PRECOLLAPSE EVOLUTION OF GLOBULAR CLUSTERS

Lyman Spitzer, Jr.
Princeton University Observatory
Princeton, New Jersey 08544 USA

1. INTRODUCTION

The dynamical evolution of a cluster in which N, the number of stars, is large, results primarily from encounters between pairs of stars; these tend to modify the system so that it approaches a more probable configuration. In statistical equilibrium, the distribution of stars among states of different energy, E_i, per unit mass would be determined to a first approximation by the probability P_i, where

$$P_i \propto g_i \exp(-BE_i) \quad ; \qquad (1)$$

here g_i is the statistical weight of each state, proportional to the amount of phase space available, and B is a constant. Evidently dynamical evolution tends a) towards stellar states which are more tightly bound by their mutual gravitational attraction and have a correspondingly lower E_i, and b) towards the free state, where the volume of available phase space is very large. The only relatively stable state for an isolated cluster is one in which a central, non-evolving massive configuration (a compact object, or tight binary, or a hierarchical multiple system) has all the initial binding energy of the cluster and all other stars have escaped.

The general tendency of large-N clusters towards a combination of collapse and disruption has been known for some years. The different mechanisms by which such a cluster evolves and the possible evolutionary sequences resulting were reviewed ten years ago, at IAU Symposium No. 69 at Besançon (Spitzer, 1975). The present paper discusses the progress made in this field during the past decade; the topics considered are essentially those related to the precollapse phase until n, the number of stars per unit volume, approaches its peak value. Work on the postcollapse phase is reviewed in the following paper by D. C. Heggie. The extensive research in these fields prior to 1974 is mostly omitted from the present review, apart from a few passing references. Some of the topics discussed here have been treated in greater detail in the review by Lightman and Shapiro (1978).

The three sections which follow this Introduction discuss certain

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physical processes which have been the subject of considerable research during the last decade. Section 2 treats the gravothermal instability, which appears to be a dominant process in a late phase of cluster evolution. The development of mass stratification, which is more important in the early phases, and which is associated with its own instabilities, is discussed in Section 3. Effects produced by the hardening of binaries are reviewed in the fourth Section. Discussion of the various additional effects which appear during the collapse phase, and presumably bring to a halt the contraction of the central core, are presented in the final Section 5.

It should be emphasized that these different physical processes are generally defined for idealized clusters. Analysis of each effect separately helps us to understand the evolution of actual clusters, and of the numerical models which approximate real clusters. The discussion here attempts to explain the phenomena observed in these models in terms of the separate idealized physical processes. However, it must be remembered that when all these processes are occurring at the same time, it may not be possible even in principle to isolate precisely the effects produced by each process.

This review is not in any sense complete, and necessarily reflects the author's special interests and background. Thus the effect of a massive black hole on cluster evolution is not discussed, despite the elegant and interesting theory of this subject developed by various authors. Similarly, several papers on the origin and early evolution of globular clusters are not included.

This survey deals essentially with isolated clusters, since there has been little work during this last decade on the effects produced by the tidal field of the Galaxy. However, one should keep in mind that evaporation of stars from many of the globular clusters is much increased by a tidal cut-off at a finite escape radius, r_t, and may be a dominant effect on the evolution of such clusters. The importance of this process is apparent in the results presented at this Symposium by Stodółkiewicz; according to his detailed model, enhanced evaporation leads roughly to the linear decrease of bound mass with time found by Hénon (1961) in his pioneering work. For some clusters, strong tidal shocks will increase the evaporation rate further. Fall and Rees (1977) have suggested that the surviving clusters we observe are those for which these processes do not lead to complete disruption in 10^{10} years.

One prominent characteristic of the research reviewed here is the wide variety of techniques which have been employed. Theoretical cluster models have been constructed using the fluid equations, several types of Monte Carlo programs, numerical integrations of the Fokker-Planck equations, N-body calculation of individual orbits, and hybrid programs making use of several of these techniques in the same models. The methods used in some of these approaches have been greatly extended and improved during the last ten years. As the following sections indicate, all this activity has substantially increased our theoretical understanding of how a globular cluster must evolve with time, though there are still major uncertainties concerning the processes that occur late in the core collapse phase.

The contact between this impressive structure of theory and astronomical observation is admittedly rather incomplete. Even during the precollapse phase, where theoretical problems seem minimal, the observational data cannot be said to verify the theory in any detail. My training under Sir Arthur Eddington, together with my own inclinations, lead me to believe that the theory, based on simple dynamical assumptions of unquestioned validity, has an independent credibility.

2. GRAVOTHERMAL INSTABILITY

2.1. Instability criteria

The numerous analyses of the gravothermal instability all stem from the classic paper by Antonov (1962), who investigated the stability of gaseous isothermal spheres bounded by a rigid, insulating outer spherical shell, with a radius denoted here as r_b. Antonov's work is based on the entropy, S, of the system, defined as

$$S = - \int f(\underline{r}, \underline{v}, t) \ell n f(\underline{r}, \underline{v}, t) \underline{drdv} \quad , \tag{2}$$

where f is the usual phase-space distribution function (to which we shall usually refer as the velocity distribution function). Equation (2) is the so-called Boltzmann entropy, consistent with the one-particle distribution function considered in equation (1). We assume here, and throughtout all of Section 2, that all stars in the system have the same mass, m, and that no binary or multiple stars are present.

We now consider what changes in S are possible when the total mass, M, energy, E, and volume, V, of the system are held fixed. At all times the velocity distribution function, f, is assumed Maxwellian and the system is taken to be spherically symmetrical, since these two assumptions maximize S; changes in the system are produced by changes in δf, corresponding to changes in particle density and local mean square velocity. In an equilibrium system δS must vanish to first order in δf, but is proportional to $(\delta f)^2$. Antonov demonstrated that if the density contrast, D, between the center and the radius $r = r_b$ is less than 709, then δS is negative for all perturbations; the equilibrium state is the most probable one, and is thermodynamically stable. For a density contrast exceeding 709 the system will be unstable for some spherical perturbations, provided that the perturbed velocity distribution remains Maxwellian.

The physical principles underlying this effect were discussed some time ago by Lynden-Bell and Wood (1968), who named it the "gravothermal instability". They pointed out the importance of negative specific heat in driving such an instability in self-gravitating systems. In a bounded isothermal sphere with a sufficient central condensation, the central core can contract and become steadily hotter as it loses heat to the outer regions.

A series of investigators have analyzed the gravothermal instability criterion from several viewpoints. Hachisu and Sugimoto (1978) reconsidered Antonov's problem, computing δS for changes from an equilibrium state, with M, E and V kept constant, and assuming both spherical symmetry and a locally Maxwellian velocity distribution. Their mathematical method is quite different, however, and yields the critical D values for modes with various numbers of nodes. For the fundamental mode, a value of 709 is again found for D.

A more direct demonstration of instability is provided by a time-dependent analysis of perturbations, demonstrating that these grow exponentially under some conditions. Such an analysis has been carried out by Nakada (1978) for the same isothermal sphere considered by Antonov, with a short mean free path assumed for the self-gravitating particles. As D increases above a critical value, essentially the same as found by Antonov, the growth rate increases above zero, and is proportional to the thermal conductivity.

Yet another technique for demonstrating the appearance of the gravothermal instability is use of the classical "linear series" method. This method, developed originally for static systems in dynamical equilibrium, is based on the nature of a series of equilibrium configurations in which some parameter, μ, is assumed to vary. A static system is characterized by a potential energy W, which will depend on μ. If with increasing μ, W increases to a maximum and then decreases, the point of maximum W is called a "turning point"; in general the system will be dynamically stable on one side of a turning point and unstable on the other (Jeans, 1929).

This same technique has been applied (Lynden-Bell and Wood, 1968) to systems in thermodynamic equilibrium, especially to the bounded isothermal sphere considered by Antonov. In this case, -S, the negative of the total entropy, replaces W. The gas is in equilibrium if S is a maximum under any redistribution of mass and energy, again keeping mass, energy and volume constant. For the parameter μ we take the density contrast, D, between the center and the gas just inside the bounding sphere, and we consider a series of equilibria all of the same mass and volume, but of different temperature and energy. For D only slightly exceeding unity, gravothermal binding is unimportant and the system is stable. At D = 709, the entropy has a minimum value, and for greater D the system is thermodynamically unstable, in agreement with Antonov's results. The mathematical basis for this method as applied to clusters and the various conditions required for its application have been discussed by various authors--see Horwitz and Katz (1978) and Katz (1978).

Since M and V are constant along the series of equilibria considered here, one can visualize a given mass moving along the series if heat is slowly added or subtracted. Since dS = TdE under these conditions, it is clear that when S is an extremum so is E; for the critical configuration at which S is a minimum, E also has its minimum value. For application of the linear series method to clusters it is generally simpler to compute E rather than S along the sequence. A plot of E against 1/T is a spiral, with each extremum in E or 1/T corresponding to the onset of some instability (Katz, 1978).

Evidently there is no question about the existence of the gravothermal instability in a bounded isothermal gas sphere. However,

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actual stellar systems differ from these idealized spheres in at least two major respects. First, the mean free path is much longer than the dimensions of the system. Second, the velocity distribution is not Maxwellian to begin with, since in the absence of any confining surface, the velocity distribution function, $f(\underline{r}, \underline{v})$, must vanish for v greater than the escape velocity.

Because of the long mean free path (LP), $f(\underline{r}, \underline{v})$ will generally be anisotropic if the mean stellar kinetic energy varies systematically with distance from the cluster center. In such systems, $f(\underline{r}, \underline{v})$ must be essentially constant along a dynamical trajectory at any one time, if the relaxation time much exceeds the dynamical crossing time. Hence if the core becomes hotter than the outer regions, as one expects when the gravothermal instability develops, then far out in the system $f(\underline{r}, \underline{v})$ for radial orbits, which pass through the core, will tend to vary less steeply with E than will the corresponding distribution function for tangential orbits. Thus the perturbations possible for a LP system differ from the perturbations with local Maxwellian distributions which are possible in the case of short mean free path (SP). As a result the entropy increase possible in such perturbations is somewhat less in the LP case than in the SP case.

The instability expected in this LP case, when the initial equilibrium is again an isothermal sphere bounded by a reflecting shell, has been studied by Ipser and Kandrup (1980) and by Inagaki (1980). The former showed that the criterion for instability was the same as for the SP case analyzed by Antonov (1962). The latter analyzed the time-dependent growth of these perturbations in the LP case, though with an isotropic velocity distribution assumed in the Fokker-Planck equation. As expected, the growth rate varied inversely as the interparticle relaxation time. This latter study, like the parallel SP analysis by Nakada (1978) discussed above, gave eigenfunctions for the instabilities, though with non-Maxwellian values for the perturbed velocity distribution function. The physical differences between the SP and LP systems do not seem to have much qualitative effect on the initial appearance of the gravothermal instability, though of course the growth rates in these two cases depend very differently on the density and the velocity dispersion.

In systems which are not confined by a bounding surface, $f(\underline{r}, \underline{v})$ must be truncated at the escape energy. In such a configuration the entropy is not a local maximum, and the system is thermodynamically unstable to begin with. It is this instability that leads to evaporation of stars and resultant contraction of the system, effects fully studied more than a decade ago. However, it is not implausible that additional contraction, related to the gravothermal instability, will appear at the turning points computed without regard to the initial non-Maxwellian character of $f(\underline{r}, \underline{v})$. Analyses of such instabilities have been carried out by Katz (1980) for various energy-truncated models, in which f vanishes above some cut-off energy. The gravothermal instability is found to set in when the escape velocity $v_\infty(0)$ from the center is about 3.8 times the central velocity dispersion $v_m(0)$. As shown in Figure 1, a comparison with the Monte Carlo computations of Spitzer and Thuan (1972) shows that the core collapse seems to change its character

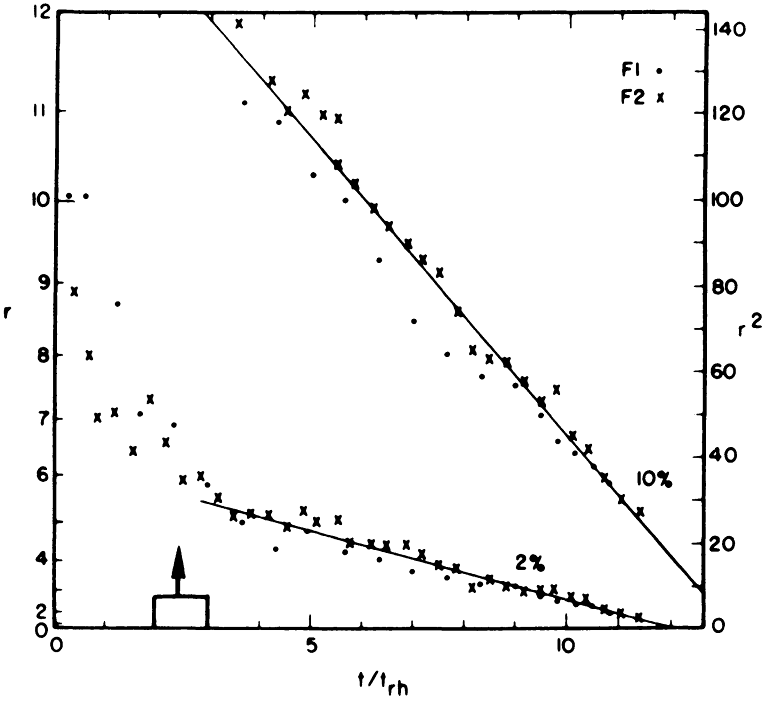

Figure 1. Comparison of Cluster Evolution with the Predicted Onset of the Gravothermal Instability. The plotted points show the radii containing 2 and 10 percent, respectively of the cluster mass for model F of Spitzer and Thuan (1972), plotted as a function of time since the cluster was born as a uniform sphere. For comparison, the arrow shows the time at which the cluster should become subject to the gravothermal instability, as computed by Katz (1980).

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at about this time, with $r_1{}^2$ subsequently decreasing linearly with time; r_1 is the radius enclosing two percent of the mass. These results seem to provide some confirmation of the gravothermal instability in realistic systems.

2.2 Self-similar collapse model

To make further progress in this field, one must analyze the detailed consequences of the gravothermal instability. Hachisu et al. (1978) have carried out a time-dependent calculation for a confined isothermal gaseous sphere, with an initial density contrast, D, much exceeding the critical value of 709. The core can either expand and cool or contract and heat up. If it expands, the sphere gradually approaches another equilibrium configuration with $D < 709$. If the core starts to contract, it continues to shrink without limit, developing a structure in which ρ is constant at small r and varies as $r^{-\alpha}$ for larger r. The value of α depends on the functional form of the thermal conductivity; if the formula for a fully ionized gas is used, α is 2.4, the same value found by Larson (1970) in his analysis based on the moment equations.

A more realistic analysis, based on an improved formula for the heat flow in a globular cluster, has been given by Lynden-Bell and Eggleton (1980). They considered a solution of the self-similar type, as in the familiar Sedov solution for a supernova shell. Thus $\rho(r)$ is a function of a scaled radius r_*, equal to $r/r_c(t)$, where $r_c(t)$ may be taken as the core radius. Hence

$$\rho = \rho_c(t)\rho_*(r_*) \quad , \tag{3}$$

where $\rho_c(t)$ is the density at $r = 0$. Analogous equations apply for $M(r)$, the mass interior to r, for v_m, the rms random velocity and for F_c, the conductive energy flux per unit area at the radius r.

Their equation for the conductive heat flux, F_c, modifies the standard formula for a perfect gas to allow roughly for the long mean free path in a cluster. In the limit of short mean free path (SP), we have the usual result

$$F_c = C_1 \rho v_m \lambda \frac{d}{dx}\left(\frac{3kT}{m}\right) \quad , \tag{4}$$

where v_m is again the rms velocity, λ is the mean free path, and C_1 is a constant of order unity. All particles are again assumed to have the same mass, m. Here $\lambda dT/dx$ is the excess temperature between the starting point of the particles and the stopping point one collision later. In the LP limit, this excess temperature becomes HdT/dx, where H is the scale height of the particle orbits. In addition, in this case the number of orbits over a distance H required for one collision is t_r/t_d, the ratio of the collision time, or relaxation time, to t_d, the dynamical crossing time; this ratio is about equal to H/v_m. Hence F_c in equation (4) must be multiplied by the factor $(H/\lambda) \times (t_d/t_r)$, yielding the result

$$F_c = \frac{C_1 \rho H^2}{t_r} \frac{d}{dx} \left(\frac{3kT}{m}\right) \quad . \qquad (5)$$

If we set H proportional to the distance $v_m/(G\rho)^{1/2}$, take for t_r the expression (Spitzer and Hart, 1971a, equation (30), with $\Psi = 1$ for only one mass component present)

$$t_r = \frac{v_m{}^3}{15.4 G^2 \rho m \ln\Lambda} \quad , \qquad (6)$$

and replace 3kT/m by $v_m{}^2$, we obtain the final equation adopted by Lynden-Bell and Eggleton (1980) for the conductive flux,

$$F_c = \frac{C_2 \rho G m \ \ln(0.4N)}{v_m} \frac{d v_m{}^2}{dx} \quad , \qquad (7)$$

where C_2 is another constant.

A second assumption is that all perturbations vanish as r_* becomes large. Thanks to this assumption the outer regions of the cluster become irrelevant, and the cluster as a whole can be an isothermal sphere, either bounded or infinite, or a more complex configuration with an inner isothermal region surrounded by a halo of stars moving in more nearly radial orbits. For the actual LP case the velocity distribution must clearly be anisotropic, as we have seen in subsection 2.1. While this anisotropy is ignored in the self-similar solution, the analysis provides a first approximation for the gravothermal collapse of the inner regions of realistic clusters.

The time dependent functions $r_c(t)$, $M_c(t)$, etc. are identical with those obtained in self-similar solutions for simple model clusters with evaporation of stars (Gurevich and Levin, 1950a, King, 1958). In particular, for $r_c(t)$ and $M_c(t)$ we have

$$r_c(t) \propto (1 - t/t_0)^{(4-2\zeta)/(7-3\zeta)} \quad , \qquad (8)$$

$$M_c(t) \propto (1 - t/t_0)^{2/(7-3\zeta)} \quad . \qquad (9)$$

Evidently t_0 is the time at which the collapse becomes singular, with r_c and M_c approaching zero together, while ρ_c becomes infinite. The parameter ζ is the ratio of the relative change of energy to the relative change of mass for the cluster core, giving

$$\frac{1}{E_c} \frac{dE_c}{dt} = \frac{\zeta}{M_c} \frac{dM_c}{dt} \quad . \qquad (10)$$

For evaporation of stars from an isolated cluster as a result of distant two-body encounters, the escaping stars have a very small kinetic energy at infinity and ζ is a small negative number, approaching zero with increasing N. Another relation for self-similar collapse is

$$v_c{}^2 \propto \rho_c{}^{(1-\zeta)/(5-3\zeta)} \quad . \qquad (11)$$

For a gravothermal collapse, $\rho_*(r_*)$ has been determined numerically (Lynden-Bell and Eggleton, 1980) and is shown in Figure 2. This

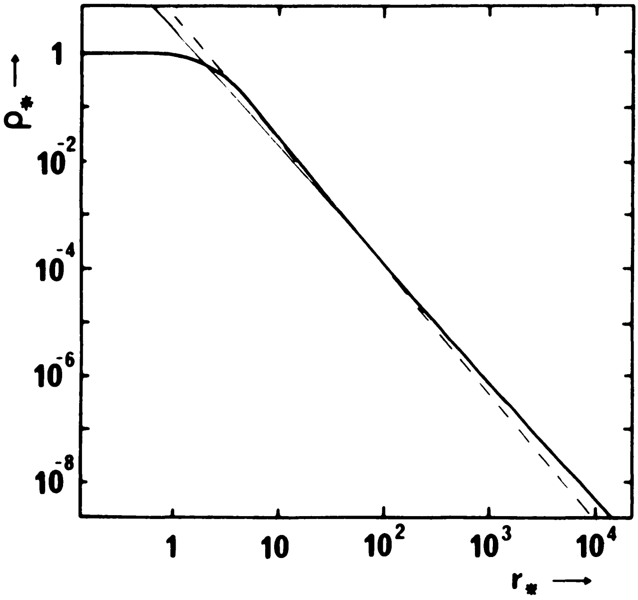

Figure 2. Density Profile of Similarity Solution for Gravothermal Collapse. The solid curve shows the density profile for the self-similar solution found by Lynden-Bell and Eggleton (1980); $\rho_* \equiv \rho(r,t)/\rho(0,t)$ is plotted against $r_* \equiv r/r_c(t)$, where $r_c(t)$ is the radius of the contracting core. The dashed line shows ρ_* varying as $r_*^{-2.4}$, while the thin straight line shows the asymptotic variation of ρ_* as $r_*^{-2.21}$.

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determination is an eigenvalue problem for the parameter, ζ, whose value is found to be 0.737. The asymptotic logarithmic slope, α, of the $\ell n \rho_* - \ell n r_*$ plot is a simple function of ζ and equals 2.208. As a result of the inwards temperature gradient, this density distribution is slightly steeper than for the isothermal sphere, in which ρ varies asymptotically as r^{-2}. The systematic material velocity, w_r, is directed inwards for small r_* and outwards for large r_*, vanishing at the radius where $\rho_* = 0.0071$, corresponding to $r_* = 16$ in Figure 2.

2.3 Comparison with realistic cluster models

As a test for the importance of the gravothermal instability we may compare this idealized first approximation with detailed cluster models. In the early Monte Carlo models by Spitzer and Thuan (1972) the scale height, κ, of the core varies with time as $(1 - t/t_0)^{0.65}$, in comparison with the exponent 0.53 found in equation (8) for $\zeta = 0.737$. Also, the macroscopic velocity, w_r, was found to reverse sign when the local density was less than $\rho(0)$ by some two orders of magnitude, in rough agreement with the theory.

A more conclusive confirmation of self-similar gravothermal collapse is obtained in later realistic cluster models, which achieve greater precision and extend to later times. The central density in these later models increases by some 3 to 4 orders of magnitude as compared with an increase of only 2 orders of magnitude in the earlier work. The Fokker-Planck equation, with f taken to depend both on energy, E, and angular momentum, J, was used by Cohn (1979) in a detailed numerical computation of cluster evolution. A new Monte Carlo method, which follows the diffusion of stellar orbits in E, J space, but with two important differences from the Hénon (1971a,b) method, was developed and applied by Marchant and Shapiro (1980). Variations in the time step used give the correct mean changes of velocity during each orbital period, and radial variations in the number of shells (Hénon's superstars) per unit mass give the necessary additional data needed to follow the evolution of a core whose mass continually decreases. These two modifications much increase the precision of the calculations, especially for late stages of core collapse.

An important result obtained with these models is the variation of velocity dispersion, v_m, in the core with changing core density. Figure 3 shows the relationship obtained in these two types of models. The value of v_m at $r = 0$ is designated here as V_0. The slope of the dashed line, which is about the same as for the plotted data, corresponds to ρ_c varying as V_0^{20}, or to an exponent 0.10 in equation (11), as compared to the value 0.094 found for $\zeta = 0.737$. For the evaporative model of core collapse, $\zeta = 0$, and the predicted exponent in equation (11) is 0.20, twice the "observed" value obtained from Figure 3. These results provide rather strong confirmation that the gravothermal instability rather than evaporation is the dominant mechanism in the core collapse of isolated clusters.

The self-similar solution of Lynden-Bell and Eggleton received further confirmation from a much more extended Fokker-Planck computation by Cohn (1980). In this work f was averaged over J, thus ignoring

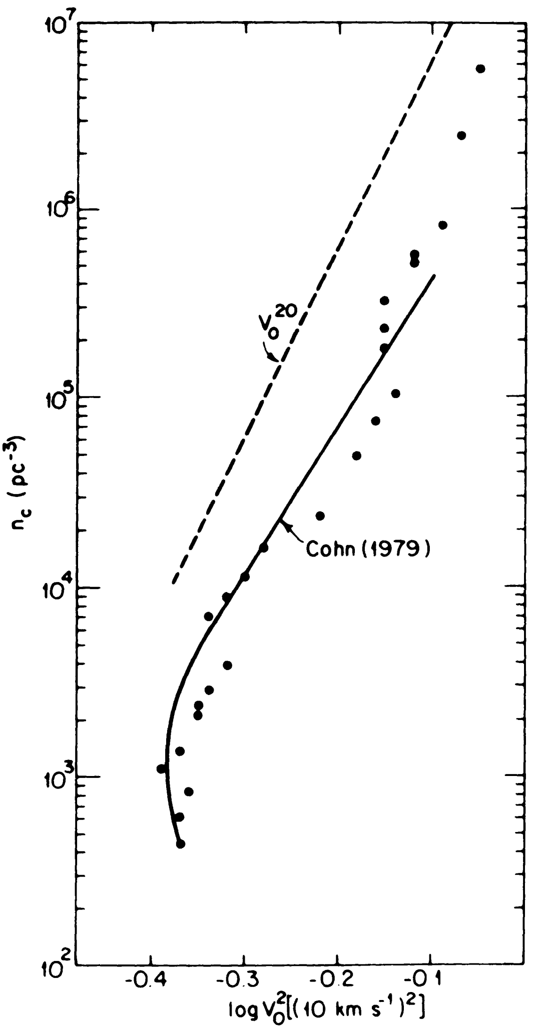

Figure 3. Variation of Central Density with Velocity Dispersion. The plotted points show n_c, the central density in the cluster core, as a function of V_0, the rms velocity dispersion at the center, computed by Marchant and Shapiro (1980) with a Monte Carlo program. The solid line shows these same quantities computed by Cohn (1979) from the time-dependent Fokker-Planck equation. For comparison the dashed line represents n_c varying as V_0^{20} during the evolution.

the anisotropy of the velocity distribution. With the simplification so achieved, the computations were extended to a central density increasing by almost 20 orders of magnitude, a somewhat formal extension, since the core mass at the end is much less than a solar mass! An interesting feature of the results is that at late times the density ρ outside the innermost core varies asymptotically as $\rho^{-2.23}$, in very close agreement with the result $\rho^{-2.21}$ obtained from the similarity solution.

Another noteworthy result of Cohn's work is that as the cluster collapses, the collapse rate, measured by $t_r(0) d\ln\rho(0)/dt$, approaches a constant value equal to about 6×10^{-3} in the first Fokker-Planck computation, with non-isotropic velocities considered, and 3.6×10^{-3} in the second one, with f(E, J) averaged over J. This difference may result in part from the increased precision in the later computations, though one would expect some difference to result between these two types of velocity distribution. In any case, the collapse is slow in that hundreds of relaxation times are required for appreciable evolution.

The anisotropy of the velocity distribution associated with the gravothermal collapse is evidenced by the Monte Carlo models of Duncan and Shapiro (1982), who used the same techniques as did Marchant and Shapiro (1980). Their result, confirming earlier less complete computations by Spitzer and Shull (1975), showed the development in late collapse stages of marked velocity anisotropies outside the core but well within the half-mass radius r_h. At a radius enclosing one fourth the total mass the mean square radial velocity exceeded half the mean square tangential velocity by 20%. It is clear physically that when the velocity dispersion of the core increases the mean square radial velocity of all stars on radial orbits will tend to increase, since f(E, J) is constant along a dynamical trajectory. One of the ways in which kinetic energy is transmitted to the outer regions is by collisional deflections of stars in radial orbits, which increase the mean square tangential velocity of stars in the outer regions of the cluster. Not unexpectedly, for r within $3r_c$ the velocity distribution remains nearly isotropic. Evidently this complex process is not considered in the similarity solution obtained by Lynden-Bell and Eggleton.

Despite this appearance of velocity anisotropies in their detailed models, Duncan and Shapiro (1982) find that in all other respects their collapse solution is very similar to that obtained with isotropic velocities assumed. In particular, the density varies about as $\rho^{-2.2}$ for radii between r_c and r_h, in close agreement with the results noted above for isotropic velocity distributions, both in the self-similar solution and in Cohn's isotropic model. We have already seen that the variation of ρ_c with v_c in the anisotropic Monte Carlo models is in relatively close agreement with the self-similar solution, in which the velocity distribution is isotropic. Apparently within the half-mass radius r_h, the approximation of isotropic velocities makes very little difference in overall behaviour of a cluster during the collapse phase.

The effect of close encounters during the collapse phase has been studied by Goodman (1983), who finds that large-angle deflections do not make any important difference, even when the number of stars in the core is small. Similar conclusions for the evaporation of stars from clusters at earlier stages were reached by Spitzer and Thuan (1972), who compared their Monte Carlo results with Henon's analysis (1960) of close encounters, and also by Retterer (1979).

3. MASS STRATIFICATION

3.1 Instability criteria

An idealized theory showed some time ago (Spitzer, 1969) that an isolated cluster with two mass components, m_1 and m_2, cannot achieve kinetic equilibrium in the core if the total mass, M_2, of the heavier stars, is more than a certain critical fraction of the total mass, M_1, of the lighter stars. The physical basis for this result is that when $\rho_2(0)/\rho_1(0)$, the ratio of densities of the two components at the cluster center, much exceeds unity, the self-attraction of the heavier stars tends to give them an appreciable mean square random velocity v_{m2} which is greater than the equipartition value $(m_1/m_2)^{1/2}v_{m1}$. An approximate theory gave as a condition of equilibrium the following limit on M_2/M_1

$$\mathscr{S} \equiv \frac{M_2}{M_1}\left(\frac{m_2}{m_1}\right)^{3/2} < 0.16 \quad . \tag{12}$$

Vishniac (1978) has found that with a continuous mass spectrum a relation similar to equation (12) can be derived, with m_1 equal to the mass of the lightest stars present, provided that the total fraction of the cluster mass in stars of mass exceeding m_2 replaces M_2/M_1. The critical value of $\mathscr{S}$ so redefined is about 2; this order of magnitude increase over the corresponding value in equation (12) results in part from the fact that m_1 is substantially less than the mean mass of the stars lighter than m_2.

The equilibrium of two-component systems has received considerable attention in the last decade, with some half dozen papers published in this area. This work generally indicates that when equipartition is in fact present in a system confined by its own gravitational field, the computed equilibrium configurations become unstable if the mass fraction in the heavier stars exceeds some critical value, roughly comparable with that found in equation (12). Stable equilibria seem possible with greater mass fractions of the relatively heavy stars, but such equilibria are either not self-confined or are not characterized by equipartition.

Analyses of two-component isothermal systems confined by a rigid bounding shell of radius r_b have been carried through by Saito and Yoshizawa (1976), Lightman (1977) and Yoshizawa et al. (1978). These analyses determine the minimum energy, E_c, for different values of m_2/m_1 and M_2/M_1. If M_2/M_1 is either very large or very small this minimum energy, measured in terms of GM^2/r_b, has the same value -0.335 as for a one-component bounded isothermal sphere, with a density contrast, D, equal to 709. For intermediate values of M_2/M_1, E_c is positive and no equilibria of negative energy exist. For M_2/M_1 small, the condition that $E_c = 0$ corresponds to $\mathscr{A}$ about equal to 0.6, some four times greater than the upper limit in equation (12). For $\mathscr{A}$ equal to 0.16 and M_2/M_1 again small, $E_c = -0.25$. As E_c increases above this value, the confinement of the system by the pressure on the outer rigid shell presumably becomes increasingly important, and the system becomes less and less like an isolated cluster.

To analyze isolated systems a non-Maxwellian velocity distribution must be assumed, with f(E) vanishing for E positive, corresponding to stars which escape. In the well known models by King (1966), f(E) is truncated according to the law

$$f(E) = f_0(e^{-BE} - 1) \text{ for } E > 0, \quad f = \text{otherwise} \quad . \tag{13}$$

As in equation (1), E is the energy per unit mass. To give a finite system, the gravitational potential is assumed to vanish at a finite cut-off radius, r_t. Two-component systems based on this distribution have been analyzed by Merritt (1981) and by Kondrat'ev and Ozernoy (1982). They applied equation (13) separately for each of the two components, with masses m_1 and m_2. The latter authors assume that

$$B_1/m_1 = B_2/m_2 \quad . \tag{14}$$

If the central potential $\phi(0)$ has a large negative value, so that $-B\phi(0)$ is a large positive number, the cut-off is in the far tail of the Maxwellian distribution, and equation (14) gives equipartition of energy, with $m_1 v_{m1}^2 = m_2 v_{m2}^2$. For shallow potential wells equipartition of energy no longer follows from equations (13) and (14); for sufficiently small $|B\phi(0)|$, f(E) is proportional to BE for $v < v_\infty$, and the mean square velocities are equal for the two mass components. For example, if $m_2/m_1 = 3$, the ratio of kinetic energies of heavy to light stars increases to about 1.5 if $|B_1\phi(0)| = 3$ (Kondrat'ev and Ozernoy, 1982), approaching m_2/m_1 as $|B_1\phi(0)|$ decreases towards zero.

In the models by Merritt (1981) the ratio B_2/B_1 is adjusted to preserve equality of kinetic energies between the two mass components at the cluster center. Models based on this assumption are obtained for arbitrary values of M_2/M_1. However, the models with $\mathscr{A}$ much exceeding 0.16 seems quite unrealistic; for example, for a model with $\mathscr{A} = 1$ the fraction of heavy stars within the tiny central core, to which equipartition is limited, is only about 10^{-8} ! The stability of these various two-component King models has not been investigated.

Katz and Taff (1983) have considered two-component models in which the velocity distribution function has the following form, based on one proposed by Wilson (1975),

$$f(E) = f_0(e^{-BE} - 1 + BE) \quad . \tag{15}$$

Equation (14) was assumed, relating the values of B for the two components. The detailed one-component numerical models (as, for example, the Fokker-Planck model by Cohn, 1979) show a distribution function in the central regions which is intermediate between equations (13) and

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(15). In the models by Katz and Taff (1983) the gravitational potential $\phi(r)$ was assumed zero at a boundary radius, r_t. The linear series method (§2.1) determined the critical condition at which instability sets in.

The results are shown in Figure 4, which plots the critical value of $k \equiv -B_1\phi(0)$, the dimensionless depth of the potential well, as a function of $\mathcal{S}$, defined in equation (12). Each critical value corresponds to a turning point in a plot of total energy against B_1. The different curves in Figure 4 correspond to different values of m_2/m_1; the arrow is drawn to indicate the critical value of $\mathcal{S}$ given in equation (12). Evidently if $\mathcal{S}$ increases much above 0.16, the maximum depth of the potential well consistent with the theoretical criterion for stability decreases rather steeply. As pointed out above, with decreasing $|B_1\phi(0)|$, equipartition disappears, and in the limit v_{m1} approaches v_{m2}. When $m_2/m_1 = 4$ and $\mathcal{S} = 0.54$, the deviations from equipartition at the cluster center are about 50%, and are even greater in the outer regions.

These results on the stability of two-component clusters are not entirely conclusive, in view of theoretical uncertainties concerning the linear series method as applied to systems with non-Maxwellian $f(E)$,--see § 2.1. However, the general evidence certainly points to the conclusion that in realistic clusters kinetic equilibrium cannot be established in the core if the mass fraction of heavy stars exceeds a small value, and the system of heavy stars at the cluster center must continue to shrink compared to the light stars.

3.2 Collapse of realistic clusters

Ever since the early Monte Carlo computations (Spitzer and Hart, 1971b) it has been known that the heavier stars tend to move into the central core, where there is an evident tendency toward equipartition, and that the core of heavier stars then proceeds to collapse in much the same way as do the cores in single-component clusters. Subsequent computations (Spitzer and Shull, 1975; Saito and Yoshizawa, 1976; Angeletti and Giannone, 1977; Inagaki and Wiyanto, 1984) confirm this general result. There are no simple time-dependent theoretical solutions, such as the self-similar solution for the sphere subject to gravothermal instability, with which multi-component models can be compared. As a result general conclusions are somewhat more difficult to extract from these models.

One result which is of theoretical significance, though not surprising, was obtained by Saito and Yoshizawa (1976), who used Larson's (1970) fluid dynamic technique, based on moments of the Boltzmann and Fokker-Planck equations, to follow the evolution of an isothermal sphere with two mass components confined by a rigid shell. They assumed equal velocities for the two components initially. If the system energy was well above the minimum possible, corresponding to stability against either gravothermal or mass stratification collapse, they found that equipartition developed at the expected rate and the system approached a stable equilibrium. Lightman and Fall (1978) compared early Monte Carlo results with a simple analytical theory for the overall

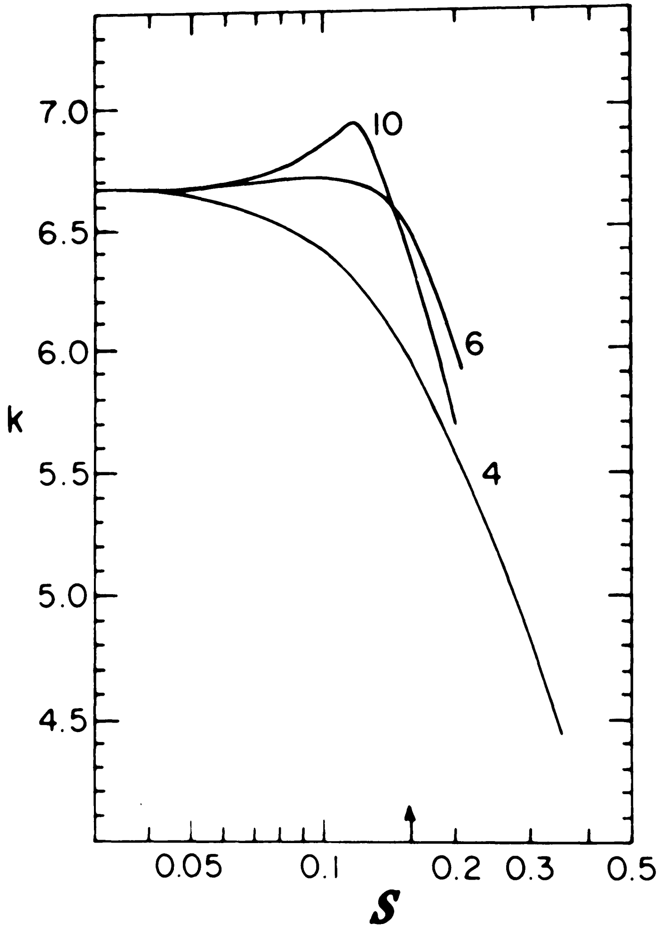

Figure 4. Theoretical Onset of Mass Stratification Instability. The curves show the maximum depth of the potential well (in dimensionless units) for which a cluster with stars of the indicated mass ratio is stable, according to the theory by Katz and Taff (1983). The quantity $\mathcal{S}$, defined in equation (16), was found to be less than 0.16 for stable equilibrium (see arrow) in an early theory by Spitzer (1969).

properties of the central core, and obtained reasonable fits. They considered two effects,--the mass stratification instability and evaporation from the core, on the assumption that the latter phenomenon is responsible for the collapse of one-component models; as we have seen, more recent computations indicate that this assumption seems invalid.

An extensive set of model two-component cluster computations was obtained by Inagaki and Wiyanto (1984), who used the time-dependent Fokker-Planck equation, with an isotropic velocity distribution, following Cohn (1980). With m_2/m_1 fixed at 2, they constructed models with M_2/M_1 equal to 0.01, 0.05, 0.11 and 1.0; the corresponding $\mathcal{S}$ values are 0.028, 0.14, 0.31 and 2.8. The results show that for the lowest $\mathcal{S}$ the central core soon approaches equipartition (with $(T_2 - T_1)/T_1$ equal to about 2%) and stays there until just before the final collapse. As $\mathcal{S}$ increases above unity, the minimum relative difference of central temperatures between the two components (i.e., the minimum relative difference of kinetic energies) increases, with $(T_2 - T_1)/T_1$ equal to about 17 percent for $\mathcal{S} = 2.8$. During the final collapse phase the core is mostly composed of heavy stars even for $M_2/M_1 = 0.01$. As shown in earlier studies, many of the heavy stars remain outside the core, and do not approach equipartition.

Inagaki and Wiyanto (1984) follow Lightman and Fall (1978) in explaining their data for core evolution by the predictions of mass stratification and the gravothermal instability, using for ζ in equation (10) the value 0.74 obtained both from the self-similar solution and the more realistic calculations for one-component clusters. In all cases, the core collapse rate, measured by $t_{r2} d\ell n\rho_2/dt$, since the core becomes composed largely of the heavier stars, is equal to the corresponding value which they find for a system composed entirely of heavy stars. This value equals about 3.5×10^{-3} (when the relaxation time they used is multiplied by 3/4 to agree with the different $t_r(0)$ used by Cohn (1980)), a rate which is essentially identical with that found also by Cohn (1980) for one-component systems with an isotropic velocity distribution. This general result strongly suggests that the final core collapse in the two-component clusters is due to the gravothermal instability. The chief effect of mass stratification in realistic clusters is to produce rather rapidly a concentration of heavy stars within the core and to shorten the time of evolution until the gravothermal stability becomes dominant. Also, since t_r varies as $1/\langle m\rangle$ for constant ρ, the dominance of heavy stars increases the final rate of collapse, though the time required for this collapse process, once it begins, is already very short.

The effect of stellar mass loss on cluster evolution has been considered by Angeletti, Dolcetta and Giannone (1980), who employed the fluid-dynamical approach developed by Larson (1970). Multi-component models, with 5 or 8 stellar groups, were considered, and stellar evolution during the cluster life was considered. The purpose of this ambitious work was to obtain a fit with the observed detailed properties of clusters. A certain qualitative agreement was obtained with M3, but the uncertainties in the initial conditions and the neglect of shock heating make the results somewhat tentative.

4. ENERGY EXCHANGE WITH BINARIES

4.1 Basic processes

The exchange of energy between single stars and binaries and its possible importance in globular clusters has been known since the pioneering work of Gurevich and Levin (1950b). These authors also pointed out the difference between "soft" and "hard" binaries. The former have net binding energies less than the translational energy of single stars and tend to gain energy and become softer as a result of gravitational interactions with passing stars. In contrast, "hard" binaries tend to lose energy and become harder. At the 1974 Besançon symposium, two important papers were in press giving much more extensive information on the interactions between single stars and binaries,--extensive numerical computations by Hills (1975a) and a very thorough analytical discussion by Heggie (1975).

The most complete numerical information on the complex interactions between single stars and binaries has since been obtained by Hut (1983), based on several hundred thousand orbital integrations. The results give definitive values for the mean exchange of energy in such encounters. Figure 5 shows the rate coefficient $\langle\sigma\Delta\rangle$ for this process, computed separately for Δ positive and Δ negative; σ is the cross section, while Δ is the relative change in binary binding energy. All stars have the same mass, m. The horizontal scale shows v_r, the velocity of the single star relative to the mass center of the binary, expressed in units of v_c, the incoming velocity for which the total energy of all three stars (regarded as mass points) vanishes. For three stars of identical mass m, $v_c{}^2 = 3Gm/2a$, where a is the semi-major axis of the binary orbit. Figure 5 shows clearly the "watershed effect", such that Δ tends to be negative for large v_r/v_c, and positive when v_r/v_c is small.

An important feature of such data is that they resolve a disagreement between Heggie and Hills in dE_b/dt, the average rate of increase of E_b, the binary's binding energy (defined as positive), in interactions with single stars. Heggie (1975) expressed his results in the form

$$\frac{dE_b}{dt} = n_s\langle v_r\sigma\Delta\rangle E_b = \frac{4(3)^{1/2}A}{35} \times \frac{n_s G^2 m_s{}^3}{v_{sm}} , \qquad (16)$$

where n_s is the particle density of stars and v_{sm} is the root mean square random velocity of the single stars. For the constant A, Heggie (1975) obtained a value about 45; the data by Hills (1975a) give a value of 17 for A. In the results by Hut (1983), plotted in Figure 5, A is not exactly constant with v_r/v_c (as indicated by the systematic change in separation between the open circles and the dashed line), but equals about 30 for a stellar kinetic energy about 10 percent of E_b.

Encounters between two binary systems can be important within an evolved cluster core, where under some conditions binaries can outnumber single stars. An asymptotic theory of this process has been developed by Spitzer and Mathieu (1980), who assume that the harder, more tightly bound binary behaves as a single star, and displaces one of the

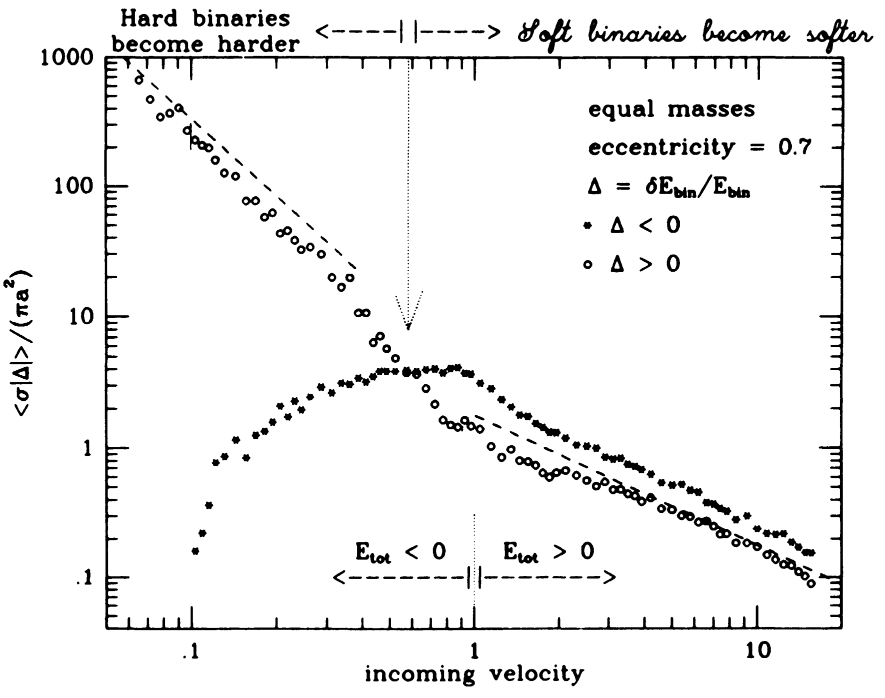

Figure 5. Energy Change in Collisions between Binaries and Single Stars. The plotted points represent average values of $\sigma\Delta$ in collisions between single stars and binaries, as a function of their relative velocity at infinity, computed by Hut (1983). The quantity σ is the cross-section, and Δ is the energy gained by the single stars. Averages over impact parameter and several angles characterizing each encounter are taken separately for Δ positive (open circles) and negative (solid circles); a is the semi-major axis of the binary. The dashed lines show theoretical results by Heggie (1975).

stars in the binary which it encounters. The resultant triple system then ejects one of the three stars, perhaps after further encounters have increased the orbital eccentricity of the loosely bound component so that it comes close to its accompanying binary, and acquires enough energy to escape. According to Heggie (1975) and Hills (1975a), the eccentricity of a binary is modified more rapidly by gravitational encounters than is the binding energy.

Direct computations of binary-binary encounters have been reported by Mikkola (1983a, 1984), who carried out roughly 5000 orbital integrations, assuming stars all with the same mass, and by Hoffer (1983), who reported some 42,000 orbits, mostly for soft binaries, with several mass groups. While these computations are not yet sufficiently detailed to yield all the cross-sections of interest, they should provide a check on various simplified models for such interactions. They agree with each other (and with Spitzer and Mathieu, 1980) in finding a high probability that in a strong interaction between two identical hard binaries, one is disrupted into two single stars; on the average, disruption results (Mikkola, 1983a) in some 60% of such collisions, with 30% leaving a three-body system. Both authors give preliminary fitting formulae for certain interaction cross-sections.

The formation of binaries by three-body encounters is, of course, a basic physical process of importance for globular clusters. It was shown early (Spitzer and Hart, 1971a) and confirmed by Heggie (1975) that for typical conditions in clusters this process is normally much too slow to have any significance. However, in a collapsing core, where the density becomes very high, but the number of stars involved is so low that the velocity dispersion remains small, this process can become significant as can the formation of binaries through tidal capture of one star by another. These two processes are discussed in Section 5.

4.2 Effect of primordial binaries on core collapse

The energy released by a hard binary, as it hardens further in encounters with passing stars, is transmitted chiefly to these single stars and increases the cluster energy (computed with each binary system replaced by a mass point). If an appreciable fraction of hard binaries is assumed to be present, produced when the stars themselves were being formed, they can provide an important source of heat to the cluster, tending to offset the collapse of the central regions. While this assumption does not seem to be supported by the limited observational data available (Trimble, 1980), we shall here explore its consequences.

A preliminary analysis of cluster evolution when primordial binaries are present was carried out by Hills (1975b), who treated the cluster core as a homogeneous sphere, and assumed that the energy released from binaries went entirely into heating the core. The chief driving force for core contraction was assumed to be evaporation. On these assumptions he found that for a mass fraction of 40% in binaries, and 10^5 stars present in the core, the density in the core initially rises slightly, but then decreases as the core expands. For somewhat

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lower mass fractions in binaries, the initial contraction is greater, but binary hardening still wins out over evaporation eventually, and the core re-expands. Somewhat similar computations have been made by Alexander and Budding (1979).

Computations for a realistic cluster containing primordial binaries have been made by Spitzer and Mathieu (1980), using a Monte Carlo method. All stars, both single and in pairs, were assumed to have the same mass. The energy released by star-binary encounters was computed from the results of Hills (1975a) and Heggie (1975), while for binary-binary encounters the asymptotic theory described in the previous section was adopted. The initial mass fraction in binaries was taken to be 20 or 50 percent in different models. Since each binary has twice the mass of a single star, mass stratification increased the fraction of binaries in the inner core to at least 90% by mass towards the end of the evolution; evidently binary-binary encounters predominated at this time.

Figure 6 shows the radius containing 2% of the system mass plotted against the time, for clusters with 20% of the initial mass in binaries. The plotted curves are for different values of A, the numerical constant in equation (16); as we have seen, the correct value is about 30. For the two models with A = 25, the total binary energy released somewhat exceeds the initial binding energy of the cluster, computed as though each binary were a mass point. Nevertheless, the collapse of the cluster core is apparently not averted, but only postponed, even though the core is composed almost entirely of binaries at the later times. This effect has two causes. First, about 60% of the energy released is carried entirely away from the cluster by escaping reaction products--mostly single stars. Second, of the energy retained in the cluster, only a fraction goes into heating the inner two percent of the mass,--the inner core where most of the binary energy is released, and where the collapse is mostly concentrated. The reaction products which are retained in the cluster have sufficient energies to give them apocenters well outside the core, and the angular momentum gained from gravitational encounters with passing stars hinders them from returning to the central core. Hence the energy of these products is gradually absorbed in an extended region of the cluster.

These models are, of course, approximate and neglect a variety of effects. The most important simplification is probably the neglect of stars with different masses. As is well known, heavy stars tend to displace lighter ones from binary systems, an effect shown clearly in computations by Hills (1975a) and by Hills and Fullerton (1980). As shown in earlier N-body computations (Aarseth and Lecar, 1975) for systems with a spectrum of masses, it is usually the heaviest stars which end up in a central binary, with other stars in elongated orbits or escaping. Evidently the rate of mass stratification would be increased by the concentration of massive stars in binaries. The net result would be a significant enhancement of core heating, since the detailed computations by Mikkola (1983b) show that the heating rate resulting from binary-binary collisions grows rapidly with increasing stellar mass.

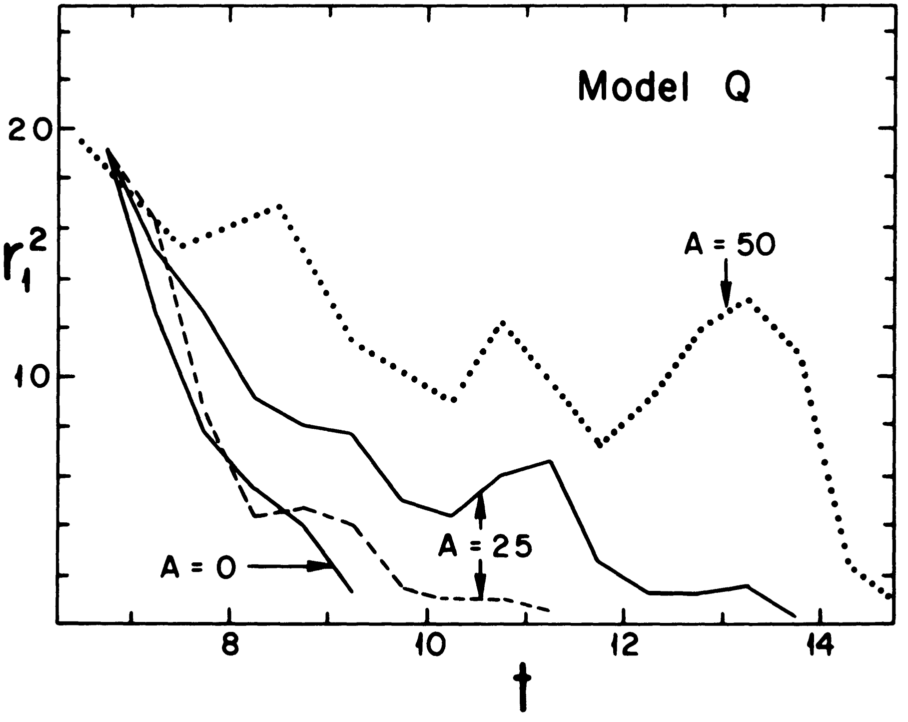

Figure 6. Evolution of a Cluster with 20 Percent of Mass in Binaries. The radius, r_1, containing the inner 2% of the cluster mass is plotted against time; both r_1 and t are in arbitrary units. Different values of A correspond to different coefficients assumed for the rate of energy gain by single stars from binaries. These computations, carried out by Spitzer and Mathieu (1980), show that binaries postpone the collapse by at most a factor two in time since the origin of the cluster.

5. PHENOMENA LATE IN THE COLLAPSE

5.1 Binary formation in three-body encounters.

As the core density increases, three-body encounters become significant and produce new binary stars. Some of these will be soft and will soon be disrupted, with a certain fraction giving up energy and becoming hard binaries. Other newly formed binaries will be hard to begin with. The formation rate, considered early by Agekyan and Anosova (1971), has been re-evaluated by Aarseth and Heggie (1976), who find that if N_c is the number of stars in a relatively uniform core, the rate of formation of hard binaries per unit time is given by

$$\frac{dN_b}{dt} = \frac{K}{N_c \ell n(0.4N) t_r(0)} , \tag{17}$$

where $t_r(0)$ is the central relaxation time and K is a constant of order 10. Since we have seen that ρ_c changes on a time scale of several hundred $t_r(0)$, it is clear that a large fraction of stars will form binaries if N_c is as low as 10^2. This tendency is evident in direct dynamical computations of cluster evolution for systems with as few as 100 stars. Binaries formed in this way (which are sometimes referred to as "three-body binaries") are not, in general, very hard, and interact strongly with single stars and other binaries, becoming harder and heating the system.

The evolution of a collapsing core, subject to the formation of three-body binaries, has been considered in detail by Stodółkiewicz 1983), using a Monte Carlo method of Henon's type with extensive modifications; in particular the time step is varied with distance from the center, as in the work by Marchant and Shapiro (1980), and interactions between binaries and single stars are followed in numerical three-body integrations, with a number of input parameters chosen at random. Binary-binary encounters are treated more crudely; following Spitzer and Mathieu (1980), such encounters are assumed to disrupt the binary of lower binding energy. A distribution of stellar masses is assumed, following the familiar Salpeter formula. As in the models by Spitzer and Mathieu (1980), almost all the interactions involving energy generation from binaries occur in the very central regions, where the density is highest.

The models computed in this way show that the core collapse ceases when the central density has increased by a factor of 10^5 to 10^6 over its value in the initial state, taken to be an n = 5 polytrope (Plummer's model). The system then re-expands. It is not obvious why this result differs qualitatively from that found by Spitzer and Mathieu (1980), who found that core collapse was postponed but not averted. Possibly the three-body binaries were sufficiently softer in Stodolkiewicz's model so that a greater function of the energy released was retained in the central core. The presence of relatively more massive binaries, as compared to the individual stars, might conceivably tend in this same direction. Further computations of such models, perhaps using different computer codes as an overall check, would be important.

In this connection, the results obtained by Bettwieser and Sugimoto (1984) are of interest. Their calculations are somewhat approximate, based on moment equations, following Larson (1970), and assume the thermal conductivity derived by Lynden-Bell and Eggleton (1980)--see equation (7). The energy generation rate per unit mass is assumed proportional to $C\rho^k v_m^{-1}$, where v_m is the velocity dispersion, ρ is the local density, and k is taken to be 1 or 2. Since the energy released is assumed to be absorbed locally, it is not surprising that the collapse is halted, again when the density has increased by about 10^6 above its initial value. More unexpected is that after the subsequent expansion, additional collapses and subsequent re-expansions are observed at intervals of a few times the half-mass relaxation time, t_{rh}. This phenomenon occurs only if the constant C in the assumed energy generation formula is less than some critical value. Consideration of the actual cluster population, including the distribution of binaries of different mass at different times, would be required to verify the reality of these non-linear oscillations, with repeated fluctuations of central density by factors as large as 10^6.

A much more realistic analysis, again giving oscillations but of much smaller amplitude, has been carried through by McMillan and Lightman (1984a,b), who used a direct N-body dynamical integration for the core ($N_c < 100$), combined with a statistical approach (equivalent to the Fokker-Planck equation) for the outer regions. All stars were assumed to have the same mass, and no binaries were assumed present initially. This pioneering but complex program indicated that core collapse followed the similarity solution of Lynden-Bell and Eggleton (1980) until the number, N_c, of stars in the core had fallen to about 25, at which point the formation and hardening of a central binary liberated energy which reversed the core collapse. When the density had decreased to about a fourth of its peak value, the central binary was ejected from the core, recoiling from a strong interaction with a single star. The actual mechanism for this process was the formation of a triple star system which subsequently disrupted; as shown by Heggie (1975), input of energy from a binary to a single star usually occurs in this way. Subsequently the core collapsed again. These successive oscillations had relatively little effect on the outer regions of the cluster.

When a realistic mass spectrum is considered, the central core may be predominantly composed of compact, relatively heavy but almost invisible objects such as white dwarfs, neutron stars and black holes. If a few heavy black holes are present, they are the most likely components of the central, most tightly bound binary. The gravitational potential resulting from a core of faint objects surrounding a central binary could produce a rather minor central peak in the observed visual surface brightness of the cluster, an effect pointed out by Illingworth and King (1977) for a core of neutron stars and by Larson (1984) for a core of black holes.

While there are uncertainties and approximations associated with these various models, there seems very little doubt that binaries formed by three-body encounters can terminate and even reverse the process of core collapse when N is very small.

5.2 Tidal captures and collisions

Because of the finite size of a star, other complex processes become important when the high density of the collapsing core increases the rate of all two-body encounters. Fabian, Pringle and Rees (1975) pointed out that tidal dissipation during a relatively close encounter between two stars could lead to their mutual capture, producing a binary system. A binary formed in this way is sometimes called a "tidal-capture binary". Such a binary is initially in a very eccentric orbit, with a periastron separation equal to the initial distance of closest approach, while the apastron separation is determined by the excess of the energy dissipated over the initial kinetic energy. Successive periastron passages will dissipate additional energy, tending to produce a nearly circular orbit, with a radius equal to twice the initial periastron separation (provided that stellar rotation makes no significant contribution to the total angular momentum).

The least distance of closest approach, R_m, for capture has been computed in detail by Press and Teukolsky (1977) for main-sequence stars each of mass M_* and radius R_*. They find that R_m/R_* decreases from 5 to 2 as the relative velocity increases from 1 to 100 $[(M_*/M_\odot) \times (R_\odot/R_*)]^{1/2}$ km/s; for globular clusters 3 is a typical value. The corresponding impact parameter is much greater, and the cross-section varies as R_m. It follows that the cross-section for an actual collision ($R_m \leq 2R_*$) is comparable with that for tidal capture (R_m/R_* between 2 and 3); the effects of such stellar collisions are treated below.

A detailed review and analysis of tidal capture and of energy losses associated with somewhat more distant collisions has been given by Ozernoy and Dokuchaev (1982). Because of their close separation, tidal-capture binaries are relatively very hard and will produce relatively little heating in interaction with other stars, except under rather extreme conditions. Their formation represents a loss of energy initially, tending to accelerate core collapse; their subsequent concentration in the core, as a result of mass stratification, releases gravitational energy which provides a source of heating for other components of the cluster.

Such tidal-capture binaries will evolve into harder systems as a result of gravitational radiation. Mass flow between the close components will also affect the evolution of these hard binaries. Occasional encounters with field stars will accelerate the hardening; also, actual collisions will much enhance mass transfer to any compact component, increasing the X-ray flux (Krolik, Meiksin and Joss, 1984).

The overall effect of tidal dissipation and tidal capture on cluster evolution has not yet been explored with detailed realistic models. Several papers have analyzed this problem, treating the core with a simple homology model. In the paper by Inagaki (1984), the gravothermal instability is taken into account; the calculations indicate that the dominant effect produced by tidal-capture binaries is the mass stratification instability, resulting from their twofold mass increase over single stars. Three-body binaries are found to dominate in only a small fraction of clusters, but this result is uncertain. It is not obvious what terminates the collapse of a core composed of hard tidal-capture binaries. Strong encounters between such binaries are likely to eject all the reaction products from the cluster. Weak encounters between three hard binaries might be expected to form complex multiple-star systems, whose subsequent evolution may provide a substantial source of energy for the cluster as a whole.

A more precise theory of these effects involving tidal capture binaries would evidently be important, but would clearly involve a number of complications. Actual collisions between field stars should also be considered since, as noted above, the rate of such collisions is about the same, or even somewhat greater, than the rate of tidal capture. Physical collisions will produce a variety of effects, starting either with a pair of stars in a common envelope or a single star formed by coalescence. Subsequent evolutionary processes could lead, perhaps, to very massive stars, to supernovae or to black holes. Such effects are discussed in the review by Lightman and Shapiro (1978). While not much research has been carried out in this area during more recent years, actual collisions are clearly of potential importance in the late evolution of a globular cluster.

These various processes will clearly be affected by the nature of the stars which have concentrated in the core. For example, if the core is made up predominantly of compact objects, as suggested above (§5.1), the rate of tidal capture will be greatly reduced. As McMillan and Lightman suggest (1984b), it is possible that because of this effect tidal-capture binaries may have no significant evolutionary importance. Regardless of their effect in stellar evolution, the formation of binaries by tidal capture seems the most likely explanation for the observed presence of strong X-ray sources in a few clusters with relatively dense cores (Lightman and Grindlay, 1982, Krolik, 1984). Evidently the collapse of globular cluster cores is still a field with many challenging problems.

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DISCUSSION

COHN: Is there no question on the basis of the calculations you reviewed that binaries must reverse core collapse?

SPITZER: There seems very little question, from the calculation I reviewed, that the formation of binaries by three-body processes, and the subsequent hardening of these binaries, can reverse core collapse.

GOODMAN: Do you see a conflict between the theoretical prediction that cluster cores should collapse rapidly and the observation that very few cores appear to have collapsed?

SPITZER: The number of cluster cores that should have collapsed, according to the computed values of t_{rh}, is not entirely certain, but could be appreciably less than half. The appearance of a cluster after collapse is also uncertain. A dense central core could still remain, but if this core is composed of neutron stars and black holes, it might produce only small effects on the distribution of visible stars. It is also possible that post-collapse expansion eliminates the relatively dense central core.

SUGIMOTO: Bettwieser and I think that there exist gravothermal oscillations of the core. When the central density is high, i.e., when the core has a cusp, the timescale of oscillation is much shorter than at the stage of less concentrated core. Therefore, in a probabilistic sense, comparable or less members of globular clusters are in the collapsed phase. This is conclusive as physics. In order to give a numerical value, however, in astronomical situations we have to take account of other effects such as escape of binaries from the system, segregation of binaries etc. Thus the detailed value in astronomical situations is an open question.

GRINDLAY: Can you elaborate further on your statement that tidal-capture binaries will not be a significant source of cluster heating and contribute to halting core collapse? I would think that the process of formation of tidal-capture binaries would be enhanced during core collapse and would remove significant binding energy from the core.

SPITZER: My statement was meant to imply that binaries formed by tidal capture would not, after their formation, be a significant source of heating for the cluster. This is because such binaries are very hard as a result of their close separation; hence, the cross-section for their close encounter with a single star is rather small, and, in addition, if a close encounter does occur, so much energy is likely to be transferred to the star that this will be ejected from the cluster and probably the binary also. On the other hand, the process of tidal capture is a dissipative one, reducing the kinetic energy of the cluster. Additional dissipation arises from close encounters between two stars which lose some of their kinetic energy but not enough to be captured. These dissipative processes tend to accelerate core collapse.

BETTWIESER: Can thermodynamic concepts be applied to N-body systems? I remember the calculations of R. Miller in 1972 showing that energy exchange between N-body systems and a foreign body can be quite the opposite you expect from thermodynamics.

SPITZER: While the conventional application of thermodynamics to N-body systems certainly involves some approximations, I believe that in most situations useful results can be obtained with this technique.