mathematicians will find it a useful preparation for grappling with the more traditional approach in other books.

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KAHANE, J.-P., *Some random series of functions* (Cambridge Studies in Advanced Mathematics 5, Cambridge University Press, 2nd ed., 1985) 305 pp., £30.00.

By the time the invitation to review this book arrived I had already bought my own copy to displace the first edition [D.C. Heath, Lexington 1968] from my desk. Harmonic analysts will not have waited to read reviews before ensuring that, at the very least, their department libraries acquire a copy.

What sort of problem does Kahane consider? Everyone knows that if we play at heads and tails repeatedly for stakes $a_0, a_1, \ldots, a_N \in \mathbb{R}$ then (provided certain regularity conditions are observed) my winnings $\sum_{j=0}^{N} \pm a_j$ will be approximately normally distributed with mean 0 and variance $\sum_{j=0}^{N} a_j^2$. Thus, if I choose signs $\pm$ at random, then, with high probability,

$$\left|\sum_{j=0}^{N} \pm a_j\right| \text{ is of comparable size to } \left(\sum_{j=0}^{N} a_j^2\right)^{1/2}.$$

It is not difficult to extend this result to two dimensions and obtain for $a_0, a_1, \ldots, a_N \in \mathbb{C}$ (subject to certain regularity conditions) that, with high probability,

$$\left|\sum_{j=0}^{N} \pm a_j\right| \text{ is of comparable size to } \left(\sum_{j=0}^{N} |a_j|^2\right)^{1/2}.$$

From this it is not hard to deduce that if $a_0, a_1, \ldots \in \mathbb{C}$ then, with high probability,

$$\sup_N \left|\sum_{j=0}^{N} \pm a_j\right| \text{ is of comparable size to } \left(\sum_{j=0}^{\infty} |a_j|^2\right)^{1/2}$$

and so, if $\sum_{j=0}^{\infty} |b_j|^2 = \infty$, then, with probability 1,

$$\sup_N \left|\sum_{j=0}^{N} \pm b_j r^j\right| \to \infty \quad \text{as } r \to 1-.$$

It follows that if $\sum |c_j|^2 = \infty$ and $|\omega| = 1$, then, with probability 1,

$$\sup_N \left|\sum_{j=0}^{N} \pm c_j \omega^j r^j\right| \to \infty \quad \text{as } r \to 1-.$$

But (thanks to the ideas of Borel, Lebesgue and Kolmogorov) we can state that if each of a countable collection of events occurs with probability 1 then they all happen together with probability 1. Thus if $\omega_1, \omega_2, \ldots$ form a countable dense subset of the circle $|z| = 1$ we know that, with probability 1,

$$\sup_N \left|\sum_{j=0}^{N} \pm c_j \omega_k^j r^j\right| \to \infty \quad \text{as } r \to 1- \text{ for each of } j = 1, 2, \ldots.$$

We have, in fact, proved the following theorem:

Theorem A. *If* $c_0, c_1, \ldots \in \mathbb{C}$, $\sum_{j=0}^{\infty} |c_j|^2 = \infty$ and $\limsup |c_n|^{1/n} = 1$ then, with probability 1, the circle of convergence $|z| = 1$ is a natural boundary for the Taylor series $\sum_{j=0}^{\infty} \pm c_j z^j$.

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Since an event with probability 1 of occurring must (to draw a very weak consequence) be a possible event we have the following corollary:

Corollary B. *If* $c_0, c_1, \ldots \in \mathbb{C}, \sum_{j=0}^{\infty} |c|^2 = \infty$ and $\limsup |c_n|^{1/n} = 1$ then there exist $\lambda_0, \lambda_1, \ldots \in \{-1, 1\}$ such that the circle of convergence $|z| = 1$ is a natural boundary for the Taylor series $\sum_{j=0}^{\infty} c_j z^j$.

At this point I would advise readers to try and prove the corollary in the special case $c_j = 1$ for all j by non-probabilistic means. Since this is not too difficult they could also try the following problem:

Problem. For which α does there exist a $C(\alpha)$ with

$$\sup_{t \in \mathbf{R}} \left| \sum_{j=1}^{N} \sin m_{jN} t \right| \leq C(\alpha) N^{\alpha}$$

for suitably chosen integers $0 < m_{1N} < m_{2N} < \cdots < m_{NN}$?

In Section 6 of Chapter 6 Kahane describes Bourgain's proof that we can take $\alpha = 2/3$. At a much more modest level the reader is invited to anticipate the typically elegant and typically probabilistic argument by which in Section 3 of Chapter 4, Kahane improves on our Theorem A.

Theorem A′. If $\limsup |c_n|^{1/n} = 1$ then, with probability 1, the circle of convergence $|z| = 1$ is a natural boundary for the Taylor series $\sum_{j=0}^{\infty} \pm c_j z^j$.

Probabilistic methods in analysis have a fairly long history starting with Borel's 1896 study of the questions discussed above. Important results were obtained by Paley and Zygmund, Littlewood and Offord, Wiener and others. However even in the 1960s few analysts kept probabilistic methods in their mathematical tool box and one of the purposes of the first edition was to make them think again. Since then many important new results and proofs have been obtained by probabilistic methods by mathematicians both within and without Kahane's mathematical sphere of influence.

Wisely, however, Kahane has not chosen to rewrite from scratch so we still get the zeal of a "missionary preaching to cannibals" which, according to Littlewood, distinguished Hardy's *Pure Mathematics*. Instead he has rewritten a few sections to take account of new developments (often inspired by the first edition) and added a selection of new results. (Incidentally, anyone teaching a first course on Brownian motion will find Section 4 of Chapter 16 repays thoughtful attention.) Altogether about a third of the material is new. One or two results present in the first edition have gone but, so far as I can see, only the reviewer and a couple of other specialists will miss them.

It is, I think, true that the very greatest mathematics requires the same kind of effort of the reader as a mountaineering expedition requires of its participants. Hours or days of hard and often tedious toil are required to attain a magnificent vista. Kahane's book is more like a ramble through fine countryside. At every mile one is rewarded with a singing waterfall or an old farmhouse and at virtually every step with a new wayside flower. Cambridge University Press is to be congratulated on obtaining this second edition for us.

T. W. KÖRNER

TOMKINSON, M. J. *FC-groups* (Research Notes in Mathematics 96, Pitman, 1984), 188 pp. £8.95.

In the study of infinite groups, it is necessary to impose conditions on the groups in order to obtain satisfactory structural results. These conditions are normally either that the group has some sort of commutativity, or that it has some sort of finiteness, or a combination of both. Such a class of groups which has received a good deal of attention in the last thirty years or so is the subject of this book, and the author has played a substantial part in the development of the theory during the latter half of this period.

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