# OBITUARY

## ARTHUR GEOFFREY WALKER 1909–2001

### 1. *Life*

Arthur Geoffrey Walker (he used the given name Geoffrey) was born in Watford, Hertfordshire, on 17 July 1909. He attended Watford Grammar School, and from there in 1928 he won an Open Mathematics Scholarship to Balliol College Oxford. In 1930 he won a Junior Mathematical Exhibition, and in 1931 he graduated with first-class honours and distinction in differential geometry. He said later that it was reading L. P. Eisenhart's *Riemannian geometry*, published in 1926, that was the turning point in his mathematical life. From 1931 to 1933 he studied with Edmund Whittaker (who was knighted in 1945) in Edinburgh; Walker took a doctorate there in 1933, his external examiner being Sir Arthur Eddington. He then returned to Oxford, and worked with E. A. Milne from 1933 to 1935. He was awarded the Harmsworth Scholarship at Merton College in 1932, and the University's Senior Mathematical Scholarship in 1934, the same year that he was elected a Fellow of the Royal Astronomical Society. Walker worked closely with Milne, and was for a time an enthusiastic, but also critical, follower of Milne's 'Kinematic relativity' (see, for example, [**6**, **9**, **19**, **23**, **43**, **50**, **55**] and Section 2 below). Milne and Walker exchanged extensive correspondence: there are over 500 letters from Milne to Walker preserved in Balliol College.

The climate of the 1930s made it hard even for very able people to begin an academic career. Walker was a 'Demonstrator' (temporary lecturer) at Imperial College, London – then known as the Royal College of Science – from 1935 to 1936, when he was appointed Assistant Lecturer at the University of Liverpool, being promoted to Lecturer Grade IIb in 1937, and Grade IIa in 1941. Apart from working in pure mathematics, he collaborated during this period with J. R. Daymond of the Department of Civil Engineering at Liverpool on the problem of outflow of tidal water from a tank receiving constant inflow, and then inflow disturbed by an isolated surge or periodic surges [**14**, **18**].

Walker remained on the teaching staff throughout the war, but during this time he was incapacitated by illness for more than a year. From 1945 he had the title 'Lecturer in Differential Geometry', and in April 1947 he moved to the University of Sheffield as Professor and Head of the Department of Mathematics. During this period, he was awarded a D.Sc. from Edinburgh in 1945, was elected to a Fellowship of the Royal Society of Edinburgh in 1946, and was awarded its Keith Medal and Prize in 1950. In 1947 he was the first recipient of the London Mathematical Society's Junior Berwick Prize (later renamed simply the 'Berwick Prize').

In 1952 Walker was appointed Professor of Pure Mathematics at the University of Liverpool and Head of the Department of Pure Mathematics, following the resignation of J. M. Whittaker, who moved to the University of Sheffield as Vice-Chancellor. It was decided by the selection committee, 'in view of the fact that full information about the field of candidates would be readily available', not to advertise the chair, but to consult external advisors (J. E. Littlewood, H. Davenport, E. C. Titchmarsh and M. H. A. Newman). Apart from these, W. V. D. Hodge and H. S. Ruse were among his referees. One of his referees wrote as follows.

> He has the two qualities which I regard as most important for a successful career in mathematics, namely, a good technique and the vision necessary to see what lines of research it is important to follow. His particular field is differential geometry and relativity. This went through a rather sterile period between about 1930 and 1945, but Walker managed to produce a substantial series of papers during this period containing many valuable contributions to the subject, for which he was very properly awarded the Berwick Prize for younger mathematicians of the London Mathematical Society. The subject has in the last few years taken immense strides forward, and is becoming one of the most important in modern mathematics, and Walker is one of the foremost in this country in contributing to it.

The 'strides' referred to are the move from local to global methods; see Section 4 below. Walker was elected a Fellow of the Royal Society in 1955, and served on its Council from 1961 to 1962.

Walker continued his long association with the London Mathematical Society (having been elected to the Society in 1932), becoming its President from 1963 to 1965, which included the Society's centenary year. (The Society under its present title was founded in 1865, but has its origins in the Spitalfields Mathematical Society which was founded in 1717.) During his time at Liverpool, Walker enjoyed travelling, and spent a whole academic year 1959–1960 in Seattle visiting C. B. Allendoerfer, as well as six months in 1966 at the University of California at Berkeley. The book

*Harmonic spaces*, written with H. S. Ruse and T. J. Willmore [**67**], originated in lectures Walker gave in Rome, and *Introduction to geometrical cosmology* [**71**] is based on lectures given in the University of Arizona at Tucson. In addition to this, he developed a close relationship with the Chinese University of Hong Kong: he was External Examiner there, and also helped to attract a number of postgraduate students to Liverpool.

At his departmental retirement dinner in 1974, attended by about fifty staff and partners, Walker stated that when he moved to the Liverpool department in 1952, the total complement – including secretaries – was five. Walker was largely responsible for turning this handful into one of the most successful departments of Pure Mathematics in the country. At the time when one member of staff was appointed in 1959, the Department was 'at the top of the list to which J. H. C. Whitehead said I should apply.' A particularly significant appointment was that of Terry Wall in 1965 to a second chair of Pure Mathematics. This did much to enhance the department's reputation, and Wall in fact remained at Liverpool until his own retirement in 1999. The roll-call of staff appointed during Walker's tenure at Liverpool, who were subsequently appointed to Chairs elsewhere, is impressive: it includes Gavin Brown (Sydney), Ronald Brown (Bangor), Tom Flett (Sheffield), Geoffrey Horrocks (Newcastle), John McCutcheon (Heriot–Watt), William Moran (Adelaide), Bob Odoni (Glasgow), Stewart Robertson (Southampton), Peter Scott (Ann Arbor), Rolph Schwarzenberger (Warwick), and Tom Willmore (Durham). In addition, Andrew Casson (Yale) was on the teaching staff as a Research Fellow in the late 1960s.

Walker was a highly respected and popular Head of Department (in those days a permanent appointment), who ruled by consultation and benevolent dictatorship. He was always kind and considerate, especially when the personal circumstances of members of staff impinged on their work, and – though sorely tried on occasion – was never known to lose his temper. He created a civilised atmosphere, where voices were not raised and problems were sorted out without fuss. He had a dislike of excessive paperwork, and in this he differed from his long-term colleague and Head of the Applied Mathematics Department, Professor Louis Rosenhead. Rosenhead was in the habit of sending messages to Walker, typed by his secretary. Walker would scribble his comments and return the message, after which the secretary was called upon to type the comments for passing back to Rosenhead. There is another side to this: while he was on leave in Seattle, Walker entrusted administrative matters to Tom Willmore, and wrote to him periodically. Willmore used to say that he needed to leave these letters by his bedside for at least a week in order to decipher them. Fortunately, the letters were always sent well ahead of the time needed for action! Walker served as Dean of the Faculty of Science from 1962 to 1965 and, being a very able administrator, served on most of the University's major committees. He had a happy gift for 'reading down the diagonal' as he put it. This meant that when presented with a massive document, he could extract the essential features in a very short time.

In 1939, Walker married Phyllis Freeman, daughter of Sterry B. Freeman, CBE. She had been a secretary in the Department of Mechanical Engineering at Liverpool, and during the second world war acted as his secretary in the Pure Mathematics Department. Phyllis and Geoffrey were accomplished ballroom dancers – Walker once surprised a friend by saying that he had won more prizes for dancing than for mathematics. Walker was also a fine table tennis player, and took pleasure in beating students less than half his age at Liverpool's Easter vacation reading parties. The Walkers were gracious hosts at their home in Thornton Hough, on the Wirral, where Geoffrey was a keen gardener. Members of the growing department were invited, together with wives or husbands, in groups of six or eight and on a regular basis. This hospitality applied to 'all ranks', from secretaries to professors. The couple displayed a genuine interest in the welfare and combined happiness of the department.

On his retirement in 1974, Walker became Emeritus Professor at Liverpool. He and Phyllis moved to Sussex, where they lived until his death on 31 March 2001 at the age of 91.

## 2. *Relativity*

A. G. Walker is remembered in the field of general relativistic cosmology (see, for example, ⟨**13**, p. 470⟩) for his role in formulating the four-dimensional spacetime metric for a universe consisting of a homogeneous and isotropic distribution of matter [**12**].

This model is a good representation of the universe on the very large scales for which inhomogeneities due to galaxies may be averaged over. The metric describing this spacetime was simultaneously proposed by the American H. P. Robertson of Princeton ⟨**8**, **9**, **10**⟩, and is today known as 'the Robertson–Walker metric', or sometimes as the 'Friedmann–Robertson–Walker (FRW) metric', as the same metric had been derived much earlier by Alexandre Friedmann in 1922 ⟨**3**, **4**⟩. (Robertson himself was killed in a car accident during a 'Space Age Astronomy Symposium' sponsored by the Douglas Aircraft Company in 1961.)

The Robertson–Walker metric is the most general spacetime metric with a spherically symmetric 3-space, and is expressed in terms of the invariant line element as:

$$ds^2 \equiv g_{\mu\nu}\, dx^\mu\, dx^\nu = dt^2 - R(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \right). \qquad (1)$$

Here, $\theta$ and $\phi$ are angular coordinates measured by an observer with world-line given by $r = 0$, and $k$ may take the values 1, 0 or $-1$ according to the curvature of 3-space. The factor $R(t)$ is known as the 'scale factor', and describes the expansion of the universe. Robertson and Walker derived the form of the metric purely on symmetry grounds, based on the 'cosmological principle' that the universe should look the same from every point, whereas Friedmann obtained it by solving the field equations of general relativity, given the homogeneous and isotropic matter distribution. Modern 'Big Bang' cosmology is based on solving the field equations of general relativity for $R(t)$, given the form of matter in the universe. The success of the Robertson–Walker metric is confirmed by calculation of the primordial densities of light nuclei ('Big Bang nucleosynthesis'), which agrees precisely with observations; see ⟨**5**, p. 87⟩.

In the 1930s, around the same time as he was deriving the Robertson–Walker metric, Walker became a supporter of E. A. Milne, who formulated an alternative to general relativity, called 'kinematic [or kinematical] relativity'. This theory was introduced in 1935, in a book reviewed by Walker in [**11**]; a later and more comprehensive text is Milne's 1948 book ⟨**6**⟩, reviewed by Walker in [**50**]. The theory was motivated by a desire to avoid the domination of physics by geometry in general relativity. Here is the way in which Walker describes the underlying principles in his (highly critical) review [**13**] of a book by G. C. McVittie.

> Considering a hydrodynamical system idealized as a set of fundamental particles, he [Milne] constructs a model universe satisfying certain physical principles which are generally acceptable. Care is taken to avoid introducing undefinable concepts and to avoid making any purely geometrical assumptions, and to this end it is laid down that all quantities should be observables, the observers, having only temporal experience, being attached to the fundamental particles. It is now possible to formulate the Cosmological Principle which states that all observers see exactly similar world-pictures. A consequence is that the observers are equivalent and can, by means of light signalling, choose similar clocks and adopt similar coordinate systems for describing distance events. It follows that there must be a group of transformations relating the coordinate system of one observer with that of any other observer, and it is this group which becomes the chief mechanism for further discussion and replaces the geometry of General Relativity. Many problems now reduce to finding scalar and vector invariants under the group of transformations, and it is at this stage that geometry can be employed with advantage. For it can be shown that a group of the kind required by kinematical theory always leaves invariant a quadratic differential form, which may be taken to define a Riemannian space, and invariants under the group are geometric invariants for this space.

A number of the articles from *The Observatory*, in particular the discussion in [**19**] between Milne, Walker and McVittie, and the book review by Dingle in ⟨**2**⟩, amount to a lively and sometimes bitter argument between Milne, his supporters and his critics. Kinematic relativity ran into many problems, and Milne sometimes defended it by claiming (with much sarcasm) that his opponents, in particular McVittie (who appears to have been a balanced and objective critic throughout), were unable to grasp the theory, or to accept new ideas.

Walker was much interested in the logical, indeed axiomatic, foundations of relativity. In [**12**] he took Milne's general principles, without assuming the special Lorentz transformations relating observers in three spatial dimensions, and obtained the general solution to the metric based on these and some other hypotheses. This solution included all systems of general relativity and Milne's system as special cases. Using Fermat's principle and a principle of symmetry – 'each fundamental observer sees himself to be at the centre of spherical symmetry', something that he regarded in [**23**] as more fundamental than Milne's cosmological principle – he derived the metric above. In [**46**], he divided theories into physical, mathematical and logical, and concluded that kinematic relativity is physical, but with a smaller basis than general relativity. This makes it easier to find a purely logical, axiomatic, basis for kinematic relativity. He found that there are six axioms needed to make the physical basis of kinematic relativity logically sound, and proved from them that temporal order (which might in principle be described by a much more complicated ordered set than the real numbers) is in fact defined by the real continuum. (Walker was led to a number of analytical investigations by kinematic relativity; see, for example, [**35**] on ordered sets.) Walker was well aware of the limitations of kinematic relativity, and attempted to overcome some of them. For example, in his paper [**55**] in *Nature*, he started by noting that Milne's theory predicts a perihelion motion of Mercury in the wrong direction, and no gravitational deflection of light. He went on to say that the theory lacked a principle of least action, but that this could be remedied by starting with a variational principle having Lorentz invariance. From this he was able to deduce that the orbit of a particle has advancing perihelion, and that light should be deflected towards the sun, exactly as in general relativity. In another paper [**33**] he shows that general relativity and kinematic relativity 'agree formally on the large'; that is, they make the same predictions when applied to cosmology. In this case kinematic relativity is equivalent to a Robertson–Walker metric with $R(t) \propto t$ and $k=-1$. Somewhat ironically, given that Milne was an opponent of general relativity, his name is now associated with this particular general relativistic solution, which is known in textbooks as the Milne model; see ⟨**7**, p. 116⟩.

Walker wrote a number of other papers, based on kinematical relativity, on the properties of 'extra-galactic nebulae', in particular their spiral form and orientation. We now know these to be other galaxies, a fact not appreciated at the time when Walker's papers were written. To his credit, in [**32**], Walker casts doubts on the Milne theory's ability to account for their spiral structure.

## 3. *Local differential geometry*

The theory of harmonic manifolds – that is to say, riemannian manifolds that admit at each point a locally radial solution of Laplace's equation – is a good example of a topic ideally suited to Walker's extraordinary ability to handle the complicated technical machinery of classical local differential geometry; he chose not to embrace the powerful Cartan calculus of exterior differential forms.

The theory of harmonic manifolds originated as follows. In euclidean space $\mathbf{R}^n$, a harmonic function (satisfying the Laplace equation $\Delta f=0$) that depends only on the distance from the origin is given by $f(x)=\|x\|^{2-n}$ if $n>2$ and $f(x)=\log\|x\|$ if $n=2$. In 1930, H. S. Ruse attempted to find on any riemannian manifold $(M,g)$ a local solution of the Laplace equation depending only on the geodesic distance of a point from an given basepoint in $M$. When this attempt failed, a riemannian manifold came to be called *completely harmonic* when solutions of this type exist. When the solution exists for only a single basepoint, the word 'completely' was dropped, and when there is a solution $f$ as above, Walker [**20**] used the phrase 'simply harmonic'. It is well known that in euclidean space a harmonic function has its mean value on every sphere equal to its value at the centre of the sphere. This property can be used to define harmonic riemannian manifolds when the metric is positive definite. In [**20**, **29**], Walker obtained a sequence of conditions on the curvature tensor of a harmonic manifold. In particular, it follows that such a manifold is an Einstein space. Lichnerowicz had conjectured that harmonic manifolds must be locally symmetric in the sense that the covariant derivative of the curvature tensor is zero ($R_{hijk,l}=0$). The curvature conditions enabled Lichnerowicz and Walker [**30**] to show that a completely harmonic manifold of any dimension $n$ and normal hyperbolic type (signature $n-2$) is of constant curvature, and that every simply harmonic manifold of this type is flat, thereby verifying the conjecture in these cases. Walker was interested also in properties of harmonic manifolds with an indefinite metric. Here the situation is radically different: he showed [**48**] that there exist, for dimension $n=4$, harmonic manifolds that are not isometric to a symmetric space.

There are several related investigations in Walker's work. In [**51**], he studied certain 'riemannian manifolds $K_n$ of recurrent curvature' (where $n$ is equal to the dimension), introduced by H. S. Ruse. Ruse suggested generalizing the symmetry condition to consider spaces for which the curvature tensor satisfied the condition $R_{hijk,l} = R_{hijk}\xi_l$ for some covector $\xi_l$. These are the $K_n$. Those actually studied by Ruse were called by Walker 'simple $K_n$ spaces': these are characterized by the property that they have $n-2$ independent parallel vector fields (and hence a trivialization of a subbundle of the tangent bundle with fibre dimension $n-2$). Walker also described another collection of 'non-simple' $K_n$, $n \geqslant 4$, that possess at least two orthogonal null parallel vector fields. Although Walker found a harmonic space that was recurrent but not symmetric, the example used an indefinite metric, and therefore provided no further counterexamples to the Lichnerowicz conjecture.

In [**62**], he considered almost-complex differentiable manifolds; that is, even-dimensional manifolds whose tangent spaces admit an endomorphism $J$ satisfying $J^2 + I = 0$, where $I$ is the identity. Walker constructed a method for associating with any tensor field of type $(p,q)$, a tensor field of type $(p,q+2)$, using only $J$ and its partial derivatives. So far, this 'torsional derivation' has received little attention in the literature.

## 4. *Parallel fields of planes*

While much research in differential geometry was expressed in terms of local coordinates, and was often concerned with purely local phenomena, there was growing interest in the years following the second world war in the relation between the geometry of a manifold and its topological structure in the large. Among the many notable results of this period is the theorem of de Rham ⟨**1**⟩. This concerns the structure of a complete connected riemannian $n$-manifold $M$, on which there is defined a field of tangent $k$-planes, where $0<k<n$, that is *parallel* with respect to the riemannian connection of $M$. Such a field of planes $\phi$ is integrable, and so forms a *parallel foliation* $\mathcal{F}$ of $M$, whose leaves are $k$-dimensional submanifolds. It follows also that the complementary field $\phi^{\perp}$ of tangent $(n-k)$-planes orthogonal to $\phi$ is also parallel, and so there is a second foliation $\mathcal{F}^{\perp}$ by $(n-k)$-dimensional leaves orthogonal to $\mathcal{F}$. What are the implications for the global topology of $M$? De Rham answered this question by showing that if $M$ is simply connected, then it is isometric to the orthogonal product $F \times F^{\perp}$ of any two leaves $F \in \mathcal{F}, F^{\perp} \in \mathcal{F}^{\perp}$. The theorem of de Rham appeared in 1952. At this time, Walker [**54**, **57**] was studying the same problem from the point of view of fibre bundles, motivated by an earlier attempt by T. Y. Thomas ⟨**12**⟩ to obtain a global 'product theorem'. Walker considered the case where the leaves of $\mathcal{F}$ fibre $M$, giving a definitive description of the structure of such bundles. The two pieces of work are closely related, although de Rham's formulation tackles the general case explicitly. Walker also discussed the fibring of $M$ in the sense of the recently introduced concept of fibre bundles.

Walker was interested too in the above problem not just for riemannian manifolds, but for pseudo-riemannian or semi-riemannian manifolds, where the symmetric bilinear form at each point has signature $(j, n-j)$, for some $j$ with $0<j<n$. In this case, the situation is potentially more complicated, since the relation between the $k$-plane $\phi(x)$ and the null-cone of the form in $T_xM$ affects the way in which the orthogonal field $\phi^{\perp}$ lies across $\phi$. Thus, if the intersection of $\phi(x)$ with the tangent plane to the null cone in $T_xM$ has dimension $q>0$ (the *nullity* of $\phi$), then $\phi(x) \cap \phi^{\perp}(x)$ has dimension $q$, and $\phi(x) + \phi^{\perp}(x)$ is a proper subspace of $T_xM$. Walker recognised that a full understanding of the global structure of $M$ in the general

https://doi.org/10.1112/S0024609303002765 Published online by Cambridge University Press

case was out of reach, and instead concentrated on devising coordinate systems in which the metric tensor took a simple *canonical form*, yielding information on the pseudogroup of coordinate transformations for the structure, and hence some insight into global questions. In [**47**] he began to explore the new features of the semi-riemannian case. In [**52**] Walker studied parallel fields of null planes ($q = k$), and obtained a local canonical form for the metric.

The case $n = 2q = 2k$ is of special interest, and is the subject of [**56**]. In 1964, Wu ⟨**14**⟩ considered the case $q = 0$ at the other end of the range, and succeeded in showing that de Rham's theorem still holds, with the obvious modifications to the statement. In two later papers [**60**, **63**] there are global results on the existence of affine connections with respect to which one or more given distributions are parallel. Walker's work remains of great interest as a starting-point for the study of the general case. For some pointers to the way in which Walker's results can be exploited to obtain further information of a global character, see ⟨**11**⟩ and its references.

*Acknowledgements.* I thank the following for their invaluable help in the writing of this obituary notice. John McDonald, Stewart Robertson and Tom Willmore largely wrote the more technical sections; besides these, Ronnie Brown, Herbert Burchnall, Michael Butler, Jim Message, Ann Newstead and Terry Wall helped with the details of Walker's life, and Nigel Hitchin provided details missing from the bibliography, which was compiled from papers held at the University of Liverpool, together with MathSciNet. (The references '*MR*' in the bibliography are to *Mathematical Reviews*.)

## *References*

⟨**1**⟩ G. de Rham, 'Sur la réductibilité d'un espace de riemann', *Comment. Math. Helv.* 26 (1952) 328–344; *MR* 14,484a.
⟨**2**⟩ H. Dingle, review of *Time, knowledge and the nebulae* by Martin Johnson (Faber and Faber, London, 1945), in *The Observatory* 66 (1945) 187–190.
⟨**3**⟩ A. Friedmann, 'Über die Krummung des Raumes', *Z. Phys.* 10 (1922) 377–386.
⟨**4**⟩ A. Friedmann, 'Über die Möglichkeit einer Welt mit konstanter negativer Krummung des Raumes', *Z. Phys.* 21 (1924) 326–332.
⟨**5**⟩ E. W. Kolb and M. S. Turner, *The early universe* (Addison-Wesley Publishing Co., 1990).
⟨**6**⟩ E. A. Milne, *Kinematic relativity* (Oxford University Press, 1948); *MR* 10,578d.
⟨**7**⟩ J. V. Narlikar, *Introduction to cosmology* (Cambridge University Press, 1993).
⟨**8**⟩ H. P. Robertson, 'Kinematics and world structure, I', *Astrophys. J.* 82 (1935) 284–301.
⟨**9**⟩ H. P. Robertson, 'Kinematics and world structure, II', *Astrophys. J.* 83 (1936) 187–201.
⟨**10**⟩ H. P. Robertson, 'Kinematics and world structure, III', *Astrophys. J.* 83 (1936) 257–271.
⟨**11**⟩ S. A. Robertson and P. M. D. Furness, 'Parallel framings and foliations on pseudo-riemannian manifolds', *J. Diff. Geom.* 9 (1974) 409–422; *MR* 50 #14787.
⟨**12**⟩ T. Y. Thomas, 'The decomposition of Riemannian spaces in the large', *Monatsh. Math. Phys.* 47 (1939) 388–418; *MR* 1,271f.
⟨**13**⟩ S. Weinberg, *Gravitation and cosmology* (John Wiley and Sons Inc., 1972).
⟨**14**⟩ H. Wu, 'On the de Rham decomposition theorem', *Illinois J. Math.* 8 (1964) 291–311; *MR* 28 #4488.

## *Works of Arthur Geoffrey Walker*

**1.** 'The second curvature of a sub-space', *Quart. J. Math. Oxford* 3 (1932) 291–298.
**2.** 'Relative coordinates', *Proc. Royal Soc. Edinburgh* 52 (1932) 345–353.
**3.** 'On small deformation of sub-spaces of a flat space', *Proc. Edinburgh Math. Soc.* 3 (1932) 77–86.
**4.** 'Spatial distance in general relativity', *Quart. J. Math. Oxford* 4 (1933) 71–80.

https://doi.org/10.1112/S0024609303002765 Published online by Cambridge University Press

**5.** 'Distance in an expanding universe', *Monthly Notices of the Royal Astronomical Society* 94 (1933) 159–167.
**6.** 'The principle of least action in Milne's kinematic relativity', *Proc. Royal Soc. A* 147 (1934) 478–490.
**7.** 'The Boltzmann equations in general relativity', *Proc. Edinburgh Math. Soc* 4 (1934–6) 238–253.
**8.** 'On Riemannian spaces with spherical symmetry about a line, and the conditions for isotropy in general relativity', *Quart. J. Math. Oxford* 6 (1935) 81–93.
**9.** 'On the formal comparison of Milne's kinematic system with the systems of general relativity', *Monthly Notices of the Royal Astronomical Society* 95 (1935) 263–269.
**10.** 'Note on relativistic mechanics', *Proc. Edinburgh Math. Soc* 4 (1936) 170–174.
**11.** 'Relativity, gravitation and world structure', essay review of book by E. A. Milne, *Oxford Magazine* 54 (1936) 485–487.
**12.** 'On Milne's theory of world structure', *Proc. London Math. Soc* 42 (1937) 90–127.
**13.** 'Cosmological theory', review of a book by G. C. McVittie (Methuen, 1937), *The Observatory* 61 (1938) 190–195.
**14.** (with J. R. Daymond), 'On a hydraulic problem involving discharge into tidal water I', *Philos. Magazine* 28 (1939) 520–531.
**15.** 'Relativistic mechanics', *Proc. London Math. Soc* 46 (1940) 113–154.
**16.** 'The orientation of the extra-galactic nebulae', *Monthly Notices of the Royal Astronomical Society* 100 (1940) 623–630.
**17.** 'Certain groups of motions in 3-space of constant curvature', *Quart. J. Math. Oxford* 11 (1940) 81–94; *MR* 2,6e.
**18.** (with J. R. Daymond), 'On a hydraulic problem involving discharge into tidal water II', *Philos. Magazine* 31 (1941) 222–230.
**19.** 'Kinematical relativity; a discussion with E. A. Milne and G. C. McVittie', *The Observatory* 64 (1941) 17–23. [The whole discussion is pp. 11–25.]
**20.** 'Note on a distance invariant and the calculation of Ruse's invariant', *Proc. Edinburgh Math. Soc.* (2) 7 (1942) 16–26; *MR* 4,171g.
**21.** 'The inextensible string', *Edinburgh Math. Notes* 33 (1943) 13–17.
**22.** 'Relativistic mechanics III: Mechanics in 4-space, and conservative fields of force', *Proc. London Math. Soc.* (2) 48 (1943) 161–179; *MR* 5,218c.
**23.** 'Foundations of kinematical relativity' (letter to the Editors), *The Observatory* 65 (1944) 242–243.
**24.** 'Note on integral functions', *J. London Math Soc.* 19 (1944) 106–107; *MR* 6,263a.
**25.** 'Completely symmetric spaces', *J. London Math. Soc.* 19 (1944) 219–226; *MR* 7,265c.
**26.** 'Complete symmetry in flat space', *J. London Math. Soc.* 19 (1944) 227–229; *MR* 7,265d.
**27.** 'Note on pseudo-harmonic functions in space of constant curvature', *J. London Math. Soc.* 20 (1945) 32–39; *MR* 7,448e.
**28.** 'A particular harmonic Riemannian space', *J. London Math. Soc.* 20 (1945) 93–99; *MR* 7,529i.
**29.** 'On completely harmonic spaces', *J. London Math. Soc.* 20 (1945) 159–163; *MR* 7,529j.
**30.** (with A. Lichnerowicz), 'Sur les espaces riemanniens harmoniques de type hyperbolique normal', *C. R. Acad. Sci. Paris* 221 (1945) 394–396; *MR* 7,395c.
**31.** 'A model of a hyperboloid of one sheet and its asymptotic cone', *Edinburgh Math. Notes* 35 (1945) 20–23; *MR* 7,164h.
**32.** 'The spiral form of extra-galactic nebulae', *The Observatory* 66 (1946) 215–217.
**33.** 'Theoretical cosmology', *The Observatory* 66 (1946) 285–288.
**34.** 'Time-scales in relativity', *Proc. Roy. Soc. Edinburgh A* 62 (1946) 221–228; *MR* 8,411f.
**35.** 'A theorem on ordered sets', *J. London Math. Soc.* 21 (1946) 9–10; *MR* 8,255a.
**36.** 'Symmetric harmonic spaces', *J. London Math. Soc.* 21 (1946) 47–57; *MR* 8,351b.
**37.** (with J. S. Batty), 'Non-integral functional powers', *Quart. J. Math. Oxford* 17 (1946) 145–152; *MR* 8,199h.
**38.** 'Commutative functions I', *Quart. J. Math., Oxford* 17 (1946) 65–82; *MR* 8,19d.
**39.** 'Commutative functions II', *Quart. J. Math., Oxford* 17 (1946) 83–92; *MR* 8,19e.
**40.** 'A theory of regraduation in general relativity', *Proc. Roy. Soc. Edinburgh. Sect. A* 62 (1946) 164–174; *MR* 7,531h.
**41.** 'Note on pseudo-harmonic functions II', *J. London Math. Soc.* 22 (1947) 101–104; *MR* 9,352d.
**42.** 'Durées et instants', *Revue Sci.* 85 (1947) 131–134; *MR* 9,222d.
**43.** 'The invariants of kinematical relativity', *Philos. Mag.* (7) 38 (1947) 316–324; *MR* 9,213d.
**44.** (with F. W. Bradley), 'Existence theorems for non-uniform power-sets', *Quart. J. Math. Oxford* 18 (1947) 97–106; *MR* 9,82a.
**45.** 'Finite projective geometry', *Edinburgh Math. Notes* 36 (1947) 12–17; *MR* 8,596f.
**46.** 'Foundations of relativity I, II', *Proc. Royal Soc. Edinburgh* 62 (1948) 319–335.
**47.** 'On parallel fields of partially null vector spaces', *Quart. J. Math. Oxford* 20 (1949) 135–145; *MR* 11,460f.

https://doi.org/10.1112/S0024609303002765 Published online by Cambridge University Press

**48.** ‘On Lichnerowicz’s conjecture for harmonic 4-spaces’, *J. London Math. Soc.* 24 (1949) 21–28; *MR* 10,739b.
**49.** (with J. D. Weston), ‘Inclusion theorems for the eigenvalues of a normal matrix’, *J. London Math. Soc.* 24 (1949) 28–31; *MR* 10,501h.
**50.** ‘Kinematic relativity’, reviews of ⟨**6**⟩, *Math. Gazette* 33 (1949) 304–306 and *Oxford Magazine* 68 (1949) 82–83.
**51.** ‘On Ruse’s spaces of recurrent curvature’, *Proc. London Math. Soc.* (2) 52 (1950) 36–64; *MR* 12,283b.
**52.** ‘Canonical form for a Riemannian space with a parallel field of null planes’, *Quart. J. Math. Oxford* (2) 1 (1950) 69–79; *MR* 11,688d.
**53.** ‘Canonical forms II: Parallel partially null planes’, *Quart. J. Math. Oxford* (2) 1 (1950) 147–152; *MR* 12,205c.
**54.** ‘Sur la fibration des variétés riemanniennes’, *C. R. Acad. Sci. Paris* 232 (1951) 1465–1467.
**55.** ‘Gravitation in kinematic relativity’, *Nature* 168 (1951) 961–962.
**56.** (with E. M. Patterson), ‘Riemann extensions’, *Quart. J. Math. Oxford* (2) 3 (1952) 19–28; *MR* 13,985g.
**57.** ‘The fibring of Riemannian manifolds’, *Proc. London Math. Soc.* (3) 3 (1953) 1–19; *MR* 15,159g.
**58.** ‘Riemann extensions of non-Riemannian spaces’, *Convegno Internazionale di Geometria Differenziale, Italia, 1953* (Edizioni Cremonese, Roma, 1954) 64–70; *MR* 16,169b.
**59.** (with S.-T. Tsou), ‘Metrisable Lie groups and algebras’, *Proc. Roy. Soc. Edinburgh.* Sect. A 64 (1955/1957) 290–304; *MR* 20 #5821.
**60.** ‘Connexions for parallel distributions in the large’, *Quart. J. Math. Oxford* (2) 6 (1955) 301–308; *MR* 19,312e.
**61.** (with W. F. Newns), ‘Tangent planes to a differentiable manifold’, *J. London Math. Soc.* 31 (1956) 400–407; *MR* 18,821a.
**62.** ‘Dérivation torsionnelle et seconde torsion pour une structure presque complexe’, *C. R. Acad. Sci. Paris* 245 (1957) 1213–1215; *MR* 19,680h.
**63.** ‘Connexions for parallel distributions in the large. II’, *Quart. J. Math. Oxford* (2) 9 (1958) 221–231; *MR* 20 #6135.
**64.** ‘Distributions and global connexions’, *Colloque Geom. Diff. Globale* (*Bruxelles, 1958*) (Centre Belge Rech. Math., Louvain, 1959) 63–74; *MR* 23 #A590.
**65.** ‘Axioms for cosmology. The axiomatic method. With special reference to geometry and physics’, *Proceedings of an International Symposium held at the Univ. of Calif., Berkeley, Dec. 26, 1957 – Jan. 4, 1958*, Stud. Logic Found. Math. (ed. L. Henkin, P. Suppes and A. Tarski, North-Holland Publishing Co., Amsterdam, 1959) 308–321 ; *MR* 21 #6270.
**66.** ‘A particular class of symmetric spaces’, *Ann. Mat. Pura Appl.* (4) 55 (1961) 23–35; *MR* 24 #A3609.
**67.** (with H. S. Ruse and T. J. Willmore), *Harmonic spaces* (Consiglio Nazionale delle Ricerche Monografie Matematiche, 8 Edizioni Cremonese, Rome, 1961); *MR* 25 #5456.
**68.** ‘Almost-product structures’, *Proc. Sympos. Pure Math.*, Vol. III (Amer. Math. Soc., Providence, RI, 1961) 94–100; *MR* 23 #A1314.
**69.** ‘Note on metrisable Lie groups and algebras’, *1963 Calcutta Math. Soc. Golden Jubilee Commemoration Vol.* (*1958/59*), *Part I* (Calcutta Math. Soc., Calcutta, 1963) 185–192; *MR* 29#567.
**70.** ‘A certain conformal structure’, *Géométrie différentielle* (*Colloq., Univ. Santiago de Compostela, Santiago de Compostela, 1972*) Lecture Notes in Math. 392 (Springer, Berlin, 1974) 98–110; *MR* 51 #1687.
**71.** *Introduction to geometrical cosmology*, Department of Mathematics, University of Arizona, Tucson, Ariz., Lecture Notes (Department of Mathematics, University of Arizona, Tucson, Ariz., 1975); *MR* 54 #4548.
**72.** ‘Note on locally symmetric vector fields in a Riemannian space’, *Topics in differential geometry* (*in memory of Evan Tom Davies*) (Academic Press, New York, 1976); *MR* 53 #9066.

*Department of Mathematical Sciences*
*The University of Liverpool*
*Liverpool L69 3BX*

Peter Giblin