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Less-is-more effects without the recognition heuristic

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Abstract

Inferences consistent with "recognition-based" decision-making may be drawn for various reasons other than recognition alone. We demonstrate that, for 2-alternative forced-choice decision tasks, less-is-more effects (reduced performance with additional learning) are not restricted to recognition-based inference but can also be seen in circumstances where inference is knowledge-based but item knowledge is limited. One reason why such effects may not be observed more widely is the dependence of the effect on specific values for the validity of recognition and knowledge cues. We show that both recognition and knowledge validity may vary as a function of the number of items recognized. The implications of these findings for the special nature of recognition information, and for the investigation of recognition-based inference, are discussed.

Keywords: less-is-more effect, fast and frugal judgment.

1 Introduction

Investigations of the recognition heuristic (RH) typically involve participants making judgments about items about which they have limited knowledge, such as the relative sizes of cities in the USA. For example, a participant might be presented with the two cities *San Diego* and *San Antonio* and asked which is bigger. In the classic work of Goldstein and Gigerenzer (2002), it is assumed that the participant will guess if they recognize neither of the items, they will use whatever additional knowledge is available to make a decision if they recognize both of the items and, crucially, if they recognize only one of the items, they will choose this item as the larger without consulting any other cues or searching for further information (the *Recognition Heuristic* or *RH*). This is because items of larger size are more likely to be encountered, hence more likely to be recognized (the recognition-magnitude correlation). Recognizing one of the two items is thus a useful cue for choosing the recognized item. If both items are recognized, however, additional knowledge is needed to make the decision and such additional knowledge may be very limited. Recognition-driven inference can give rise to the *less-is-more effect (LiME)*, whereby individuals who recognize many of the items often perform worse than individuals who recognize fewer of the items (Goldstein & Gigerenzer, 2002).

The LiME is a counter-intuitive finding, predicted to occur under given circumstances if the RH is applied (Goldstein & Gigerenzer, 2002; McCloy, Beaman & Smith, 2008). The counter-intuitive nature of the LiME prediction allows for a strong test of the RH and has been used as a rhetoric device to promote the heuristic (Borges, Goldstein, Ortmann & Gigerenzer, 1999; Gigerenzer, 2007; Schooler & Hertwig, 2005). Evidence for the LiME has also been observed empirically (Frosch, Beaman & McCloy, 2007; Goldstein & Gigerenzer, 2002; Reimer & Katsikopoulos, 2004) but, counter to this, failures to observe the effect have also been cited in attempts to refute the RH (e.g., Boyd, 2001; Dougherty, Franco-Watkins & Thomas, 2008; Pohl, 2006). At least as originally introduced, a LiME is a mathematical necessity (given certain assumptions) rather than a proof of recognition-based inference. Nevertheless, the consensus appears to be that the observation of a LiME implies that the recognition heuristic was employed (Pachur, Mata & Schooler, 2009), and that the use of knowledge will dilute or reduce the size of the LiME (e.g., Hilbig, Erdfelder & Pohl, 2010). Here we explore whether LiMEs are also mathematical necessities if those assumptions are altered somewhat — specifically if inference is no longer recognition-based but instead makes reference to some form of knowledge.

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LiMEs need not appear only when the RH is studied in isolation. They are also predicted by formal models of knowledge-based inference if those models exploit the recognition principle. Gigerenzer and Goldstein (1996) used the appearance of the effect as part of their comparison of five integration algorithms with the Take The Best (TTB) algorithm (Gigerenzer & Goldstein, 1996; pp. 656–661). TTB and all of the integration algorithms were implemented such that, in each case, recognition was used as a cue if only one item was recognized (p. 657). Unsurprisingly, all six algorithms produced a non-monotonic relationship between recognition and correct inference (Gigerenzer & Goldstein, 1996, Figure 6). However, as we will demonstrate, LiMEs can be produced by knowledge-based decision-making processes which use neither recognition-driven inference nor the related speed-of-retrieval inference that Schooler and Hertwig (2005) have shown produces similar advantageous effects for moderate over lesser forgetting rates. The first aim of this paper is to prove by analytical means that LiMEs can be produced by knowledge-based decision-rules. They are not unique to recognition-driven inference and cannot therefore be viewed as providing unconditional support for this hypothesis. Our second aim is to examine, using the basic framework developed, how both recognition and knowledge validities vary as a function both of the correlation between recognition and magnitude and the number of items recognized.

1.1 Moderators of the recognition-magnitude correlation.

In Goldstein and Gigerenzer's original (2002) formulation of the RH, additional knowledge is used only as a tie-breaker to decide between two recognized items. When a single item is recognized, inference is purely recognition-driven. This aspect has aroused much interest and has proven controversial (Gigerenzer & Brighton, 2009; Hilbig & Pohl, 2008; Hilbig, Pohl & Bröder, 2009; Newell & Fernandez, 2006; Newell & Shanks 2004; Pachur & Hertwig, 2006; Pachur, Bröder & Marewski, 2008; Pohl, 2006; Richter & Späth, 2006). In an alternative formulation, limited knowledge can be used even when only one item is recognized. This alternative formulation is worth examining because a number of accounts, generally favorable to the RH, have seemingly relaxed the criteria for its application. For example, Volz et al. (2006, p. 1935) conclude, on the basis of neuroimaging evidence that, "the processes underlying RH-based decisions go beyond simply choosing the recognized alternative." Additionally, the discrimination index proposed by Hilbig and Pohl (2008) led them to conclude that a substantial number of recognition-consistent choices were informed by further information other than recognition alone.

The relationship (whether positive or negative) between the recognition of an item and its magnitude is clearly central to the RH. It works because, in the tasks to which this approach has been successfully applied, larger items are more prominent (more newsworthy, more important, etc.) than smaller items and this leads to larger items being more likely to be recognized. However, if the question related to the relative size of pairs of birds and the single recognized item was a house-sparrow, the Recognized → Larger inference makes much less sense than when the same options are presented but the question relates to the relative population size of the two birds.[1] This highlights the fact that recognition actually correlates with prominence, which may not itself correlate with all forms of magnitude *per se*. The prominence-recognition correlation also may not hold — or at least, it may vary in size — if the items experienced as prominent vary between individuals. One potential moderating factor is sampling bias. The newspaper example given by Goldstein and Gigerenzer (2002) is a good case. In this example, it is suggested that a city may be recognized if it is frequently mentioned in a newspaper, and that a larger city is more likely to be so mentioned. The individual receiving the newspaper is implicitly assumed to be a fairly passive processor of the information contained within the newspaper. No consideration is given to the potential difference between an individual who actively seeks out a newspaper and one who does not, or to potential differences between choice of reading matter. These may have very different content (e.g., the *New York Review of Books* versus the *National Enquirer*), and each of which might be sought out, or passively encountered, to different degrees by different individuals or groups of individuals. Calculating the recognizability of a city from the relative frequency with which it is mentioned in any one publication may be misleading if applied to a group of individuals who disproportionately sample from another publication or from different sections of the same publication (e.g., the sporting pages versus the "style" section). Overall, biased sampling of this type may be good or bad for the performance of the heuristic, depending on whether a disproportionate number of "large" items are sampled, which would enhance the validity of recognition (e.g., a soccer fan will recognize more towns with premier league soccer teams) or whether sufficient "small" items are sampled to reduce the magnitude-recognition correlation (e.g., a golf fan will recognize more towns with famous golf courses, but such towns do not on the whole tend to be large in size).

A basic premise in what follows is that, for any given individual, there are several subgroups of items which

[1] Thanks to Pete Bibby for this example

the individual is able to recognize and about which they may also have partial knowledge. This is particularly likely if they are local to the individual in some way or if they form part of a set of items of special interest to that individual. For example, the third author has observed anecdotally that the only German citizens of her acquaintance who reliably recognize the Yorkshire city of Leeds are football fans. Coincidentally, British citizens of her acquaintance show the same pattern for the Nordrhein-Westfalen city of Leverkusen. Hence, anecdotally at least, it appears that football fans and those uninterested in the game may have differential access to subsets of European cities. Special access to information regarding subgroups may also vary with the choice domain, a point which is easily confirmed using existing empirical data. For example, by-item analysis of data taken from an experiment by McCloy, Beaman, Frosch and Goddard (2010), in which a group of 40 participants were asked to indicate which of a group of famous individuals they recognized, found no significant effect of the gender of the participant on the overall recognition rate, $F(1, 43) = 2.3$, $p = .14$ but a significant effect of the reasons why the individuals rose to fame (as either sports personalities, fashion and show-business professionals, rock stars or business people), $F(3, 43) = 13.48$, $p < .001$, and a significant interaction between this factor and the gender of the participant, $F(3, 43) = 13.44$, $p < .001$. Males recognized, on average, sports personalities 78% of the time (females = 55%) and rock stars 75% of the time (females = 66%). In contrast, females recognized fashion and show-business professionals 57% of the time (males = 33%) and the two genders were both poor at recognizing business people, males = 16%, females = 11%. Thus, gender is a factor which provides, or at least contributes to, differential access to different subsets of rich and famous people. In what follows, we consider similar situations where, for an individual within the environment, there is no simple correlation between recognition and magnitude because subsets of the items are prominent for reasons unconnected to magnitude (e.g., the age, gender or special interests of the individual).

2 Study 1: Models predicting the LiME

To formally examine the appearance of LiMEs, we suppose a pool of N items, split into several subsets A, B, C, ... Within each subset the participant is able to recognize u v, w, ... items, respectively. In a typical test of recognition-driven inference, the experimenter selects items quasi-randomly from the pool. Since the constraints on the experimenter are unknown, a random selection from N is assumed. In the basic case, pairs of items are chosen, and the participant's task is to say which is larger. For purposes of exposition, we restrict attention to situations with just three subsets. The models can easily be extended to other cases (e.g., the participant is asked to choose between more than two items [Frosch et al., 2007; McCloy et al., 2008] and/or the pool is split into more than three subsets).

2.1 The basic framework

Suppose that, when presented with a two-alternative forced choice task, an individual recognizes from among the two alternatives i items from subset A, j items from subset B, and k items from subset C. On a given trial, only two items are presented, so i, j, k range from 0 to 2, with $i + j + k \leq 2$. That is, the number of items recognized on any trial could vary from 0–2 for any of the three subsets but the total number recognized obviously cannot exceed the two items presented. p_{ijk} is the probability that this event occurs. (For example, if $i = 1$, $j = 1$, and $k = 0$, p_{ijk} is the probability of recognizing one item from subset A and one from subset B.) p_{ijk} is obviously dependent on how many items the participant can recognize in each of the subsets, but is independent of the decision rule adopted. α_{ijk} is the probability of success, given the recognition of i, j and k items from their respective subsets. This parameter is dependent on the decision rule the participant adopts and is the only thing that distinguishes the models we consider. The overall probability of success $P(u, v, w)$ is given by:

$$P(u, v, w) = \Sigma_{ijk}\alpha_{ijk}p_{ijk} \tag{1}$$

Having outlined the basic framework, we can present the models. The RH model requires little introduction, the alternative against which it is to be compared we refer to as LINDA (Limited INformation and Differential Access).

2.1.1 The Recognition Heuristic (RH) model

The distinguishing feature of the RH model is that the participant chooses the recognized item when only one item is recognized. So $\alpha_{000} = 0.5$ (no item recognized, pure guess); α_{100}, α_{010}, and α_{001} reflect the success of the recognition heuristic (they should be greater than chance if the recognition heuristic has some validity, and should be quite large for the clearest LiMEs); α_{110}, α_{101}, α_{011}, α_{200}, α_{020}, α_{002} reflect use of knowledge (two items are recognized, so additional knowledge is used to discriminate them; LiMEs should be clearest if these knowledge probabilities are close to chance).

2.1.2 The Limited INformation and Differential Availability (LINDA) model

As the name implies, this model requires two basic assumptions:

1. *The limited information assumption*. For each recognized item, the individual has relevant but limited information about its size (e.g. that the size is above the population median). For the sake of simplicity, this is presented as if it were criterion-based knowledge rather than inference from cues. However, recent data show little impact on the use (or non-use) of recognition-based inference when criterion knowledge is available (Hilbig et al., 2009). Reanalysis of data by Hilbig et al. (2009) also shows that — at least for the domain they examined (the size of cities in Belgium) — participants showed something approximating median knowledge. Hilbig et al. recorded participants' estimates of the populations of each of the cities they were asked to consider so it is possible to calculate, per participant, the probability that they correctly judged whether the city in question was above or below the sample median.[2] This information may not be totally reliable, and we use parameters p_A, p_B, ... for the probabilities that information regarding a recognized item belonging to subsets A, B, ... is accurate. From Hilbig et al.'s data, participants were judging an item to be the correct side of the sample median on 69% of occasions on average (s.d. = 13%). In what follows, for the most part, we assume $p_A = p_B = p_C = 1$ as this is the simplest case but varying this parameter (for example changing it from 1 to .7, or 70% of cases correct) only alters the magnitude of the effects observed and does not affect the general conclusions.

2. *The differential availability assumption*. Some subsets are more accessible than others so that, for a given individual, more items may be recognizable within one subset than within another. Note that this assumption does not necessarily imply no correlation between magnitude and recognition, but it allows the extent of this correlation to be manipulated by varying the relative recognizability of the subsets.

The limited information assumption assumes that there is, at the least, some information available at the time of decision-making against which to evaluate the usefulness of choosing the recognized item in any given case. The reliability of this information may also vary. Either the information may be incorrect or (potentially) it may be misapplied in some way. For simplicity, these possibilities are both reflected in the value of a single parameter, as noted in assumption 1. The differential availability assumption states merely that, within any set, the items within some subsets are more or less recognizable than the items within some other subset.

2.1.3 Numerical example

For the LINDA model described above, consider the situation where individuals have what we will term *median knowledge* of items from pool N, i.e. they accurately know whether each recognized item is above or below median. Subset A includes items in the top quartile of the size distribution, subset B includes items in the second highest quartile of the size distribution, and subset C contains all the remaining items. The Appendix gives the derivations of explicit expressions for all the terms in Equation (1). In the first example, it is assumed for purposes of exposition that median knowledge is perfect, i.e., that the median knowledge about a recognized item is accurate with no chance of error ($p_A = p_B = p_C = 1$). This assumption is relaxed in later examples.

In order to formally compare the RH model with the LINDA model, the models are designed to perform equally well when all items are recognized. In the current simple example, where all items are recognized, u and v are the number of items recognized from subsets A and B, which constitute the top two quartiles of the distribution, respectively, and w is the number of remaining recognized items (subset C), so if $u = v = 25$ and $w = 50$, the total number recognized, then $n = N = 100$. The probabilities of a correct inference when recognizing 2 items in any of the possible combinations that may occur (e.g., 2 from u, or 1 from u and 1 from v, and so on) are given by the equations presented in sections 1 and 3 of the Appendix. LINDA's performance with full recognition is the sum of these probabilities, which works out as 0.7525, so in the RH model probabilities of success when both presented items are recognized were also set to 0.7525. The size of the pool from which the test items are drawn is set at 100 but the same pattern of results is obtained for all large values of N. The key prediction is the relation between the proportion of correct decisions (P in equation (1)) and n, the number of items in the pool the participant can recognize.

To examine how these models interact with the recognition of items from different subsets, consider the cases where there is a close link between the recognition of items and the subsets from which they are drawn. The notation ABC means that items from subset A are all more recognizable than the items from subset B, which in turn are all more recognizable than the items from subset C. This strict ordering of recognition is obviously unrealistic

[2] Thanks to Ben Hilbig for making these data available.

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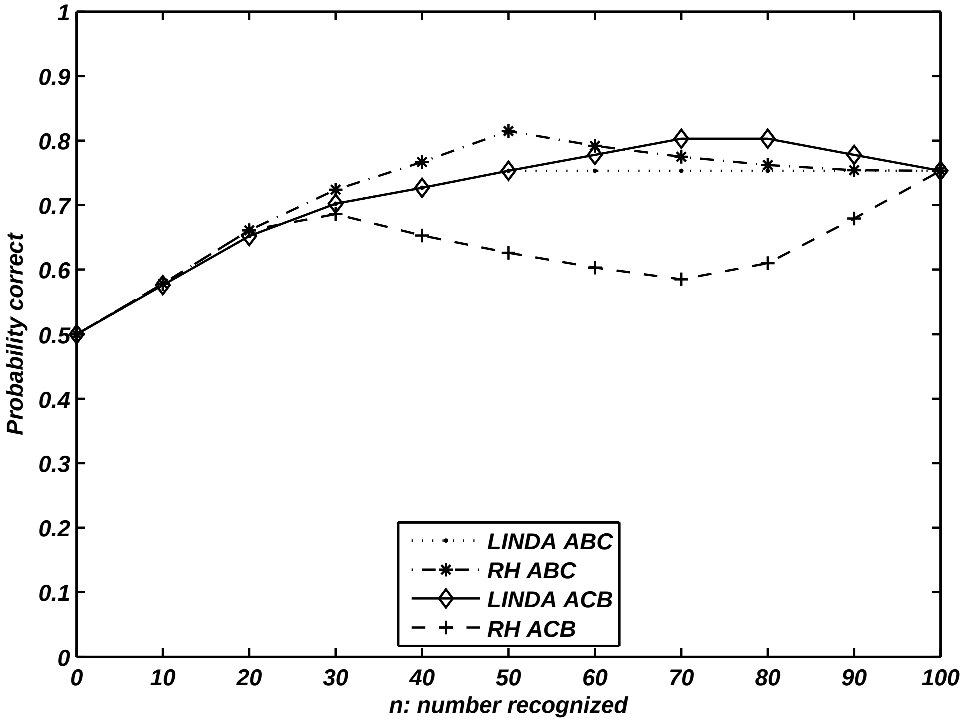
Figure 1: Proportion correct using LINDA and the RH for different orderings of subsets (and hence different recognition-magnitude correlations). ABC ordering is equivalent to a recognition-magnitude correlation of $\rho = .919$ and ACB ordering is equivalent to $\rho = .306$.



but is useful to demonstrate relations between recognition and the properties of the two models and could easily be relaxed to allow some overlap between the recognition of items from different subsets. If this constraint is enforced, and the equations given in the Appendix calculated accordingly, then the results shown in Figure 1 are obtained.

Figure 1 shows the performance of LINDA and the RH model for two different magnitude-recognition orderings: ABC (items in the top quartile of the size distribution are most recognizable and items below median are least recognizable) and ACB (items in the top quartile are most recognizable, then items from below the median and finally items from the second quartile). ABC ordering corresponds to a strong magnitude-recognition correlation ($\rho = .919$) and ACB ordering to a smaller, but still positive, correlation between magnitude and recognition ($\rho = .306$). The plausibility of such an ordering of recognition might be queried, but it is fairly easy to generate scenarios in which particularly large items are most recognizable, then particularly small items. For cities, as already mentioned, the possession of a good golf course enhances its recognizability (in the UK: Carnoustie, Lytham St Annes, St Andrews, Sunningdale, Turnberry) but good golf courses are not, for the most part, associated with large cities because of the space they require. The ABC ordering produces effects we would expect from the literature. The RH model, using the recognition heuristic, shows the expected LiME, while the knowledge-based LINDA model shows a monotonic relation between proportion correct and number of recognizable items.

The situation is quite different for the ACB ordering: here it is LINDA that produces an inverted-U shaped function and a LiME. LiMEs therefore cannot necessarily imply use of the recognition heuristic — even given a positive magnitude-recognition correlation — but may occur for other reasons. The inverted-U shaped functions that characterize the LiME indicate that a task becomes more difficult once the number of recognizable items passes a certain level. In the case of the RH model and the ABC ordering, this is because "easy" decisions (select the recognized item when only one item is recognized) are gradually outnumbered by "difficult" decisions (choose between items, both of which have been recognized) as the number of recognizable items increases. In the case of LINDA and the ACB ordering, moderate levels of recognition produce many easy decisions (discriminating a recognized item drawn from subset A from a recognized item drawn from subset C) but the decisions become more difficult when items of intermediate size, from subset B, begin to join the pool of recognizable items as the number of recognizable items increases. If

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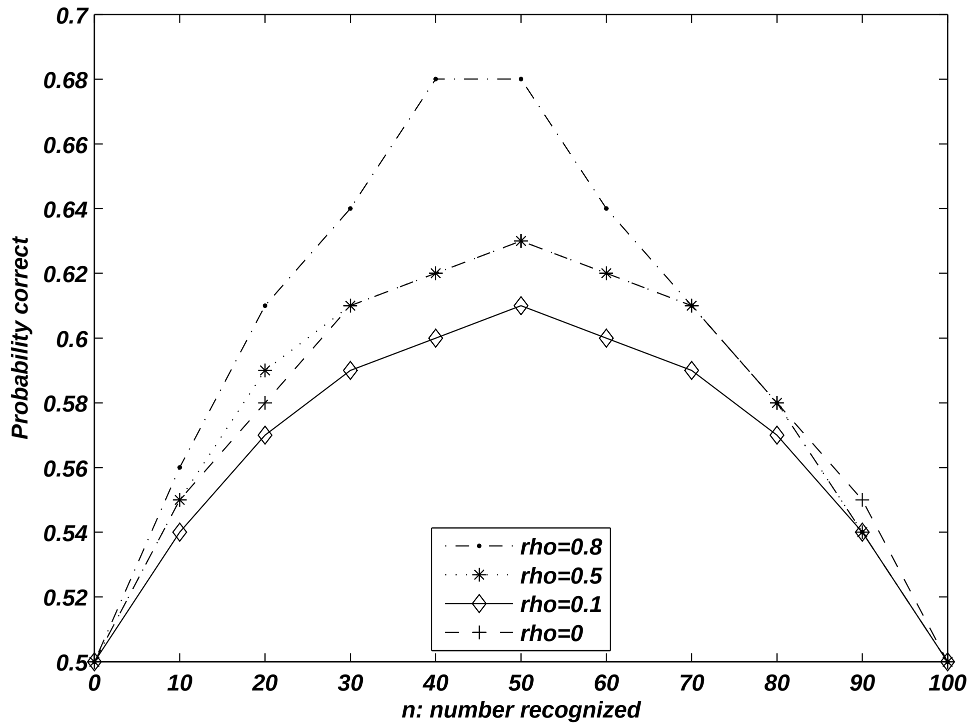
Figure 2: Proportion correct for the LINDA model when discrimination between two recognized items is at chance. The same calculations can be made for the RH but are not given here. A spreadsheet to simulate the RH was produced by McCloy et al. (2008) and can be used for calculating the RH's predictions for situations corresponding to those in this Figure depicted for LINDA. The spreadsheet is available to download from http://www.personal.rdg.ac.uk/~sxs98cpb/philip_beaman.htm although note the calculations in this spreadsheet assume automatic application of the RH, even when recognition is not a good cue.



the size of the LiME is defined as the maximum proportion correct minus the proportion correct when $n = N$ (e.g., McCloy et al., 2008) then the effect size for the RH and for LINDA is similar when LINDA has totally reliable information (for ordering ABC, RH effect size = .06, for ordering ACB, LINDA effect size = .06). The size of the effect is reduced if LINDA's information is less reliable (e.g., if $p_A = p_B = p_C = 0.7$, effect size for ordering ACB = .03) but increases if the assumption is made that LINDA has difficulty with discriminations when both items are recognized.

In calculating LINDA's predictions, we previously assumed no extra difficulty was involved in having to choose between two recognized items, but this assumption might not be realistic: choosing between two recognized items may, in some instances, be extremely difficult. An extreme version of this is shown in Figure 2. Here it is assumed that LINDA makes decisions in the way already outlined when only one item is recognized, but does not have the capacity to make a decision when both items are recognized, and so is obliged to guess. The situation resembles one outlined in Goldstein and Gigerenzer (2002, pp. 84–85) in which German participants were experimentally exposed to the names of US cities without being presented with any further information which might be of use, and is also comparable to Schooler and Hertwig's (2005) ACT-R implementation of the recognition heuristic, which also assumed chance level performance when both items were recognized (Schooler & Hertwig, 2005, p. 614).

Figure 2 shows clear LiMEs also appear for this version of LINDA. Interestingly, unlike the RH model, which requires quite large magnitude-recognition correlations to allow recognition validity to exceed knowledge validity, LINDA shows LiMEs for all values of ρ, although the largest LiMEs occur for the largest values of ρ. No "recognition validity" parameter was built into LINDA *a priori* (although clearly the validity of recognition is to some extent reflected in the values of ρ) so these results are not subject to the criticism that it is trivial to show LiMEs if knowledge validity is set sufficiently low relative to recognition validity (McCloy et al., 2008). Once again, then, a knowledge-based decision model produces LiMEs, and thus — once again – LiMEs are not a unique prediction of the RH model.

2.2 Discussion

Whilst the RH and LINDA give LiMEs in different circumstances, the effects are produced for essentially the same reasons. When relatively few items are recognizable, the task is easier than when many items are recognizable. In the case of the RH model, when an intermediate number of items are recognizable the individual is more frequently confronted with the easy decision of selecting the one item recognized, and this position is reversed when many items are recognizable. For the LINDA model, performance for intermediate levels of recognition is good because the participant is often asked to make the easy discrimination between an item drawn from top quartile (subset A) and an item drawn from the bottom quartiles (subset C). Adding items from the second highest quartile (subset B), makes the task more difficult and leads to a drop in performance. Natural examples of highly recognizable subsets comprised of small items (C) are required to make this analysis plausible. In addition to the examples of cities with famous golf courses already mentioned, there are numerous remote towns famous for being inaccessible (and therefore necessarily small): Alice Springs, Lerwick, Machu Picchu and Spitzbergen, and in other domains, e.g., the population sizes of various animal species, there are animals famous for being endangered (e.g., Giant Panda, Gorilla) which are more immediately recognizable than animal species with sustainable but by no means large populations.

The fluency rule, discussed by Schooler and Hertwig (2005), also produces similar results to LINDA and, once again, for similar reasons. In the context of the fluency rule, the "less" of less-is-more refers to forgetting rates rather than recognition rates as in the RH. In that case, intermediate rates of decay allow for better discrimination between items than low rates of decay (items retrieved more quickly are presumed to be larger). This leads to the only other "knowledge" based LiME of which we are aware. Crucially, however, the fluency rule does not use or require further knowledge beyond the fact of fast retrieval. Thus, although it produces LiMEs of a kind, these are arguably recognition rather than knowledge-driven. Knowledge about the item itself is never consulted, only knowledge pertaining to the act of retrieval or recognition. Regardless of the validity of this argument, our results nevertheless suggest that LiMEs might be both more prevalent, and more difficult to ascribe to a single strategy, than previously assumed. LINDA demonstrates that LiMEs can occur for knowledge-based decisions and also that, when discrimination between two recognized items is sufficiently difficult, these effects can occur regardless of the recognition-magnitude correlation.

2.2.1 Reasons for the elusiveness of less-is-more

The above argument seems to imply that LiMEs should be observed empirically far more readily than seems to be the case. However, whilst the effect has been empirically verified on some occasions (Borges, Goldstein, Ortmann, & Gigerenzer, 1999; Frosch et al., 2007; Goldstein & Gigerenzer, 2002; Reimer & Katsikopoulos, 2004; Snook & Cullen, 2006) it has not been observed universally (Boyd, 2001; Pachur & Biele, 2007; Pohl, 2006). One reason for this may be that LiMEs occur in different situations for different reasons. Whilst it is possible to find a LiME under circumstances where a LINDA-like decision-rule might be operating, such an effect would be easier to discover if the magnitude-recognition correlation was moderate rather than large, and when the information was particularly reliable, or the discrimination between two recognized objects particularly difficult (see Figure 2). Consequently, it would be relatively easy to miss such an effect if the experimental situation was deliberately designed to maximize the magnitude-recognition correlation, as many have been (e.g., Pohl, 2006). There is a clear difference between a model showing a LiME "in principle" when all factors are under control and a LiME appearing in a standard experimental design which may be statistically underpowered to show a small LiME in a noisy environment. One way around this might be to partition subjects into groups based upon how much knowledge they appear to employ to inform nominally "recognition-based" inferences (using e.g., the methods developed by Hilbig and Pohl (2008) or Hilbig et al. (2009)). It might then be possible to examine whether the appearance or size of any LiME is negatively associated with knowledge used (as proponents of the RH might propose) or if the relation is more complicated (as LINDA would predict).

A second and more interesting possibility is that insufficient attention has been paid to some of the parameters that need to be controlled for a situation to arise where LiMEs would be expected. For example, the key prerequisite of the LiME produced by the RH is that recognition validity should exceed knowledge validity. Reliable manipulation of the recognition and knowledge validity parameters can be problematic, however. In Goldstein and Gigerenzer's (2002) account, it is implicit that both recognition and knowledge validity are, or can be, independent of n, the number of recognizable items in the pool of items from which the stimuli are drawn. For example, Figure 2 (p. 79) of their account illustrates the LiME by holding recognition validity constant and varying knowledge validity and number recognized independently (between and within hypothetical individuals, respectively). This is important because n may not be under experimental control, hence *a priori* estimates of recognition and knowledge validities may be misleading. Later in their paper, Goldstein and Gigerenzer (2002, p. 80) acknowledge that, "recognition and knowledge validities usually vary when one individual learns to recognize more and more objects from experience" but they also appear to endorse the view that, if multiple individuals are involved, who recognize different numbers of objects, it is possible that "each individual has roughly the same recognition validity" (Goldstein & Gigerenzer, p. 80). However, in situations where the recognition-magnitude correlation is high, an individual who recognizes only a few items from the pool of items will mostly recognize very large items. Hence, on any given trial, a recognized item for that individual is likely to be larger than the unrecognized item. In contrast, an individual who recognizes more items from the pool will encounter more trials when the single item they recognize is not larger than the unrecognized item.

It thus seems *a priori* unlikely that recognition validity can be independent of n, where n varies among individuals. Similarly, when both items are recognized and individuals are obliged to use their knowledge, an individual who only recognizes a few items from the pool is likely to encounter items of a similarly large magnitude when both are recognized. Such items may be less discriminable than the pairs of items — drawn from a greater range of sizes — encountered by an individual able to recognize many items. Hence it also seems *a priori* unlikely that knowledge validity can be independent of n.

To formally test the specific question of whether recognition and knowledge validities can be independent of n, it is possible to derive values associated with both recognition and knowledge validity and examine the effect of varying n upon these values. First, consider recognition validity. Goldstein and Gigerenzer (2002) present two computer simulations (pp. 80–82) that partially address this by varying recognition validity varied as a function of either n (number recognized) or N (the size of the pool from which the stimuli are taken). Their results, however, are presented only in terms of overall accuracy (the percentage or proportion of correct inferences calculated across all choices, including those informed by knowledge or the result of guesswork) rather than directly examining the effects upon recognition validity itself. Using the previously presented notation, the probability of being correct given that only one of the two presented items is recognized is as follows:

$$\frac{\alpha_{100}p_{100} + \alpha_{010}p_{010} + \alpha_{001}p_{001}}{p_{100} + p_{010} + p_{001}} \qquad (2)$$

The probability expressed in this equation is obviously equivalent to recognition validity and can be calculated for both LINDA and the RH model according to the method outlined in the Appendix. Figure 3 shows recognition probabilities, conditional on recognizing one item of a stimulus pair (i.e., "recognition validity"), for two versions of LINDA (high quality knowledge with $p_A = p_B = p_C = 1$, and low quality knowledge with $p_A = p_B = p_C = 0.7$) and for the RH model. Note that for LINDA, the "recognition validity" represented by these graphs represents only the validity of recognition-consistent inference because LINDA always uses some (albeit limited) knowledge, whereas for the RH model the values so expressed represent the validity of recognition-driven inference. Three correlations (low, medium and high) between recognizability and size were obtained, as previously.

As expected, the RH model's performance when just one of the two items is recognized improves with ρ. This is also true for the high quality knowledge version of LINDA ($p_A = p_B = p_C = 1$) and the same effect is present but in a weaker form for the low quality knowledge version of LINDA ($p_A = p_B = p_C = 0.7$). Crucially, the performance of both models varies with n. These results show formally that observed recognition validity, as assessed from actual performance, can vary according to other aspects of an individual's knowledge. This effect is particularly marked for large ρ. Next, consider knowledge validity. Similarly to recognition validity, the conditional probability of a correct inference given that both items are recognized can be derived and is expressed in our notation as follows:

$$\frac{\alpha_{110}p_{110} + \alpha_{101}p_{101} + \alpha_{011}p_{011}}{p_{110} + p_{101} + p_{011}} \qquad (3)$$

Figure 4 shows probabilities of correct inference, conditional on recognizing both items of a stimulus pair (knowledge validity), for LINDA, varying recognition-magnitude correlations. This is the high quality knowledge with $p_A = p_B = p_C = 1$, a lower quality knowledge version with $p_A = p_B = p_C = 0.7$ produces lower levels of performance overall but almost identical patterns in response to the same variations in n and ρ. Knowledge validity for situations in which the RH is the object of attention is often set at an arbitrary value (e.g., Goldstein & Gigerenzer, 2002; Schooler & Hertwig, 2005) but we would expect it to vary as a function of n and ρ for many knowledge-based heuristics, as it does for LINDA, although the specifics will depend upon the exact nature of the inference rule.

In conclusion, finding LiMEs is dependent not only upon identifying the decision rule and circumstances under which they are expected but also upon accurately estimating — or manipulating — n in order to obtain the recognition- and knowledge-validity parameters required. Given this, it is perhaps less surprising than it initially appeared that such effects, which would appear

Figure 3: Probability correct, given only one of two items are recognized according to recognition (RH) and knowledge-based (LINDA) models. This is equivalent to Goldstein and Gigerenzer's (2002) concept of recognition validity for the RH model and to the validity of recognition-consistent inference for the LINDA model. The x-axis only runs from 10–90 items recognized (out of a possible 100) because the graph plots probability correct given that exactly one of the two presented items is recognized.

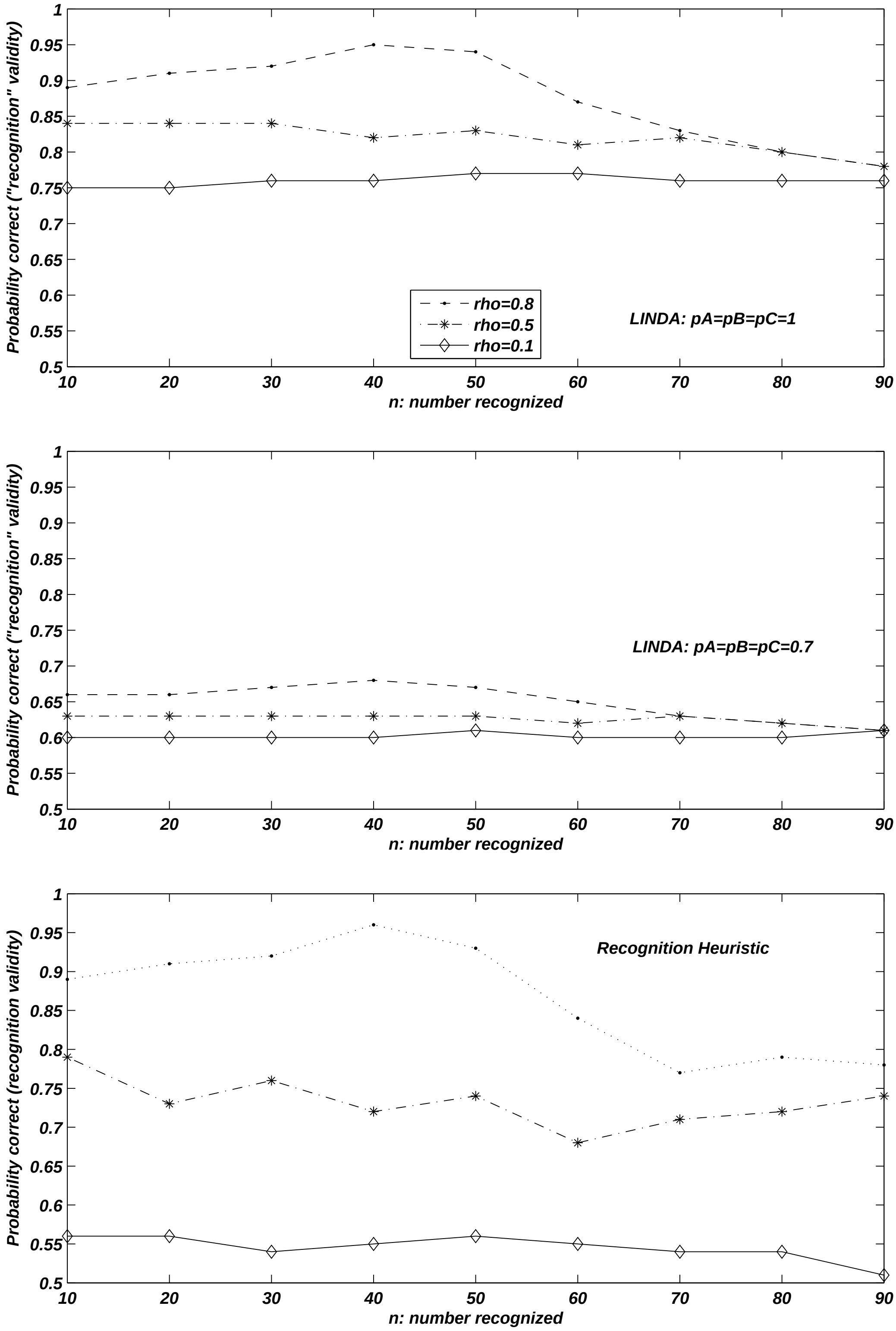


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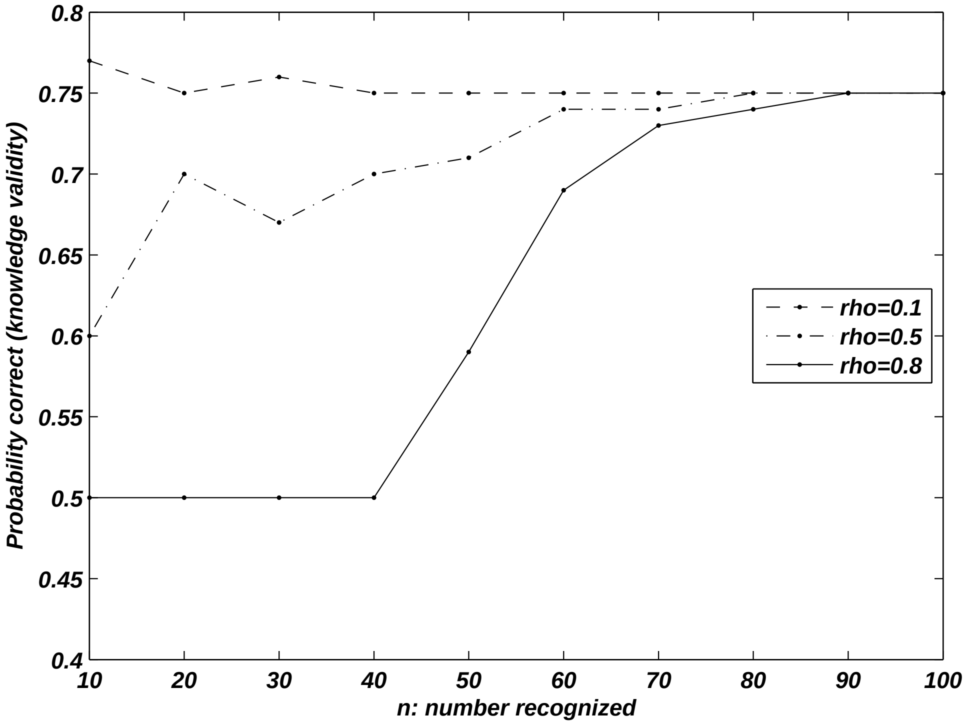
Figure 4: Probability correct, given both items are recognized, for LINDA as a function of n and ρ. This is equivalent to Goldstein and Gigerenzer's (2002) concept of knowledge validity.



to be a mathematical necessity, may sometimes be elusive when investigated empirically.

3 General discussion

The aim of the current paper was not to present unequivocal support for LINDA as in some way a better, more accurate, or more comprehensive model of decision-making than the RH, or to refute the RH as a model (indeed, it has proven far more productive than its underlying simplicity might lead one to believe). We have instead attempted to meet the rather more modest aim of giving an existence-proof that, generally, disentangling recognition from other forms of information is more difficult than it may first appear. In this context, LINDA is best viewed as an analytical tool to enable us to make these arguments in a mathematically rigorous way. The counter-intuitive nature of LiMEs was previously viewed as providing a strong test of recognition-driven inference given that LiMEs are predicted by the RH. This position is weakened by the demonstration that LiMEs can easily be produced using a set of assumptions in which recognition-only inference plays no part.

Criticisms of LiMEs as a means of promoting recognition-driven inference could perhaps be interpreted as an argument against the proposal that inference might sometimes be recognition-driven. It should be emphasized that this was not the intent. Rather, we wished to provide a demonstration that findings which initially seem favorable to such a position may not necessarily be as conclusive as they first appear. The absence, as well as the presence, of LiMEs is also less informative than some have assumed (e.g., Boyd, 2001; Dougherty et al., 2008; Pohl, 2006), and for similar reasons. The recognition validities for both recognition-consistent and recognition-driven inferences are similarly dependent upon variations in n, which is not ordinarily under experimental control. For at least one form of knowledge-based inference (that of LINDA) knowledge validity itself is also a function of n. It is possible therefore that both published demonstrations of LiMEs and published failures to obtain such an effect employed different *de facto* recognition and knowledge validities than those assumed *a priori*. A positive contribution therefore is to suggest that future studies along these lines will need to take such factors into account.

Finally, LINDA can be applied either in tandem or in opposition to the RH. For example, the rule "Apply knowledge (e.g., LINDA) if both items are recognized and apply the RH if only one item is recognized" is standard procedure for many heuristics (e.g., Gigerenzer & Goldstein, 1996, examined six different procedures that made use of the recognition principle when knowledge failed and only one item was recognized). However, "Apply LINDA whenever possible but if LINDA does not provide usable information for this item, apply the RH" is also a valid strategy and one which might prove superior if LINDA is particularly reliable. This latter statement reduces to the assertion that a minimal level of confirmation or refutation will be sought when only one item is recognized and that "mere" recognition will be employed only if and when this minimal test fails to produce usable knowledge. This assertion is consistent with recent data by Hilbig and Pohl (2008).

In the current formulation, LINDA always has access to median knowledge for the recognized items (though this information may not always be correct). Other LINDA-like models could be developed where some recognized items may not have median knowledge associated with them, although we do not go into detail about such items here. The key difference between LINDA and the recognition heuristic is that sometimes LINDA recognizes items which it believes are below median. This enables it to guess correctly, in situations where only one item is recognized, that the recognized item is the smaller of the pair. In contrast, provided the magnitude-recognition correlation is positive, the RH always guesses that the recognized item is larger (the converse also applies: where the magnitude-recognition correlation is negative, the RH will always guess that the recognized item is smaller whereas LINDA will sometimes know better). In circumstances where LINDA believes all the items it recognizes are above median, LINDA and the RH make identical predictions. There is nothing magical about using median knowledge in our modeling, it is simply a tractable way of characterizing limited information. Any model that has the property that it knows that some of the items it recognizes are small, but in general has very limited information, is likely to behave in a LINDA-like manner.

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Appendix

1. Derivation of the values of p_{ijk} in Equation (1): A total sample of N = 100 items is assumed, of which n are recognized. n is systematically varied between 0–100 in all the studies reported here.

u, v and w are the numbers of items recognized from each of the subsets A (comprising items only from the top quartile), B (second quartile) and C (below median). These can be used to calculate n.

The probabilities associated with recognizing 0, 1 or 2 items from u, v and w on any given trial (p_{ijk}) can then be calculated as follows:

Probabilities associated with recognizing none of the items from u, v or w:

$$\begin{aligned} p_{000} &= \frac{N-u-v-w}{N} \cdot \frac{N-u-v-w-1}{N-1} \\ &= \frac{(N-u-v-w)(N-u-v-w-1)}{N(N-1)} \end{aligned}$$

Probabilities associated with the recognition of only one item:

$$p_{100} = \frac{2u}{N} \cdot \frac{N-u-v-w}{N-1} = \frac{2u(N-u-v-w)}{N(N-1)}$$

This is the probability that only one of the two items is recognized and it is in the top quartile (a member of u).

Probabilities that the item recognized is from the second quartile, or is below the median, and that the other item is not recognized can be calculated by substituting v or w, respectively, for u in the first term, giving:

$$p_{010} = \frac{2v(N-u-v-w)}{N(N-1)}$$

$$p_{001} = \frac{2w(N-u-v-w)}{N(N-1)}$$

Probabilities associated with the recognition of both items:

$$p_{110} = \frac{2uv}{N(N-1)}$$

(for u and v, so one item is in the top quartile and one item is in the second quartile)

$$p_{101} = \frac{2uw}{N(N-1)}$$

$$p_{011} = \frac{2vw}{N(N-1)}$$

(as above, substituting v and w where appropriate)

$$p_{200} = \frac{u(u-1)}{N(N-1)}$$

(where both items are in the top quartile, both are members of u)

$$p_{020} = \frac{v(v-1)}{N(N-1)}$$

$$p_{002} = \frac{w(w-1)}{N(N-1)}$$

(as above, substituting v and w where appropriate).

2. Parameters for the Recognition Heuristic model. These represent the calculated probabilities of success associated with recognizing 0, 1 or 2 items where the appropriate probabilities of recognizing 0, 1 or 2 items are given by the equations calculated in section 1 of this appendix. Overall performance of each of the strategies (the RH and LINDA) is then given by equation (1)

Recognize none:
$\alpha_{000} = 0.5$ (Chance)

Recognize one item (which happens to be in the top quartile, i.e., a member of u):

$$\begin{aligned} \alpha_{100} &= 0.5 \cdot \frac{0.25N-u}{N-u-v-w} + \frac{0.75N-v-w}{N-u-v-w} \\ &= \frac{0.875N-0.5u-v-w}{N-u-v-w} \end{aligned}$$

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For recognition of members of v and w, the chances of success are similarly:

$$\alpha_{010} = 0 + 0.5 \cdot \frac{0.25N - v}{N-u-v-w} + \frac{0.5N - w}{N-u-v-w} = \frac{0.625N - u - v - w}{N-u-v-w}$$

The recognized item is in the second quartile.

$$\alpha_{001} = 0 + 0.5\frac{0.5N - w}{N-u-v-w} = \frac{0.25N - 0.5w}{N-u-v-w}$$

The recognized item is below median.

For all these cases which involve recognition of both items

$\alpha_{110} = \alpha_{101} = \alpha_{011} = \alpha_{200} = \alpha_{020} = \alpha_{002}$

It is assumed knowledge can be used with a certain probability of success. This probability is chosen to make the LINDA and RH models "equivalent" in our examples, in the sense that they both produce the same probability of success when all items are recognized.

3. Parameters for the LINDA model. These represent the calculated probabilities of success associated with recognizing 0, 1 or 2 items where the appropriate probabilities of recognizing 0, 1 or 2 items are given by the equations calculated in section 1 of this appendix. The means of deriving these equations is basic probability theory similar to that used to obtain the corresponding values for the Recognition Heuristic, although the equations themselves are necessarily more complex and therefore explained in a little more detail. Overall performance of the LINDA model is given by Equation 1.

Recognize none: $\alpha_{000} = 0.5$ (Chance).

Recognize one item:

$$\begin{aligned}\alpha_{100} = p_A \cdot 0.5 \cdot \frac{0.25N - u}{N-u-v-w}2+ \\ p_A \cdot \frac{0.75N - v - w}{N-u-v-w}+ \\ (1-p_A) \cdot 0.5 \cdot \frac{0.25N - u}{N-u-v-w} + 0 \\ = \frac{0.5 \cdot (0.25N - u) + p_A \cdot (0.75N - v - w)}{N-u-v-w}\end{aligned}$$

The participant recognizes one item, which is from the top quartile. With probability p_A they believe it to be above median and choose it.

The first term is then the probability that the non-recognized item is also in the top quartile, times the probability of success (chance).

The second term is the probability that the non-recognized item is in the second quartile or lower, times the probability of success.

With probability $1 - p_A$ the participant believes the recognized item is below median, and so does not choose it.

The third term is the probability that the non-recognized item is in the top quartile, with chance probability of being correct.

The fourth term is the probability that the non-recognized item is in the second quartile or lower, with no chance of being correct.

The probability of choosing correctly if one item from u is recognized is the sum of these terms and the second line of the equation rewrites the calculation for overall probability of success into a more succinct form. The equations for choosing correctly when a single item from v or w is recognized take similar form:

$$\begin{aligned}\alpha_{010} = 0 + p_B \cdot 0.5 \cdot \frac{0.25N - v}{N-u-v-w}+ \\ p_B \cdot \frac{0.5N - w}{N-u-v-w}+ \\ (1-p_B) \cdot \frac{0.25N - u}{N-u-v-w}+ \\ (1-p_B) \cdot 0.5 \cdot \frac{0.25N - v}{N-u-v-w} + 0 \\ = \frac{0.375N - u - 0.5v + p_B \cdot (0.25N + u - w)}{N-u-v-w}\end{aligned}$$

The recognized item is in the second quartile (from v). With probability p_B they believe it is above median.

$$\begin{aligned}\alpha_{001} = 0 + (1-p_C) \cdot 0.5 \cdot \frac{0.5N - w}{N-u-v-w}+ \\ p_C \cdot \frac{0.5N - u - v}{N-u-v-w}+ \\ p_C \cdot 0.5 \cdot \frac{0.5N - w}{N-u-v-w} \\ = \frac{0.25N - 0.5w + p_C \cdot (0.5N - u - v)}{N-u-v-w}\end{aligned}$$

The recognized item is below median (from w). With probability $1 - p_C$ the participant believes the item is above median.

Two items are recognized.

With reference to u, v, w, the possible combinations in which this might occur are: 110, 101, 011, 200, 020, 002.

If two items are recognized and one item is in the first quartile (from u) and the second item is in the second quartile (from v). With probability p_A the participant believes the first item is above median and with probability p_B they believe the second item is above median. This then gives the following derivation:

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$$\begin{aligned}\alpha_{110} &= 0.5p_A p_B + p_A(1-p_B)+\\ &\quad 0 + 0.5(1-p_A)(1-p_B)\\ &= 0.5 + 0.5p_A - 0.5p_B\end{aligned}$$

Calculations for the other ways in which two items might be recognized (e.g., one item from the top quartile and one from below the median) can similarly be combined with the parameters p_A, p_B and p_C as follows:

$\alpha_{101} = 0.5 + 0.5p_A - 0.5(1-p_C) = 0.5p_A + 0.5p_C$

(As above, substituting $1 - p_C$ for p_B)

$\alpha_{011} = 0.5p_B + 0.5p_C$

(As above, substituting p_B for p_A)

$\alpha_{200} = \alpha_{020} = \alpha_{002} = 0.5$

Both items are from the same subset, and so cannot be distinguished, performance is chance.