be calculated in this way. In the binomial case even the above argument is not required. For the N-queue system the state space can be redefined as $F \equiv$ all queues free and the complementary event $\bar{F}$. The system is then an alternating renewal process between the states F and $\bar{F}$, with the dwell time in F being exponential with mean $1/N\lambda$. Assume, without loss of generality, that the system begins in F at $t = 0$, then standard renewal theory arguments (Cox [2], p. 83) give the explicit form of the Laplace transform of the busy period. If only the mean is required, from [2], p. 84,

$$\bar{p}_0 = \frac{1}{(1+\rho)^N} = \frac{1/N\lambda}{E[B] + 1/N\lambda}.$$

Although the queuing systems differ, an heuristic argument for the mean waiting time in (3.13) is at least possible using the analogy between systems. Input and output rates are the same for both systems. For the N-queue system the waiting time before starting service is 0 and the mean time spent in the system is $1/\mu$. The value of $E[w] = 1/\mu$ could thus perhaps be anticipated because of the identical birth and death processes.

The above observations are offered very much in the spirit of the comparisons used in [1]. Alternative probabalistic arguments are given which appear to be useful, as well as being of interest in their own right.

References

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Yours sincerely,

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Dear Editor,

Comments on a paper by V. Giorno, C. Negri and A. G. Nobile [1]

The model described in [1] appears to be a new interpretation of a model which is well known in queueing theory. Some of the definitions in the paper seem inconsistent and the tables contain some errors. May we draw the authors' attention to the following points.

1. The mathematical model is already known in queueing theory as described in textbooks (e.g. Kleinrock [2], pp. 107–108 or Cooper [3], pp. 108–111) for $n = s$. The actual physical interpretation is new.

2. The time-dependent solutions are well known. They are given for example in Jensen [4], pp. 51–52 and Syski [5], section 5.2.3.4.

3. The definition of the effective arrival rate λ^* appears to be erroneous. This definition causes the effective mean interarrival time $E\{t\}$ to be different from $1/\lambda^*$. A more correct and natural definition of the effective interarrival rate is

$$\lambda^{**} = \sum_{j=0}^{N} \lambda(N-j)p_j.$$

By this definition we get

$$E\{T\} = 1/\lambda^{**}.$$

4. The paper postulates that we must require $\lambda/\mu = \rho < 1$. This is not necessary. Any value $\rho > 0$ is acceptable in a queueing system with a finite number of customers. With the natural definition of the effective interarrival rate λ^{**} we always get

$$\rho^* < 1$$

for any value of ρ.

5. We believe all values in Table 1 should be multiplied by a factor of 100.

We hope that the authors will find our comments helpful.

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