Multiple Periodic Variable Stars
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OBSERVATIONAL ASPECTS OF MULTIPERIODICITY IN THE β CANIS MAJORIS STARS

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I. Introduction

Multiperiodicity has been recognized as an important aspect of β Canis Majoris-type variability ever since the intensive observations of the "classical" members of this group by Struve and his coworkers in the early 1950's. About half the stars observed by Struve exhibited a radial-velocity variation whose period was several hours, but whose amplitude was modulated with a period of the order of days. This quickly became known as the "beat phenomenon," and was interpreted as an interference between two nearly equal short periods.

The number of stars considered to be certainly or very probably members of the β Canis Majoris class has roughly doubled since the early 1950's (although the number of stars involved remains relatively small), but the percentage of variables having two or more short periods has remained about the same. Among newly discovered as well as among "classical" variables, about half the stars seem to have only one short period, with no periodic modulation. In the present discussion, we shall call these the Class I variables. In a few additional cases, all involving members of binary systems, the one short period seems to be superimposed on a longer-period variation that can be attributed to orbital motion. We shall call these systems Class II variables. Finally, the truly multiperiodic variables -- those having two or more short periods, whether or not a binary period is present -- will be called Class III variables.

II. The Observations

The observational data for the Class I stars are given in Table 1, where the stars are listed roughly in the order of decreasing quantity of available information. With the exception of BW Vul, to which we shall return later, these stars all appear to be relatively simple objects (which is not to say that they are simple to explain!). None of them is known to be a binary star. All have a single, well-determined period which, at least in the well-observed cases of δ Cet and ξ^1 CMa, does not show secular variations. The light and velocity curves are nearly sinusoidal, and there are no large changes in the line-width (like those observed by Struve in the multiperiodic variables), nor phase differences in the velocity

Table 1

Single-Period Stars

Star	P(d)	2K (km/s)	Δv (mag)	dP/dt (s/cent)
γ Peg	0.1517495	7.0	0.017	
δ Cet	0.1611380	12.6	0.025	0.0
ξ^1 CMa	0.2095755	36.0	0.034	0.0
BW Vul	0.2010249 (1925)	160 (var)	0.2 (var)	+4
HR 6684	0.1398903	22.6	0.028	
τ^1 Lup	0.177365	10.6	0.027	
V986 Oph	0.2907 ?	33 ?	0.014 ?	
o Vel	0.131977	7	0.03	
δ Lup	0.16547 ?		0.0035	

curves as derived from lines of different elements (van Hoof effect). However, recent high-resolution work such as that of Le Contel (1969) on γ Peg suggests that even these stars may undergo subtle line-profile variations.

BW Vul is an exceptional member of this class, since its period is increasing at a rate of about 4 seconds per century, its velocity amplitude is also secularly increasing, and it shows large spectrum variations. However, since the existence of a single short period seems to be firmly established, we shall include BW Vul in Class I as the exception that proves the rule.

Table 2 gives the observational data for the Class II variables. With the exception of α Lup, which is included in this class by analogy, all are known to be members of binary systems. It seems likely that they would be ordinary Class I variables, were it not for this circumstance. (However, most of them do show small profile variations, and the van Hoof effect has been observed in α Vir and α Lup.) In α Vir, the well-determined orbital period produces light variation as well as velocity varia-

Table 2

Single-Period Stars with Modulation

Star	Short P(d)	2K (km/s)	Δv (mag)	dP/dt (s/cent)	Long P(d)	2K (km/s)	Δv (mag)
β Cep	0.1904881 (1968)	45 (var)	0.0365	+1.2	10.893? 6?	6.6	
α Vir	0.1737853 (1969)	17	0.029 (var)	-1	4.01454	228	0.06
β Cen	0.157	14.4	0		352	32	
λ Sco	0.2137015	17	0.0231		10.1605		0.0079
α Lup	0.259882	20 (var)	0.02 (var)		10 y ?		

tion, because of the ellipticity of the components. There is also long-period light variation in λ Sco, although its period is nearly twice the orbital period. No light variation, of either short or long period, has been observed for β Cen.

The case of β Cep is complicated. Struve considered it to be a single-period variable, despite changes in the velocity amplitude. Long-period variations of 11 days and 6 days have been reported by Fitch (1969) and Fischel and Sparks (1972), respectively. But although β Cep has two known companions, one of them discovered by speckle interferometry, neither of these is close enough to the primary to have a short enough orbital period to produce the observed variations.

The Class III stars are presented in Table 3, where P_I refers to the short-period oscillation with the largest radial-velocity amplitude, P_{II} to the one with the second-largest amplitude, and P_b to the beat period between them. The first five stars in this list were well observed by Struve, who usually determined the principal period and the beat period, and then derived the secondary period from these. Struve's periods have generally been confirmed and refined by later observers, and in some cases new short-period ($P < 1$ day) and/or long-period variations have also been found. These are listed under "Other Short P" and "Other Long P," respectively. The latter may be connected with binary orbital motion. The last six stars in Table 3 have been discovered more recently, and there is usually some uncertainty as to the value of their secondary period, whose existence is inferred from changes in the principal oscillation.

Many of the Class III variables are known binaries (σ Sco, 16 Lac, θ Oph, β Cru, and κ Sco), and Fitch (1969) has suggested that ν Eri and 12 Lac are binaries on the basis of their long-period modulation. It has not been proven that _all_ Class III stars are members of binary systems.

Table 3

Multiperiodic Stars

Star	P_I(d)	2K(km/s)	Δv(mag)	P_{II}(d)	2K(km/s)	Δv(mag)	P_b(d)	Other Short P(d)	Other Long P(d)	dP_I/dt (s/cent)
β CMa	0.25002246 (1940)	10.6	0.0044	0.2513003 (1940)	6.6	0.0210	49.17 (1940)	0.23904		$\{+0.5(P_I)$ $-0.5(P_{II})$
σ Sco	0.2468429 (1960)	100	0.040	0.2396710	15	0.021	8.2		33.1	+2.3
16 Lac	0.169165	29.6	0.05	0.170845	9.0	0.04	17.15		12.097	
ν Eri	0.1735089	49.0	0.114	0.1779	16	0.067	6.9808		15.79	+0.2
12 Lac	0.19308883	37.5	0.078	0.197358	17.5	0.029	8.9252	0.18	25.85	$\{+0.1$
								0.16	39	−0.2
								0.15	8.876	
KP Per	0.201753	20(var)	0.072	0.1982	5?	0.036	10	0.01		
θ Oph	0.140531	12	0.04				4-6			
β Cru	0.160474	4.4	0.04				6	0.2365072	7-8 y	
								0.121383		
15 CMa	0.184557	6.5	0.02				2	0.20 or 0.17		+20
								0.19296		
κ Sco	0.1998303	5.8	0.0087	0.205430		0.0038	7.3316	0.189512	2.951	
ε Cen	0.169608		0.0084	0.17696 or 0.2150		0.0034				

All the well-observed Class III stars show strong line-width (i.e. profile) variations, and many of them also have a van Hoof effect (among the well-observed stars, only ν Eri and KP Per do not). Secular changes in the principal period, which are rare in the Class I and Class II stars, have been reported for a number of Class III variables -- but the exact value and the interpretation of these changes remain debatable in most cases.

The prototype star, β CMa, is undoubtedly the most unusual object in Class III. Its two short periods can be observed independently, without recourse to the beat period or a periodogram analysis, because one of them (which we have called P_I) carries most of the radial-velocity variation while the other (our P_{II}) carries most of the light variation as well as the line-profile variations. Moreover, Shobbrook (1973a) has shown that these two periods have secular variations with opposite signs. Finally, there is no evidence that β CMa is a member of a multiple system.

Notes and references to the data on individual stars, with some alternative values, are given in the following paragraphs. The authors' original nomenclature regarding the ordering of periods is followed in these notes, rather than the notation of Tables 1-3.

γ Peg. P = 0.1517495d according to Sandberg and McNamara (1960). Le Contel (1969) found line-profile variations, although McNamara (1953, 1956) did not. There is no van Hoof effect according to McNamara (1956). McNamara (1955) gives 2K = 7.0 km/s. Jerzykiewicz (1970) gives ΔV = 0.017 mag. Struve (1955) lists dP/dt as inconclusive.

δ Cet. Jerzykiewicz (1971a) gives P = 0.1611380d, ΔV = 0.025 mag. McNamara (1955) observed 2K = 12.6 km/s, with no profile variations. According to van Hoof (1968), dP/dt = 0.

ξ^1 CMa. Shobbrook (1973b) gives P = 0.2095755d, dP/dt = 0, Δv = 0.034 mag. McNamara (1955) observed 2K = 36.0 km/s, with no profile variations. Van Hoof (1962e) suggested that the observed period is actually a combination of two interference waves, implying that four overtones are excited. However, Shobbrook (1973b) disagrees with this interpretation.

BW Vul. Petrie (1954) found that the period was 0.2010249d in 1928, but it increases at an average rate of about 4 s/cent (Petrie 1954, Odgers 1956, Percy 1971). The velocity amplitude also increases at a rate of 0.7 km/s per year (Petrie 1954); its value was 160 km/s in 1954 (Odgers 1956). There are complicated line-profile variations (Struve 1954, Petrie 1954, McNamara *et al*. 1955, Odgers 1956). Although Petrie (1954) found no van Hoof effect, McNamara *et al*. (1955) observed a velocity discrepancy between the hydrogen lines and those of other elements. The light range is 0.162-0.210 mag (Walker 1954a).

HR 6684. This variable was discovered by Jerzykiewicz (1972). Pike (1974) gives 2K = 22.6 km/s. Earlier, McNamara and Bills (1973) had found 2K < 5 km/s. However, the light curve is so stable that there is probably no beat. The period is 0.1398903d, according to Morton and Hansen (1974), who observed Δv = 0.028 mag.

τ^1 *Lup*. In his discovery paper, Pagel (1956) reported P = 0.177365d and 2K = 10.62 km/s, with no profile variations. Although the residuals were rather large, Pagel attributed them to observational errors rather than to a beat. Watson (1971) observed Δv = 0.027 mag.

V986 Oph. Jerzykiewicz (1975) gives P = 0.2907d, ΔV = 0.014; but this period does not fit all his observations, nor does a hypothetical interference between two short periods. Periods found by other observers are 0.2890d (Lynds 1959), 0.28465d (van Hoof 1967), and 0.2859d (Hill 1967). The radial velocity is known to be variable (Plaskett and Pearce 1931), but no detailed study has been published.

o Vel. The variability of this star was discovered by van Hoof (1972), who found $P = 0.131977d$, $2K \sim 7$ km/s, $\Delta V = 0.03$ mag. Morton and Hansen (1974) state that o Vel probably has a single period.

δ Lup. The discoverer is Shobbrook (1972), who finds $\Delta v = 0.0035$ mag. The most probable value of the period is 0.16547d, but 0.14273d is also a possibility.

β Cep. Gray (1970) gives $P = 0.1904881d$, $\Delta V = 0.0365$ mag. Struve et al. (1953) found $dP/dt = +1.2$ s/cent (not confirmed by Gray), and 2K variable between 18 and 46 km/s. Fitch (1969) found a long-period modulation of $P = 10.893d$ which could be binary motion (with $2K = 6.6$ km/s), but this modulation was also not observed by Gray. Van Hoof (1962c) found 10 overtone periods in β Cep. Fischel and Sparks (1972) observed a 6-day variation of the ultraviolet C IV multiplet, while Goldberg and Walker (1974) observed line-profile variations with the short period. Speckle interferometry (Gezari et al. 1972) shows that β Cep has a companion at 0.3 arcsec with $\Delta m_V = 5$ mag, in addition to its visual companion.

α Vir. The short period is 0.1737853d according to Shobbrook et al. (1972), with $2K \sim 17$ km/s and Δv decreasing from 0.029 mag in 1969 to essentially 0 in 1971. The orbital period of 4.01454d (first described by Struve et al. 1958) is reflected in both the light curve and the velocity curve, with $2K = 248$ km/s, $\Delta v = 0.06$ mag (Shobbrook et al. 1969). Smak (1970) found $dP/dt = -5$ s/cent, but Shobbrook et al. (1972) and Lomb (1975a) favor -1 s/cent. Lomb (1975a) finds a van Hoof effect and profile variations. The other short periods originally proposed by Shobbrook et al. (1972) are now considered statistically insignificant (Lomb 1975a). Dukes (1974) found four short periods, with slightly different frequencies for the light and radial-velocity variations.

β Cen. Lomb (1975b) finds P = 0.157d, 2K = 7.2 km/s. He also observes profile variations. There is no light variation (Breger 1967, Shobbrook 1973b). There is a long period of 352d associated with a velocity range of 2K = 32 km/s (Breger 1967, Shobbrook and Robertson 1968); this could be a binary period. Beta Cen has a visual companion at 1.3 arcsec (Δm = 3) and is also a spectroscopic binary with equal-brightness components (Breger 1967, Hanbury Brown et al. 1974, Lomb 1975b).

λ Sco. Shobbrook and Lomb (1972) give P = 0.2137015d, Δv = 0.0231 mag. They also find the first harmonic, 1/2 P, to be present in the light curves. Lomb and Shobbrook (1975) found the same two periods in the radial velocities, with 2K = 17 km/s and 6.0 km/s, respectively.
A long period of 10.1605d is found from the variation in mean magnitude (Δv = 0.0079 mag). But this is probably not the orbital period, for which Shobbrook and Lomb (1972) suggest 5.6d. The intensity interferometer (Hanbury Brown et al. 1974) shows that λ Sco is a binary with equal-brightness components.

α Lup. Pagel (1956) discovered that α Lup varies with a period of 0.259882d. He observed a radial-velocity amplitude of 2K = 14 km/s and no light variation (Δv < 0.005 mag). However, more recent observations place 2K as high as 20 km/s (Milone 1962, Rodgers and Bell 1962) and Δv as high as 0.02-0.03 mag (van Hoof 1965b). Van Hoof (1964, 1965b) suggests a 10-year modulation period. There is an indication of profile variations and van Hoof effect (Rodgers and Bell 1962).

β CMa. Struve (1950a) found P_1 = 0.25002246d with K_1 = 5.8 km/s, P_2 = 0.2513003d with K_2 = 2.0 km/s, and P_b = 49.1695d. P_1 was constant while P_2 (the line-broadening period) was decreasing in both period and amplitude, so that P_b increases. These findings were essentially confirmed by Struve et al. (1954) and Milone (1965). Shobbrook (1973a) found

P_1 = 0.25003d, P_2 = 0.2512985d, and P_3 = 0.23904d (as well as 1/2 P_1 and 1/2 P_2) in both radial-velocity and photometric data. In his most recent data, the ranges were 0.0044 mag and 10.6 km/s for P_1, 0.0210 mag and 6.6 km/s for P_2, and 0.003 mag and 2.2 km/s for P_3. Shobbrook found that P_1 increases at a rate of 0.5 s/cent while P_2 decreases at the same rate, so that P_b had increased to 49.20d by 1941. Van Hoof (1962d) found the fundamental radial mode, two overtones, and interference oscillations in Struve's data. Line-profile variations were observed by Struve (1950), equivalent-width variations in the Balmer lines by Kupo (1965), and the van Hoof effect by van Hoof and Struve (1953).

σ Sco. Struve et al. (1955) found P_2 = 0.246844d, $2K_2$ = 100 km/s; P_1 = 0.255 or 0.239d, $2K_1$ = 15 km/s; P_b = 8.0d. They computed that P_2 was increasing at a rate of 2.3 s/cent. Line-profile variations were observed by Levee (1952) and Huang and Struve (1955); the profiles varied with period P_2. Struve et al. (1955) also observed a van Hoof effect. Struve et al. (1961) give P_2 = 0.2468429d, P_b = 8.0d, and an orbital period (from the variation of the mean velocity) of 33.19 or 34.23d. Van Hoof (1966) found P_2 = 0.2468406d, with a light range of 0.040 mag; P_1 = 0.2396710d with a light range of 0.021 mag; and P_b = 8.252d. He did not confirm Struve's conclusion regarding the secular increase of the period. He computed the orbital period as 33.008d, exactly $4P_b$. Fitch (1967) recomputed the orbit and found P_{orb} = 33.13d with K = 34.7 km/s.

16 Lac. The best values of the period and amplitude appear to be P_2 = 0.169165d, K_2 = 14.8 km/s, Δv_2 = 0.05 mag; P_1 = 0.170845d, K_1 = 4.5 km/s, Δv_1 = 0.04 mag; P_b = 17.15d (Struve et al. 1952a; Walker 1952a, 1954b; McNamara 1957). There is also a periodic variation of the mean radial velocity with a period of 12.097d (Struve et al. 1952a), which does not appear in the photometry (Miczaika 1952, Walker 1952a). Small line-

profile variations were observed by Struve et al. (1952a), and a van Hoof effect by van Hoof et al. (1954). Starting with the principal oscillation (Struve's P_2) and the orbital period, Fitch (1969) computed two more short periods and some linear combinations between the short periods and the orbital period.

ν Eri. From the observations of Struve et al. (1952b) and Walker (1952b), we have $P_2 = 0.1735089d$, $K_2 = 24.5$ km/s, $\Delta v_2 = 0.114$ mag; $P_1 = 0.1779d$, $K_1 = 8$ km/s, $\Delta v_1 = 0.067$ mag; $P_b = 6.9808d$. Line-profile variations were observed by Struve et al. (1952b), Struve and Abhyankar (1955), and Laskarides et al. (1971). The latter authors report that there is no van Hoof effect. Van Hoof (1961a,b) found a fundamental oscillation and four overtones, as well as interference waves, in both the photometric and radial-velocity data. Opolski and Ciurla (1962a), Fitch (1969), and Kameswara Rao (1969) all suggested that the data could be represented by one short-period oscillation (Struve's P_2) combined with a long modulation period of 15.79d. The long period could be an orbital period. Struve (1955) gives dP/dt = +0.2 s/cent.

12 Lac. Struve and Zebergs (1955) give $P_2 = 0.19308883d$, $2K_2 = 37.5$ km/s; $P_1 = 0.197358d$, $2K_1 = 17.5$ km/s; $P_b = 8.9252d$. Barning's (1963) analysis of the photometry from the 1956 international campaign confirmed these two periods and added $P_3 = 0.182127d$ and $P_4 = 25.85d$. Other short periods suggested have been 0.1558 and 0.162d (de Jager 1957), and 0.1541, 0.16 and 0.18d (Sato 1973). Sato gives 0.078 mag for the light range of the primary oscillation, and 0.029 mag for the secondary oscillation. He finds that the primary period is decreasing at a rate of 0.2 s/cent, while Struve (1955) wrote that it was increasing at a rate of 0.1 s/cent. Struve (1951) proposed a long period of 39d; Fitch (1969) suggested an orbital period of 8.876d, based on the work of Opolski and Ciurla (1961,

1962b). Line-profile variations were observed by Struve (1951), Struve and Zebergs (1955), and Struve et al. (1957). The van Hoof effect is mentioned by Struve and Zebergs (1955) and Beres (1966).

KP Per. Jerzykiewicz (1971b) gives P = 0.201753d, and confirms the 10-day beat period of Klock (1965). Klock found the secondary period to be 0.1982d with a light range of 0.036 mag, while the light range of the principal oscillation was 0.072 mag. Klock also suggested a 10-15 min variation which was not confirmed by Jerzykiewicz. Slightly different values of the two periods are given by Joshi (1966). Struve and Zebergs (1959) observed variable line profiles but no van Hoof effect.

θ Oph. P = 0.140531d (van Hoof 1962b, Briers 1967), but van Hoof gives P_b = 6d while Briers gives P_b = 3.9d. Van Hoof and Blaauw (1958) found 2K = 12 km/s for the main oscillation, and observed spectral variations. They state that θ Oph is a member of a spectroscopic binary system. The light amplitude is about 0.04 mag (Briers 1967). Van Hoof (1962b) found the fundamental mode, two overtones, and interference waves to be present.

β Cru. Pagel (1956) discovered the variability with a period of P = 0.160474d, a velocity range of 2K = 4.4 km/s, and a light range of 0.04 mag. His period was confirmed by Heintz (1957), who suggested a long period of 7-8 y. But van Hoof (1962a) considered the fundamental period to be 0.23650d, with Pagel's period being the first overtone. Van Hoof also found the second overtone at P = 0.121383d, and a beat period of 6d between the fundamental and the interference oscillation of the fundamental with the second overtone. Hanbury Brown et al. (1974) report that β Cru is a binary with Δm = 2.9 mag.

15 CMa. Shobbrook (1973b) gives P_1 = 0.184557d, with the harmonics 1/2 P_1 and 1/3 P_1 also present. He finds no other short period in his data, despite the variability of the amplitude. On the other hand, he confirms

the idea that there was a 2-day beat period in van Hoof's (1965a) data, giving P_2 = 0.2033 or 0.1690d in 1963-4, and 0.2037 or 0.1681d in 1965. The radial velocity data of Lynds et al. (1956) yield P_1, 1/2 P_1, and P_3 = 0.19296d in Shobbrook's analysis, where $2K(P_1 + 1/2\ P_1) = 9.7$ km/s, and $2K(P_3) = 4$ km/s. The total light range is 0.01-0.02 mag (Watson 1971, van Hoof 1965a, Walker 1956). Shobbrook (1973b) finds that the principal period is not constant, but the form of its variation is doubtful -- the rate of change could be as high as +20 s/cent.

κ Sco. Lomb and Shobbrook (1975) give P_1 = 0.1998303d with a radial-velocity amplitude of 5.8 km/s, P_b = 7.3316d, P_2 = 0.205430d. Another short period, P_4 = 0.189512, may also be present. They state that κ Sco is a spectroscopic binary. Earlier, Shobbrook and Lomb (1972) had found the same two principal periods (with light ranges of 0.0087 mag and 0.0038 mag, respectively), as well as P_3 = 2.951d. They noted that $P_3 = 2P_b/5$, where $2P_b$ could be a "long period" of 14.74d.

ε Cen. Shobbrook (1972) gives P_1 = 0.169608d and P_2 = 0.17696d, with light ranges of 0.0084 and 0.0034 mag, respectively. An alternative value for P_2 is 0.2150d.

III. Discussion

In Table 3 we adopted the notation P_I and P_{II} to denote the strongest and second-strongest oscillations in the stellar radial velocity. But in the past, different observers have tended to use different nomenclatures which reflected both their method of determining the periods and their interpretation of the periods they found. Struve (1950b), for example, designated as P_2 a short radial-velocity period accompanied by changes in the line profiles; a short radial-velocity period without profile variations was called P_1. Furthermore, he denoted by P_3 the beat period between P_1 and P_2, by P_4 the period of amplitude variation of P_2 (seldom if ever observed), and by P_5 the period of variation in the mean velocity of P_2 (this often turned out to be a spectroscopic binary period). Struve believed that either P_1 or P_2 was the orbital period of a very small satellite, while the other short period was either the rotation period or the pulsation period of the primary star. This theory has been largely discarded, because of the obvious difficulty of maintaining a satellite in orbit with the required period.

Van Hoof, on the other hand, in his long series of articles on individual β CMa stars, proposed that these stars are purely radial oscillators in which the fundamental and several overtones, as well as the interference oscillations between them, are excited. He therefore identified the principal period as P_0, indicating that it was the radial fundamental mode; the other, shorter periods he found were called P_1, P_2, etc. to indicate that they were radial overtones. The interference periods were given labels of the type $P_{i,i+2}$; one of these was most often identified with what we have called the secondary oscillation. Although the periods found by van Hoof sometimes gave plausible values for the ratios of overtone to fundamental periods (P_i/P_0) in a simplified stellar model (e.g., a polytrope with $n = 3$ and $\Gamma = 1.52$), the existence of the "overtone" periods is very dubious because van Hoof did not apply any prewhitening to the data before searching for these small oscillations by a periodogram technique.

Wehlau and Leung (1964), in their paper on the application of the periodogram method to observations of short-period variable stars, point out the problems of "diffraction" peaks (due to the total observing "window") and aliases (due to the spacing of individual observations), which must be minimized by use of an apodizing function and by subtracting out the oscillations already found (prewhitening) before a search for smaller oscillations can be made. Gray and Desikachary (1973) showed how to carry out the prewhitening in the frequency domain.

Fitch (1967) suggested a refinement to the periodogram technique, in which the data are first searched in short segments to find approximate periods, thus reducing the extent of the frequency domain that must be searched at very high resolution with the full data set. In addition, Fitch (1967, 1969) studied a number of β CMa stars that either are known spectroscopic binaries (σ Sco, 16 Lac) or exhibit long-period modulation of the mean light and/or radial velocity of their primary oscillation (ν Eri, 12 Lac, β Cep). He denoted the "binary" period by P_0 and the principal short period by P_1, and expressed all the other periods found by the periodogram technique as linear combinations of these (and sometimes of higher overtones). The rationale for this procedure is the theory that the principal oscillation in a binary β CMa star is perturbed by the variation in tide-raising potential as its companion moves in orbit. Those linear combinations of orbital frequency and pulsation frequency are preferentially excited which have natural resonance with another pulsation overtone. Although this is a possible analysis of the multiple periods in some β CMa stars, especially the known binaries, it is doubtful that this theory can explain all cases of multiperiodicity.

Recent observers have frequently used the least-squares method introduced by Barning (1963), a variation on the periodogram technique in which one minimizes the residuals rather than maximizing the amplitude of the period being sought. The application of this method is described by Shobbrook and his coworkers (Shobbrook _et al_. 1972, Shobbrook and Lomb 1972, Lomb 1975c). Since the least-squares and periodogram methods do not _require_ the determination of a beat or other long period, the current tendency is simply to label periods in order of decreasing amplitude: P_1, P_2, etc., a scheme similar to that adopted in Tables 1-3.

Although some authors occasionally attempt to identify a radial or non-radial mode, fundamental or overtone, on the basis of the velocity-to-light amplitude ratio, such identifications must remain doubtful until a theoretical study is made of the expected values of this ratio for various radial and non-radial modes in realistic stellar models. Moreover, as Watson (1971) has pointed out, the correct observational quantity to use in the comparison is not the observed velocity-to-light amplitude ratio, but the ratio of percentage changes in radius and in luminosity. Derivation of this quantity from the observations requires a precise knowledge of the bolometric correction and its variation over the light cycle.

Finally, although this paper has not attempted to deal with the purely theoretical aspects of multiperiodicity in β CMa stars, we may point out some of the pitfalls lying in wait for the theorist. Any successful theory must explain both the 50% of β CMa stars that are multiperiodic .. . and the 50% that are not! It must explain multiperiodicity both in binary stars ... and in single stars like β CMa itself. And this without introducing any assumptions contrary to the observations, such as excessive rotation speeds. It seems likely that the origin of multiperiodicity is intimately connected with the basic instability mechanism for β Canis Majoris stars, and that until the instability is definitely identified, no final conclusion can be reached concerning multiperiodicity.

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Discussion to the paper of LESH and AIZENMAN

VALTIER: I would like to know if BW Vul presents changes in amplitude.

LESH: Yes, Petrie and Odgers observed a secular increase of 0.7 km/sec per year in the radial velocity amplitude of BW Vul.

COX: Are you able to divide the β CMa stars into groups on the H-R diagram and position them so that one might tell if single period and multiple period stars are in definitely different evolutionary states?

LESH: This is an interesting idea, but unfortunately there is no obvious separation by spectral type (or, equivalently, by log T_{eff} and M_{bol}) between the single period and multiple period β CMa variables.

LE CONTEL: Could you confirm the fact that the best observed stars are the stars you classify in class III (multiple period stars)?

LESH: No, some of the single period stars, including γ Peg, δ Cet, and ξ^1 CMa, have been very well observed.

BAGLIN: Can you comment on the fact that there seem to be variables and non-variables in the same domain of the H-R diagram?

LESH: Morris Aizenman and I have shown that the "instability strip" for β CMa variables is a region of the H-R diagram that is crossed three times by a massive star in the course of its normal post-main-sequence evolution-once in the core-hydrogen-burning phase, once in the secondary contraction phase, and once in the shell-hydrogen-burning phase. Both variable and non-variable stars are present in the "instability-strip." We believe that the simplest explanation for this phenomenon is to assume that the variable and non-variable stars are in different evolutionary stages - most likely the non-variable stars are in the core-hydrogen-burning phase, while the variable stars are in either the secondary contraction or the shell-hydrogen-burning phase.

MOLNAR: In your Table 2, you give a photometric amplitude due to a binary motion for α Vir and λ Sco. Is this due to an occultation or something else such as reflection?

LESH: This is an aspect effect. That is, it is believed that these stars are tidally distorted. If there is an eclipse in the α Vir system, it is so shallow that it has a negligible effect on the total light variation.