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Initial Test of a Bayesian Approach to Solar Flare Prediction

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Abstract: A test of a new Bayesian approach to solar flare prediction is presented. The approach uses the past history of flaring together with phenomenological rules of flare statistics to make a prediction for the probability of occurrence of a large flare within an interval of time, or to refine an initial prediction (which may incorporate other information). The test of the method is based on data from the Geostationary Observational Environmental Satellites, and involves whole-Sun prediction of soft X-ray flares for 1976–2003. The results show that the method somewhat over-predicts the probability of all events above a moderate size, but performs well in predicting large events.

Keywords: methods: statistical — Sun: activity — Sun: flares — Sun: X-rays, gamma rays

1 Introduction

The space weather effects of large solar flares motivate flare prediction. For example, the soft X-ray flux due to large flares causes increased ionisation of the upper atmosphere, which can interfere with high frequency radio communication. There is considerable interest in knowing when such short-wave fadeouts are likely to occur, and Australia's Ionospheric Prediction Service (IPS) issues predictions on this basis.[1] Other agencies issuing flare predictions include the US National Oceanic and Atmospheric Administration (NOAA)[2] and NASA.[3]

Existing methods of prediction are probabilistic, and rely, for example, on the classification of physical characteristics of active regions and historical rates of flaring for regions with a given classification (McIntosh 1990; Bornmann & Shaw 1994). One weakness of classification-based approaches is that regions with a given classification may exhibit a wide range of flaring rates. The method of McIntosh (1990) also considers other information including the number of large flares already produced by an active region (the tendency of an active region which has produced large events to subsequently produce large events is called persistence), but this is done in an ad hoc way. No consideration is given to the important information in the number of small events already observed.

A new approach to flare prediction (Wheatland 2004) exploits the history of observed flaring together with simple phenomenological rules of flare statistics to make a prediction, or to refine an existing prediction. The basic method is as follows. It is well known that the size distribution of flares (e.g. the distribution of peak soft X-ray flux) follows a power law (e.g. Crosby, Aschwanden, & Dennis 1993):

$$N(S) = \lambda_1(\gamma - 1)S_1^{\gamma-1}S^{-\gamma}, \qquad (1)$$

where $N(S)$ is the number of events per unit size S and per unit time, λ_1 is the total rate of events above size S_1, and γ is the power-law index. Suppose we are interested in the probability of a large event ($S \geq S_2$) occurring in a time ΔT. The expected rate of events above S_2 is, according to equation (1),

$$\lambda_2 = \lambda_1 \left(\frac{S_1}{S_2}\right)^{\gamma-1}. \qquad (2)$$

Flare occurrence may be described on short timescales as a Poisson process in time (e.g. Moon et al. 2001) and on longer timescales as a time-dependent Poisson process (e.g. Wheatland 2001). According to Poisson statistics, the probability of at least one large event in time ΔT is

$$\epsilon = 1 - \exp(-\lambda_2 \Delta T). \qquad (3)$$

To apply these formulae it is necessary to estimate λ_1 and γ from data, and hence to estimate ϵ. We adopt a Bayesian approach, in which 'estimating' a parameter means calculating a posterior probability distribution for the parameter, given available data and any prior information (e.g. Jaynes 2003). We assume that a sequence of M events with sizes $s_1, s_2, \ldots, s_M$ (all larger than S_1) have been observed to occur at times $t_1 < t_2 < \cdots < t_M$ respectively. The power-law index γ may be approximated by the maximum likelihood value (Bai 1993)

$$\gamma^* = \frac{M}{\ln \pi} + 1, \quad \text{where} \quad \pi = \prod_{i=1}^{M} \frac{s_i}{S_1}. \qquad (4)$$

To estimate the rate we adopt a piecewise constant Poisson model. Hence we need to identify the most recent interval

[1] www.ips.gov.au
[2] www.sec.noaa.gov/ftpdir/latest/daypre.txt
[3] beauty.nascom.nasa.gov/arm/latest/

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T' during which the rate is constant, and we assume that $M' \leq M$ events occurred during that time. One approach to the determination of the interval T' is to use the 'Bayesian blocks' procedure (Scargle 1998), which is discussed in more detail below. Based on M', T', and γ^*, the posterior probability distribution for ϵ is (Wheatland 2004)

$$P(\epsilon) = C[-\ln(1-\epsilon)]^{M'}(1-\epsilon)^{(T'/\Delta T)(S_2/S_1)^{\gamma^*-1}-1} \times \Lambda\left[-\frac{\ln(1-\epsilon)}{\Delta T}\left(\frac{S_2}{S_1}\right)^{\gamma^*-1}\right], \quad (5)$$

where $\Lambda(\lambda_1)$ is the prior distribution for λ_1, i.e. the distribution we would assign to λ_1 in the absence of any data, and C is the normalisation constant, determined by the requirement $\int_0^1 P(\epsilon)\mathrm{d}\epsilon = 1$. The mean of $P(\epsilon)$ provides the estimate of the probability of at least one large event within time ΔT, and the standard deviation of the distribution provides an estimate of the associated uncertainty.

2 The Test

As a basic test of the new approach to prediction, equation (5) was applied to the NOAA Solar Event Lists of X-ray flares observed by the Geostationary Observational Environmental Satellites (GOES) for 1975–2003.[4] For each day, whole-Sun flare prediction was performed for the next day ($\Delta T = 1$ d).

The relevant measure of size, S, of the GOES events is the peak soft X-ray flux in the 1–8 Å band. The choice of threshold $S_1 = 4 \times 10^{-6}\,\mathrm{W\,m^{-2}}$ was based on inspection of the distribution of peak soft X-ray flux for the entire dataset. Figure 1 shows the distribution, plotted in differential form (upper panel) and cumulative form (lower panel). The threshold size S_1 is indicated by the vertical solid line. Events above this size are observed to be distributed approximately as a power law, and the thick solid lines in each panel indicate the power-law model, with the maximum likelihood power-law index $\gamma^* \approx 2.15 \pm 0.01$. For peak fluxes below the threshold there is departure from power-law behaviour due to problems with event selection against the time-varying soft X-ray background.

Predictions were made for each day for events above size $S_2 = 10^{-5}\,\mathrm{W\,m^{-2}}$ ('M class' and larger flares) and for events above size $S_2 = 10^{-4}\,\mathrm{W\,m^{-2}}$ ('X class' flares) using equation (5). The corresponding prediction probabilities for a given day are labelled ϵ_{M} and ϵ_{X} respectively. It should be noted that these values are not independent, since X class flares are a subset of events above M class.

The predictions used data within a window of time spanning one year prior to each day. For each day, equation (4) was applied to the one-year window of data prior to the day to determine γ^*. Then the Bayesian blocks procedure was applied to the same data to determine a decomposition into a piecewise-constant Poisson process. This procedure returns a sequence of times $t_{\mathrm{B0}} < t_{\mathrm{B1}} < \cdots < t_{\mathrm{BK}}$ at which the rate is determined to change (where t_{B0} and t_{BK} are the start- and end-time of the window), and a corresponding sequence $\lambda_{\mathrm{B1}}, \lambda_{\mathrm{B2}}, \ldots, \lambda_{\mathrm{BK}}$ of rates. The last Bayesian block was used to determine T' and M': namely $T' = t_{\mathrm{BK}} - t_{\mathrm{B(K-1)}}$ and $M' = \lambda_{\mathrm{BK}} T'$.

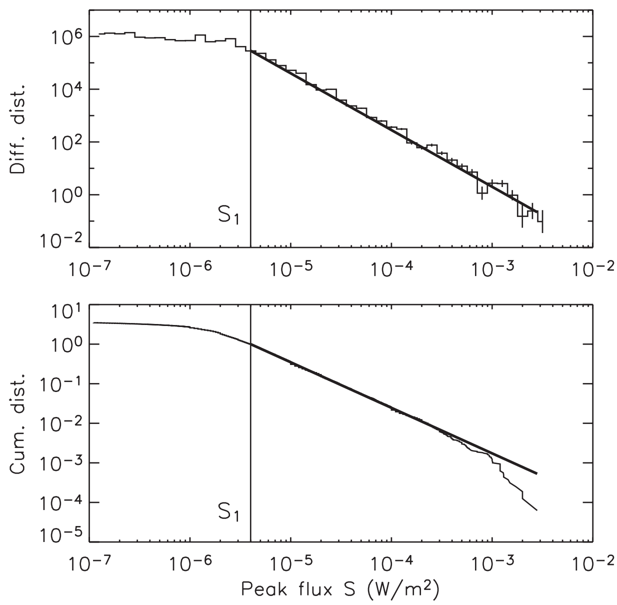

Figure 1 Upper panel: differential distribution of peak flux for GOES events 1976–2003 (histogram), and the power-law model distribution (thick line). Lower panel: cumulative distribution for all events (joined points), and the model (thick line). In both panels the threshold S_1 for power-law behaviour is shown by the vertical line.

The data in every Bayesian block but the last was used to construct the prior $\Lambda(\lambda_1)$. A model form $\Lambda(\lambda_1) = a\exp(-b\lambda_1^c)$ was chosen for the prior, and the parameters a, b, and c were determined by requiring that the first three moments of this model distribution match the first three moments of the data, estimated from the Bayesian blocks decomposition. Specifically we required that the model distribution was normalized, and that it had a mean rate and mean square rate equal to the corresponding estimates from the Bayesian blocks. These three conditions uniquely determined values of a, b, and c.

3 Results

Figure 2 illustrates the results of the test on a year by year basis for 1976–2003 (the results for 1975 are omitted because the predictions are made using less than a year of previous data). The upper panel shows the predictions for M class (and larger) flares. The histograms represent the observed number of days on which there was at least one M class flare (or larger). The diamonds represent the sum of the ϵ_{M} values for all days within the given year, which is the predicted number of days on which there should be at least one event of M class or larger. The lower panel shows the same display, but for events of X class. Uncertainties are shown for the predicted values, based on summation of the individual prediction uncertainties in quadrature.

The upper panel of Figure 2 indicates that the values of ϵ_{M} are systematically too large. More quantitatively, we find that the average value of ϵ_{M} over all days (1976–2003)

[4] ftp://ftp.ngdc.noaa.gov

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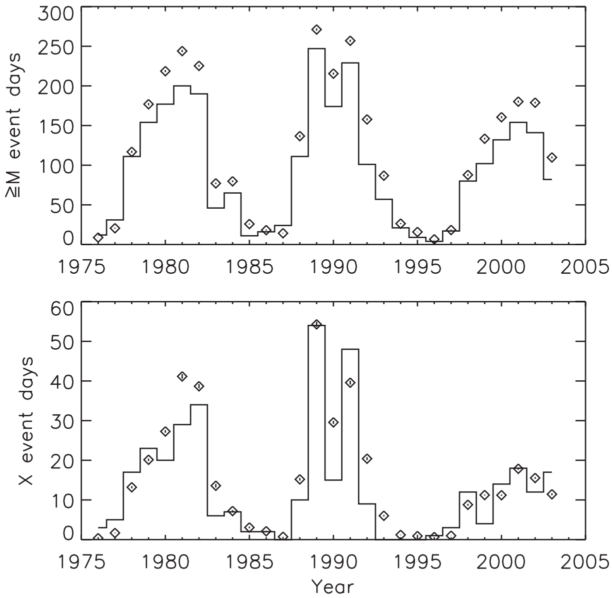

Figure 2 Upper panel: observed/predicted numbers of M (or larger) event days (histogram/diamonds). Lower panel: the same but for X event days.

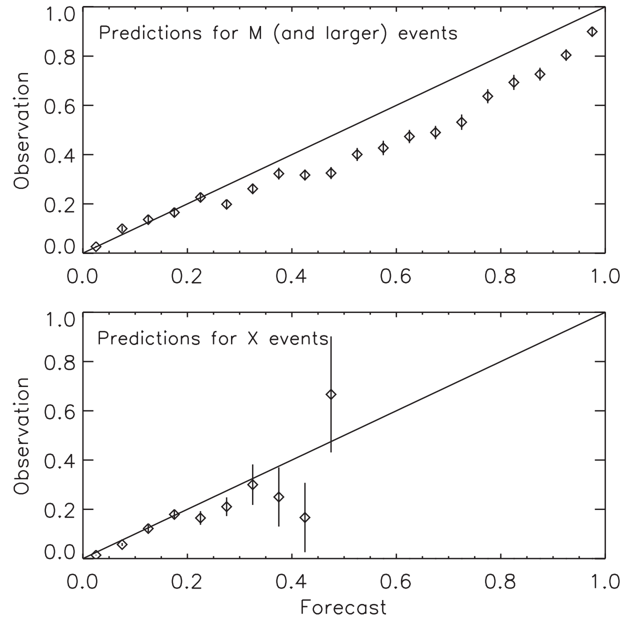

Figure 3 Upper panel: reliability plot for prediction of M events and above. Lower panel: the same but for X events.

is 0.320, whereas the observed value (the fraction of days on which there was at least one M class event) is 0.264. The lower panel of Figure 2 indicates that the method has done quite well in predicting X class events. In fact the average value of ϵ_X for 1976–2003 is 0.040, whereas the observed value is 0.036.

Figure 3 gives a more detailed display of the results of the test for all years (1976–2003) in the form of a pair of 'reliability plots', in which the horizontal axis shows the forecast probabilities ϵ_M or ϵ_X for each day (in bins of 0.05), and the vertical axis shows the true probability for flaring on the given days, estimated from the observed number of event days. This is the Bayesian estimate assuming binomial statistics and a uniform prior: if there are R days with at least one event out of a total of S days, the estimate for the probability is $p = (R+1)/(S+2)$ and the corresponding error is $[p(1-p)/(S+3)]^{1/2}$ (e.g. Jaynes 2003, p. 165). Perfect prediction corresponds to the solid 45° line on the plot. The upper panel of Figure 3 is the reliability plot for M (and larger) event prediction, and the lower panel is the reliability plot for X event prediction. The upper panel confirms that the predictions for M class events are systematically too large, although it shows that the effect is only associated with days on which ϵ_M is larger than about 0.25. The lower panel indicates that the predictions for X class flares are quite good for all values of ϵ_X, although the method is conservative, in that it does not assign values larger than about 0.5.

It is interesting to compare these results with predictions made by NOAA. Statistics are available for NOAA predictions for 1987–2002.[5] The NOAA results indicate a very serious over-prediction of X class events. During 1987–2002 the NOAA predictions imply that there should be 372.5 X event days, when in fact there are 200 such days. The present method predicts 233.8 X event days for 1987–2002, which is a considerable improvement over the NOAA result. For M (and larger) events, the NOAA results show over-prediction similar to the results obtained with the present method. A more detailed comparison with NOAA predictions will be presented in a future paper (Wheatland 2005).

4 Discussion

A Bayesian approach to flare prediction (Wheatland 2004) has been tested for whole-Sun prediction of GOES soft X-ray events, based on the NOAA Solar Event Lists for 1975–2003. The method is found to over-predict events of M class and above, but performs quite well for prediction of X class events.

There are several possible reasons for the over-prediction of events above M class. One possibility is that the method is systematically late in detecting the decline in rate associated with the decay of a large active region, or the rotation of a large active region off the disk. The method uses the Bayesian blocks procedure to detect rate changes, and is always trying to 'catch up' with what the Sun is doing. A specific limitation of the Bayesian blocks method is that it must have at least one event in a block, so that the rate is never identically zero. This limitation leads to overestimation of the rate at times of very low activity. It should also be noted that the existing Bayesian blocks procedure is not guaranteed to find an optimal decomposition — in future this procedure will be replaced by an optimal algorithm recently devised by J. D. Scargle (2003, private communication). Another possibility is that there is a bias in each individual prediction which becomes apparent in the analysis of a large number of predictions.

[5] www.noaa.sec.gov/verification/

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Such a bias will become less serious if each individual prediction is more accurate. The fractional error in each prediction goes as $(M')^{-1/2}$ (Wheatland 2004), where M' is the number of events associated with the estimation of the rate above size S_1. This error becomes smaller with increasing M', i.e. if a smaller S_1 can be chosen. It should be noted that the GOES event lists are a relatively poor choice for the present purpose because the time varying soft X-ray background means that a relatively large S_1 must be used (see Figure 1).

The GOES event lists also have other shortcomings as a basis for prediction from event statistics. Besides the departure from a power law at small sizes, it is likely the lists are incomplete above the nominal threshold S_1, e.g. due to the difficulty of distinguishing two flares occurring close together in time (Wheatland 2001). This is unlikely to be important for the test described here, provided that the size distribution obeys a power law above the threshold, since both prediction and validation of the prediction rely on the same (possibly incomplete) lists. Another problem with the GOES event lists is that the GOES peak fluxes are not background subtracted, so that the intrinsic flux due to an M class event at a time of low activity, when the background is low, is larger than the intrinsic flux due to an M class event at a time of high activity, when the background is high. However, again this is not particularly important to the present method provided that the size distribution for observed events is not distorted from a power-law form by this effect. The use of a previous year of data to make a prediction allows the possibility of incorporating variation in the power-law index with the solar cycle (e.g. due to this effect) but in fact we find no evidence of such a variation. In subsequent work the method will be applied to more accurate event catalogs.

The present test is limited to whole-Sun prediction. In the future the method will also be applied to individual active regions, and other prior information on the rate (e.g. the McIntosh classification of the associated sunspots) will be incorporated. However we note that, even in this simple form, the method has out-performed NOAA predictions for X class events.

Finally we note that automated predictions, based on this method, are now published on the web.[6] Predictions are made each day using the latest NOAA solar event lists. The web pages include a running score of how reliable the published predictions are, in the form of automatically updated reliability plots.

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[6] www.physics.usyd.edu.au/~wheat/prediction/prediction.html