35. STELLAR CONSTITUTION (CONSTITUTION DES ÉTOILES)

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1 INTRODUCTION

In previous reports, short reviews of the recent developments in a few selected major topics have revealed themselves very valuable and we go on with this useful practise. The subjects are: *Stars of very low luminosity: red, brown and white dwarfs* (F. D'Antona), *Solar models* (P. Demarque) and a short note on *The observation of the solar neutrinos* by the Kamiokande collaboration (Y. Totsuka), *The HR diagrams for massive stars in the Magellanic Clouds* (Ed. L. Fitzpatrick), *Late evolution of asymptotic giant branch stars: clues from IR and radio observations* (S. Kwok), *SN 1987A* (J.C. Wheeler), *Binary and millisecond radio pulsars and evolution of close binaries* (E. van den Heuvel), *Progresses in computational methods for stellar models* (G. Meynet). I am very grateful to the authors for providing promptly these very useful reviews.

The field of stellar constitution and evolution is fastly growing and becomes richer due to a continuous inflow of new results. New theoretical progresses are achieved in the various stellar evolutionary phases, simultaneously many new data are provided by all observational techniques from radio to X and γ-rays as well as from Particle Astrophysics. In this very lively context, it is becoming more and more difficult for a given astrophysicist, even in a well–defined field as Stellar Constitution, to keep aware of all significant results. As the division of scientific domains cannot be pursued indefinitely, the specialists must also keep some general view, which contributes to the unity of the field and favours fruitful interactions. Reports like the present one may play a useful role in this context.

In stellar structure and evolution many of the most challenging problems are brought about by new observations such as, for example, the questions related to millisecond pulsars and SN 1987A. Also, the growing possibility of observing stars in external galaxies has put a stimulating demand for new studies in stellar evolution. To be solved, the new problems require a considerable theoretical investment, the study of many physical processes and the realisation of quantitative models to be compared to the observations. These are some of the many tasks for the specialists in stellar constitution. The experience of recent years clearly shows major fascinating problems ahead of us and also an increasing number of new connexions with the other fields of astronomy.

The study of physical processes and the construction of quantitative models, i.e. numerical simulations, are the keys for our understanding of stellar structure and evolution. In that respect, the studies about stellar opacities, nuclear cross sections, stellar hydrodynamics, equation of state are of prime importance, as well as the developments of numerical techniques to solve the equations in more realistic cases. Some emphasis to this problem is also given in this report.

References are given, when possible, as their number in the Astronomy and Astrophysics Abstracts. For the readers' convenience the names of the authors are always quoted in the text. For very recent references which did not figure in the Abstracts prior to the completion of the various reports, the conventional reference system has been used.

D. McNally (ed.), Reports on Astronomy, Vol. XXIA, 421–437.

Since the last report, six IAU Symposia and eleven IAU Colloquia were held on topics of interest to Commission 35. They are:

Symposium 138 *Solar photosphere: Structure, convection, magnetic fields.* Kiev, USSR, May 15–20, 1989; Symposium 137 *Flare stars in star cluster associations and solar vicinity.* Buyrakan, USSR, Oct. 23–27, 1989; Symposium 142 *Basic plasma processes in the Sun.* Bangalore, India, Dec. 1–5, 1989; Symposium 147 *Fragmentation of molecular clouds and star formation.* Grenoble, France, June 11–15, 1990; Symposium 143 *Wolf–Rayet stars and interrelations with other massive stars in galaxies.* Denpasar, Indonesia, June 18–22, 1990; Symposium 145 *Evolution of stars: the photospheric abundance connection.* Druzba, Bulgaria, Aug. 27–31, 1990.

Colloquium 106 *Evolution of peculiar red giant stars.* Bloomington, USA, July 27–29, 1988; Colloquium 111 *The use of pulsating stars in fundamental problems of astronomy.* Lincoln, USA, Aug. 15–17, 1988; Colloquium 113 *Physics of luminous blue variables.* Val Morin, Canada, Aug. 15–18, 1988; Colloquium 104 *Solar and stellar flares.* Palo Alto, USA, Aug. 15–19, 1988; Colloquium 107 *Algols.* Victoria, Canada, Aug. 15–19, 1988; Colloquium 114 *White dwarfs.* Hanover, USA, Aug. 14–19, 1988; Colloquium 121 *Inside the Sun.* Versailles, France, May 22–26, 1989; Colloquium 122 *Physics of classical novae.* Madrid, Spain, June 27–30, 1989; Colloquium 128 *Magnetospheric structure and emission mechanisms of radio pulsars.* Lagow, Poland, June 17–23, 1990; Colloquium 129 *Structure and emission properties of accretion disks.* Paris, France, July 2–6, 1990; Colloquium 130 *The Sun and cool stars: activity, magnetism, dynamos.* Helsinki, Finland, July 17–21, 1990.

2 STARS OF VERY LOW LUMINOSITY: RED, BROWN AND WHITE DWARFS

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2.1 Introduction

The theory of intrinsically low luminosity stars has received a strong impulse, in the latest years, by the huge amount of new results of observations and surveys. The ultimate relevance of research on Very Low Mass (VLMs) stars and Brown Dwarfs (BDs) is to infer information on stellar fragmentation at the low mass end, and, consequently, on the possible role of BDs as hideplace for barionic dark matter. Importance of White Dwarfs (WDs) as indicators of the age of the Galaxy has recently been put forward and investigated. A review of the VLM-BDs can be found in Liebert and Probst (44.065.033), and a more recent one in Liebert (1990, McDonald Obs. 50th Anniversary Symposium). The IAU Colloquium 114 "White Dwarfs" held in Hanover (USA) in 1988 (49.012.055) and the reviews by D'Antona and Mazzitelli 1990 (Ann. Rev. Astr. Ap. **28**) and Koester and Chanmugam 1990 (Repts. Progr. in Phys. **53**, 837) make a recent point of the stage of research on WDs.

The present summary of highlights in this field will concentrate on some of the open questions relevant to the stellar structure of these objects.

2.2 VLMs: where is the low luminosity end of the main sequence?

This is the most clear–cut way of posing the problem of the transition between "stars" (nuclearly supported) and "brown dwarfs" (contracting and cooling objects). While the uncertainty in the hydrogen burning minimum mass can be estimated at 25% at most (passing from $\sim 0.08\ M_\odot$ for population I, D'Antona and Mazzitelli 40.065.040, to $\sim 0.1\ M_\odot$ for extreme population II, D'Antona 44.065.076), the minimum luminosity of the main sequence may vary by a factor of ten or more according, mainly, to the input atmospheric opacities. The limit for population II is at $\log(L_{\rm min}/L_\odot) \sim -3$, while the limit for population I is around –4. This general trend is confirmed by Greenstein's recent study (1989, Comments Astrophys. **13**, 303) of LHS faintest dwarfs, indicating a considerably larger minimum luminosity ($M_V \sim 15$) for high velocity dwarfs than for disk population I dwarfs ($M_V \sim 19$).

The absolute theoretical determination of the minimum main–sequence luminosity for population I stars is very poor, due to the uncertainty in the opacities. These are very difficult to compute, both because an enormous number of molecular and atomic transitions are to be considered, and because of the uncertain role of grain formation at low temperature (Lunine et al. 49.065.022), and for the fact that in the atmospheres of VLMs the gas is in highly non-ideal conditions. Recent updates of relevant opacities for solar composition have been provided by Alexander et al. (1989, Ap. J. **345**, 1014). The recent theoretical studies give $-4.3 \le \log(L_{\rm min}/L_\odot) \le -3.8$. In the writer's opinion, as the recent search for VLMs companions to M dwarfs (Henry and McCarthy 1990, Ap. J. **350**, 334) indicates a cutoff at infrared magnitude $M_K \sim 10.5$ ($\log(L/L_\odot) \sim -3.5$), this latter value must be considered the "empirical" determination of the main sequence for pop. I.

2.3 What do we know about the mass–luminosity relation of VLMs?

The derivative of the mass-luminosity relation enters into the determination of the mass function of VLMs from their luminosity function, thus is a very important quantity. Actually, theory shows clearly that this relation cannot be considered "unique". Three main factors affect the luminosity for a given mass $M \le 0.2M_\odot$: i) the age (D'Antona and Mazzitelli 40.065.076; Liebert and Probst 44.065.033); ii) the opacity (Burrows et al. 1989, Ap. J. **345**, 939); iii) the equation of state (Dorman et al. 1989, Ap. J. **342**, 1003). Further, chromospheric activity, flares and spots affect the resulting luminosity in a not clear, but probably important way, dependent on the stellar age (Giampapa and Liebert 42.065.063). Stellar activity is more modest for advanced spectral types (Fleming and Giampapa 1989, Ap. J. **346**, 299) in single stars, but mass determinations, obviously, are possible only for binaries, where, in many cases, the level of activity is independent from the age, and can be large also for very late spectral types. This problem complicates the few available observational data: as an example, the atmospheric parameters recently determined for Gliese 866A/B (Leinert et al. 1990, in press) correspond to theoretical masses (0.15 and 0.11 $M_\odot$) well below the dynamically inferred masses (0.22 and 0.16 $M_\odot$). Are the flare activity and X-ray emission of this binary connected to the underlying reason for this peculiarity?

2.4 Have any "bona fide" BDs been detected?

This question is a corollary of the previous one. Most of the impressive observational results obtained in these years are difficult to interpret because of the discussed uncertainties in the theory. A few BD candidates have been inferred by astrometric perturbations (Wolf 424, Heintz 49.118.010) and in the radial velocity variations surveys (Campbell 49.118.041; Marcy and Benitz 1989, Ap. J. **344**, 441; Latham et al. 1989, Nature **339**, 38). But the most interesting surveys, aimed to *detect* BDs, have given, up today null or controversial results. The IR speckle interferometry survey of M dwarfs provide *one* candidate on radial velocity measurements grounds (Gliese 623B, having $M = 0.067 - 0.087M_\odot$, Marcy and Moore 49.118.031) but the inferred luminosity is $\sim 10^{-3}L_\odot$, a factor 10 *larger* than the theoretically expected value. The search for IR excess around WDs by Zuckerman and Becklin provides a good candidate (GD165B, 46.118.036) at about $10^{-4}L_\odot$, which, however, could be still a star at the end of the main sequence. The claimed companion to G29–38 (44.118.028) is probably better explained by dust (Graham et al. 1990, Ap. J. **357**, 216).

There seem to be *no* uncontroversial detections of candidates for $0.01 \le M/M_\odot \le 0.08$, leading us to suspect either a correlation between fragmentation and hydrogen burning limit (very difficult to understand) or that we have overestimated the expected number of BDs on the basis of our theoretical models, as it would be the case if the main-sequence end in luminosity is large enough ($\log(L/L_\odot) \sim -3.5$).

2.5 Can the luminosity function of WDs tell us the age of the Universe?

The recent confirmation that the luminosity function for WDs drops dramatically at $-4.5 \le \log(L/L_\odot) \le -4$ (Liebert et al. 46.126.036) has prompted a series of attempts to find a motivated explanation for the lack of cool WDs. The most straightforward explanation is that the oldest WDs are still hanging at a luminosity $\sim 10^{-4}L_\odot$. If we had a clear idea of the cooling of WDs down to this limit, a simple model of galactic evolution would tell us which is the age of the disk of the Galaxy, and, consequently, we could infer the age of the Galaxy itself or even of the Universe. After the first outline of the method, (by Winget et al. 43.161.230), more complete investigations have been given by Iben and Laughlin (49.161.230), by Yuan (1990, Astr. Ap. **224**, 108) and in M. Wood's Ph.D. thesis (1990, University of Texas, see also Wood 1990, Journal R.A.S. Canada **84**, 150). Up today, the uncertainty in the "typical" cooling times reflects into an uncertainty in the determination of the disk age in the range $6 \times 10^9 \leq t_{\rm disk} \leq 12 \times 10^9$yr, so we are far from an interesting constraint on the age of the Universe, although the method can be promising. There has been a large discussion among researchers, as the first determination of the disk age by Winget et al. ($t_{\rm disk} \sim 9 \times 10^9$yr) had raised the problem of a possibly too large gap with the age of the Galaxy inferred from all the studies on globular clusters. The attempts to enlarge the disk age were not successfull (e.g. Garcia–Berro et al. 45.065.041; Barrat et al. 45.065.094; and Noh and Scalo 1990, Ap. J. **352**, 605). In addition, the models which adopt "evolutionary" WD remnant chemical stratifications (Mazzitelli and D'Antona 45.065.061) end up with very low ages for the disk ($\sim 6 \times 10^9$yr D'Antona and Mazzitelli 1989 Ap. J. **347**, 934). A possible solution of the problem has been opened by the new determinations of the critical ratio (Γ) between coulombian and thermal energy which is needed for crystallization: recent computations have pushed this parameter up to 180 – 200 (Ogata and Ichimaru 1987, Phys. Rev. A **36**, 5451, and Stringfellow, De Witt and Slattery 1990, Phys. Rev. A **41**, 1105), while the previous value was at 150. Consequently, crystallization occurs *later* in the WD evolution, and it may succeed in prolonging the cooling times, raising again the age of the disk to $11 - 12 \times 10^9$yr. Relevant computations are not yet available.

2.6 Pulsations in WDs: clocks for the cooling and mass determination.

Among the observational efforts which sure have given or are going to give important information on WD structure we must mention the "Whole Earth Telescope" (Nather, 49.036.157). A world spread bunch of observers have cared to obtain continuous observing runs of several variable WDs in order to avoid day–aliases in the determination of the periods. These observations already have provided two important hints:

1. a determination –which will be improved by future observations– of the period derivative of the ZZ Ceti star G117–B15A (Kepler et al. 1990, Ap. J. **357**, 204), placed at $P/\dot{P} = (8.2 \pm 5.0) \times 10^8$yr. The period derivative for this type of WDs provides a measure of the rate of cooling, thus posing important constraints on the inner chemical composition of WDs (Tassoul et al. 1990, Ap. J. Suppl. **72**, 335).

2. several pulsation periods for a few DO WDs have been determined. The period spacing is constant over several modes, and the periods correspond to successive values of n. In hot WDs, the spacing depends very strongly *only* on the total stellar mass, which thus has received a determination for two objects (PG 1159–035: $0.6 M_\odot$, and PG 0122+200: $0.73 M_\odot$, see Kawaler 45.126.019 and Kawaler and Hansen, 49.065.116).

Further novelties are to be expected in this field, and it is fair to remember that in this case progress is due more to the astronomers coordinated efforts than to new advanced technology or to increased telescope power.

3 SOLAR MODELS

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There continues to be a great deal of interest in the construction of solar models, primarily motivated by solar neutrino observations, and by the developing techniques of helioseismology. The latter, in particular, puts very high demands on the numerical accuracy (reproducibilty within one part in 10^4) of solar models. This has stimulated a number of sensitivity studies of the standard solar model on physical input (Bahcall and Ulrich, 45.080.102; Guenther and Sarajedini, 45.080.110; Guenther et al., 1989, Ap. J. **345**, 1022; Turck–Chièze et al., 46.080.097; Sackmann and Fowler, 1990, Ap. J., in press; GONG Newsletter, ed. F. Hill, National Solar Observatory, Tucson), and has necessitated improvements in numerical techniques as well (Guenther and Sarajedini, 45.080.110; Guenther et al., 1989, Ap. J. **345**, 1022; GONG Newsletter; Gabriel, 49.065.002).

Improvements in relative abundances of the heavy elements (Grevesse, 38.071.010; Anders and Grevesse, 49.105.032) are being incorporated in model calculations, in particular in opacity calculations, usually the

Los Alamos Opacity Library (Huebner et al. 1977, Los Alamos Sci. Lab. Rep.No. LA-6760-M). Detailed comparisons with the earlier Cox and Stewart (2.065.062) opacities have been performed. Improved approaches to opacity calculations are also being introduced by the Livermore group, as well as by another international collaboration (Iglesias et al., 44.063.111; Seaton, 45.063.102; Däppen et al., 46.065.046). These improvements in opacities are coupled with improvements in the equation of state of the solar material which are subtle, but significant in some contexts, particularly in helioseismology, because of the high sensitivity of the speed of sound to the equation of state (Guenther et al., 1989, Ap. J. **345**, 1022; Christensen-Dalsgaard et al., 46.080.051; Dziembowski et al., 46.080.004; Cox et al., 1990, Ap. J., in press).

Another important recent development has been the study of the internal rotation of the Sun. Following earlier work by Endal and Sofia (21.065.011; 29.080.016), a method for calculating the evolution of the rotating Sun from the pre-main sequence Hayashi phase to the present and beyond, has been developed which takes into account the effects of rotationally induced instabilities on the transport of angular momentum and chemical mixing (including the burning of Li and Be) during the evolution (Pinsonneault et al., 49.080.011). This approach was made possible and much strengthened by recent observations of rotational velocities and light element abundances in sun-like stars in clusters. For a different point of view on the internal rotation of the Sun, see Tassoul and Tassoul (49.080.027). Rotation decreases the predicted solar neutrino flux by only about 6 per cent. Rotational models also permit the forward calculation of p-mode splitting for direct comparison with observation (45.012.009; 49.012.034). The solar internal rotation can also be tested by helioseismology. A great deal of effort has been extended to develop inversion techniques to derive the rotation curve of the Sun. The success of this work has been hampered by the non-uniqueness of the inversion process, and by the still considerable uncertainties in observed p-mode rotational splittings (for a detailed discussion, see GONG Newsletter; 45.012.009; 49.012.034; Dziembowski et al., 49.080.008).

Another improvement to high precision solar models, required for both the interpretation of solar neutrino experiments and helioseismology (Bahcall, 49.003.110), has been the inclusion of chemical diffusion inside the Sun. This is a small but possibly significant effect. Diffusion might be particularly effective just below the convection zone (where helium and heavier elements are drained out of the convection zone, thus affecting its structure and depth), and near the center (raising the temperature and the predicted neutrino flux) (Loeb and Bahcall, 1990, Ap. J., in press; Pinsonneault and Bahcall, 1991, Ap. J., to be submitted). The solar neutrino observations are of increasing interest for our understanding of neutrino physics (45.012.009). In addition, the Sun continues to serve as a means of testing the existence of exotic particles which might modify its structure (Gilliland and Däppen, 45.080.073; Bouquet and Salati, 49.065.052; Finzi and Harpaz, 49.080.025).

4 OBSERVATION OF THE SOLAR NEUTRINOS

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The results from one thousand days of real-time, directional solar neutrino data by the Kamiokande collaboration can be summarized as follows:

1. Observation was made between December 15, 1986 and April 10, 1990 and corresponds to 1040 days of running time.
2. The minimum observable neutrino energy was 7.5 Mev.
3. The observed solar neutrino flux was 46±5±6% of the prediction of the standard solar model, where the errors are statistical and systematic, respectively.
4. The time variation of the solar neutrino flux was not found within the statistical errors (about 30%) in the time period of January, 1987 to April, 1990 during which the solar activity changed from minimum to maximum.

5 THE HR DIAGRAMS FOR MASSIVE STARS IN THE MAGELLANIC CLOUDS

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The HR Diagram is a valuable tool for studying the general properties of stellar evolution and the effects of global galactic properties, such as metallicity, on the formation and evolution of stars. In recent years, particularly since SN 1987A, it has become obvious that the results of stellar evolution calculations for massive stars are extremely sensitive – at both qualitative and quantitative levels – to the model input assumptions (i.e., mass loss rates, elemental abundances, and the treatment of convection) and that currently these assumptions are not well enough constrained to yield unique results. Observational tools like the HR Diagram offer crucial guidelines for formulating and evaluating the theoretical models. For well-known observational reasons (e.g., low reddening and well-determined distances) the Magellanic Clouds are particularly conducive to the study of the HR Diagram.

Producing an HR Diagram requires first a census of the various types of stars populating the mass range of interest, and then a conversion of the observational parameters into effective temperatures and luminosities. In this brief review, I will concentrate on recent developments in the census of massive stars in the Magellanic Clouds and then discuss the constraints imposed on evolution models from recent analyses of the HR Diagrams.

5.1 The Census

Among the massive stars – which I take here as those with initial masses $M_i > \sim 12\ M_\odot$ – the census can be broken into four subsets: 1) the blue/white supergiants; 2) the red supergiants; 3) the "exotic" stars; and 4) the O and early-B Main Sequence (MS) stars.

THE BLUE/WHITE SUPERGIANTS: For the Large Magellanic Cloud (LMC) the most complete accounting of the blue/white supergiants (i.e., the luminous O, B, A, F and G-type stars) is the catalog of Rousseau et al. (21.159.002) which contains entries for 1791 stars. The corresponding catalog for the Small Magellanic Cloud (SMC) is that by Azzopardi and Vigneau (32.002.089; AV), and contains entries for 524 stars. Additional SMC members, located outside of the boundaries of the AV catalog are listed by Sanduleak (2.159.001; 13.159.010). There have been no major new surveys of the LMC and SMC blue/white supergiants since these works, although new photometry and spectroscopy for catalog members are always being obtained.

In general, the blue/white supergiants represent the best studied subset of stars in the census, and the subset which is most securely placed on the HR Diagram. Both the LMC and SMC catalogs are probably substantially complete for the post-MS blue/white stars initially more massive than $M_i = 12\ M_\odot$ (corresponding to $M_{bol} < -6.5$). The analysis of the LMC HR Diagram by Fitzpatrick and Garmany (1990, Ap. J. **363**, in press) yields 955 such supergiants. About 280 such supergiants are found in the SMC (Garmany and Fitzpatrick, 1989, IAU Coll. 113, p. 83). The O-type supergiants are probably well-represented in both the Rousseau et al. and AV catalogs, although the counts of the less evolved and less luminous O stars are extremely incomplete (see below).

THE RED SUPERGIANTS: Surveys of the LMC red supergiant population have been published by Sanduleak and Philip (22.002.052), Westerlund et al. (29.113.013) and Rebeirot et al. (33.156.009). The surveys utilize different discovery techniques and have correspondingly different completeness characteristics. The Sanduleak and Philip survey yields 609 stars with $M_v < \sim -5.0$, which roughly corresponds to $M_{bol} < \sim -6.5$ and thus to $M_i > \sim 12\ M_\odot$. For the SMC, the most recent surveys are by Prévot et al. (34.156.001) and Sanduleak (1989, A. J. **98**, 825). The Sanduleak survey contains 372 red supergiants with $M_v < \sim -5.0$, or $M_{bol} < \sim -6.5$.

The lack of classification-grade spectroscopy and multicolor photometry for most of the red supergiants in these surveys hampers their placement on the HR Diagram in any but a schematic manner.

THE EXOTIC STARS: The "exotic" stars are identified by the presence of optical emission lines and are easily discovered with objective prism surveys. Bohannan and Epps (12.159.002) published an H-alpha emission survey of the LMC, complete to V $\sim$ 15 mag and containing 625 "stellar-like" objects. A comparable survey of the SMC is in progress (B. Bohannan and J. Doggett, in preparation).

The exotic stars include, at one extreme, such prominant objects as the Wolf–Rayet stars, the Luminous Blue Variables, B[e], and Ofpe/WN stars. Recent conferences have been devoted to these objects (IAU Coll. 113, "The Physics of Luminous Blue Variables"; and IAU Symp. 143, "Wolf Rayet Stars and Interrelations with Other Massive Stars in Galaxies") and the most current references can be found there. The precursor to SN 1987A, the star Sk – 69 202, probably also belongs under the heading of "exotic", although prior to the SN explosion it was an observationally undistinguished member of the blue/white supergiant class. At the other end of the range are essentially "normal" luminous blue/white supergiants, for which Balmer line emission is merely indicative of high luminosity. Between these extremes lie the bulk of the emission–line stars. A fascinating array of objects is probably present here, although the lack of classification–grade spectroscopy and continuum photometry for most makes this a largely unexplored class of objects.

THE O AND EARLY–B MS STARS: From the numbers of blue/white and red supergiants arising from stars with $M_i > \sim 12\ M_\odot$(and assuming that the post–core hydrogen burning phases occupy ~ 15% of the total stellar lifetime), we can crudely estimate that the LMC and SMC must contain some 10,000 and 3,500 MS stars, respectively, with $M_i > \sim 12\ M_\odot$. While very numerous, the vast majority of these stars are so faint that there is essentially no census information available for them. Indeed it is not until $M_i \sim 30\ M_\odot$(spectral type ~ 08 V and V ~ 13.5 in the LMC) that one might begin to expect some degree of completeness in the number counts. However, while some such stars are included in the Rousseau et al. and AV catalogs, as revealed in spectral classification studies such as Conti et al. (42.156.014; LMC) and Garmany et al. (43.156.018; SMC), they are in nowhere near the numbers that might be inferred from the corresponding number of supergiants.

Only relatively recently has it become appreciated that the massive young stars are found preferentially in compact groups and in regions of bright nebulosity which have been deliberately excluded, for obvious reasons, from the large scale surveys which produced the Rousseau et al. and AV catalogs. Recent observational efforts, particularly the application of CCD photometry and multiobject spectroscopy to such regions, are revealing large numbers of previously unknown O–type stars, including some of the most massive stars in the Magellanic Clouds. For example, a spectroscopic/photometric study of NGC 346 by Massey et al. (1989, A.J. **98**, 1305) has doubled the number of early O stars known in the SMC! The first O3 star in NGC 346 (and only the second in the SMC) was discovered by Walborn and Blades (41.114.102) and Niemela et al. (42.153.050). Unevolved O stars have also been detected in other dense nebular regions in the SMC (e.g., Testor and Lortet, 43.156.026).

LMC studies have been similarly active. The 30 Doradus region is particularly rich in massive stars (Melnick, 40.153.032; Walborn and Blades, 44.132.050); but other regions are also yielding evidence of recent massive star formation (e.g. Heydari–Malayeri and Testor, 42.132.001, and references therein; Garmany and Walborn, 43.156.021). A detailed investigation of the OB associations Lucke–Hodge 117 and 118 by Massey et al. (49.152.001) has identified about 50 previously uncataloged stars with $M_i > 10\ M_\odot$. The Lucke–Hodge catalog contains over 100 such associations and clusters of early–type stars which are even more compact, some of which are starting to be resolved (e.g., Heydari–Malayeri et al., 1989, A. Ap. **222**, 41).

In summary, the census of the MS O and early–B stars is currently a very active field, but much additional work is required before reliable statistics will be available for this class of objects.

5.2 Results from HR Diagram Studies

One of the primary results from study of the Magellanic Cloud HR Diagrams is the discovery that the most luminous early–type stars, with $M_{bol} < -9.5$, do not have counterparts in the red supergiant region (Humphreys and Davidson, 26.115.001). The maximum observed luminosity decreases with decreasing effective temperature until $T_{eff} \sim 10000$K, and thereupon remains roughly constant through the red supergiant region. Recent estimates of the location of the upper limit are given by Humphreys (44.115.004) and Garmany et al. (43.156.018). The direct implication of the limit is that stellar evolution tracks for stars initially more massive than ~ 40 $M_\odot$ do not extend into the red region. It is likely that stellar mass loss halts the redward progress of the evolution tracks, although it is not certain what physical process induces mass loss rates large enough to have this effect.

Current studies suggest that the SMC has approximately the same upper limit as the LMC, indicating that the process is relatively insensitive to metallicity. The calibration of the physical parameters of the

SMC stars is uncertain, however, due to their low metallicity. Also, the SMC HR Diagram is so lightly populated in the upper regions that it is difficult to accurately define the upper limits.

A second, and more recent, result from the LMC HR Diagram was reported by Fitzpatrick and Garmany (1990, Ap. J. **363**, in press). Concentrating on the distribution the blue/white supergiants initially less massive than $\sim 40\ M_\odot$, they found evidence for a steep decrease in the density of blue/white supergiants in the HR Diagram across a boundary referred to as the "ledge". The ledge is a diagonal feature, running from approximately (log $T_{eff} = 4.2$, $M_{bol} = -9$) to (log $T_{eff} = 3.9$, $M_{bol} = -6$). The upper limit corresponds to initial stellar masses of up to $\sim 25 - 30\ M_\odot$. This range includes the precursor to SN 1987A, the star Sk – 69 202. The ledge suggests a change in the evolutionary timescales during the blue supergiant stage ("faster" evolution on the cool side of the ledge), which probably occurs during core helium burning.

Fitzpatrick and Garmany suggested that the diagonal nature of the boundary is most compatible with the "blue loop" class of evolution models. Such a mode of evolution would imply that most of the blue supergiants in the LMC are in a core helium burning POST–red supergiant phase. Of course, the explosion of SN 1987A from a blue supergiant also implies that some type of blue loop occurs in the evolution of stars in the initial mass range 15–20 $M_\odot$, but not necessarily during the core helium burning stage. Given the small number of stars and uncertainty in the calibrations, it is not clear that a feature corresponding to the LMC ledge is present in the SMC HR Diagram (Garmany and Fitzpatrick, 1989, IAU Coll. 113, p. 83).

The final major items to be discussed here are the distribution of the red supergiants and the "B/R ratio" (i.e., the ratio of blue to red supergiants). Humphreys (25.159.022) has noted that in the Milky Way the typical red supergiant has a spectral type in the range M2–M3, while in the LMC and SMC the dominant spectral classes shift to M1–M2 and K–M0, respectively. If a systematic shift in the spectral type vs. T_{eff} calibration can be ruled out, this result suggests that stellar evolution tracks penetrate significantly less deeply into the red in the LMC and even less so in the SMC than in the Milky Way, perhaps as a result of the metallicity differences among the galaxies.

The B/R ratio also provides constraints for evolution models, since it is related to the relative lifetimes spent in the blue and red regions. For the most luminous red stars, with $M_{bol} < -7.5$, Humphreys and McElroy (38.155.041) find a B/R ratio of 4 for the SMC and ~ 10 for the LMC, without evidence for much luminosity dependence. Their dataset for the red supergiants, however, is clearly incomplete below $M_{bol} \sim -8$. The census data above suggest that when less luminous stars are included, the total B/R ratios for $M_{bol} < -6.5$ are of order unity for both galaxies. I suspect that incompleteness affects the lowest luminosity bins in the Humphreys and McElroy sample, but also that there is a strong luminosity dependence in the B/R ratios. Better spectroscopic and photometric data for the red supergiants are needed for a detailed determination of the B/R ratios, and to place strong constraints on the mass dependence of the evolution characteristics.

In the above discussion I have concentrated on results derived from study of the "normal" blue/white and red supergiants, and thus on constraints for stellar evolution up to, but probably not beyond, the core helium burning stage. The Magellanic Clouds' "exotic" star populations certainly contain many clues to the properties of evolution in later phases. See the IAU conferences noted in the Census section for current ideas in these areas. At the other end of the stellar evolution ladder, the LMC and SMC potentially provide the best sites for the dermination of the initial mass functions (IMF's) for massive stars. The current high rate of discovery of massive stars in the two galaxies suggests, however, that the final census of these is far from complete and that the final word on the IMF's has yet to be spoken.

6 LATE EVOLUTION OF ASYMPTOTIC GIANT BRANCH STARS: CLUES FROM INFRARED AND RADIO OBSERVATIONS

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6.1 Introduction

Conventional spectral classification schemes for oxygen–rich stars stop at about spectral class M10, with a vague notion of transition to S and C spectral types beyond. It is now recognized that there exist oxygen-rich asymptotic giant branch (AGB) stars that have evolved beyond the M10 limit. The IRC and AFGL infrared sky surveys have discovered many heavily reddened stars that have luminosities higher than Mira variables and are likely to be late AGB stars. Similar objects have also been discovered by infrared observations of stars discovered in radio OH surveys (Jones et al., 31.138.004). The infrared continuum emission is likely to have originated from a circumstellar dust envelope formed by mass loss as the stars ascend the AGB.

The recent *Infrared Astronomical Satellite (IRAS)* sky survey has extended the wavelength coverage to $\lambda \sim 100\mu$m. Based on the colour of the sources, it is estimated that $\sim 1/4$ (or 60.000) of all sources in the *IRAS Point Source Catalog* are evolved AGB stars (Chester, 41.002.092), many of which have no known associations. Ground-based observations of low colour temperatures *IRAS* sources have found that many have no optical counterparts, and their evolved-star nature can only be established by their circumstellar properties (Kwok et al., 44.133.022).

For AGB stars with faint or no optical counterparts, one has to rely on radio and infrared techniques as probes of their properties. Oxygen-rich stars generally show the 9.7 μm silicate feature either in emission or absorption in the infrared (Merrill and Stein, 19.133.004) and λ18cm OH maser emission in the radio (Herman and Habing, 40.112.088). For carbon-rich stars, the 11.3 μm SiC feature is usually present (Treffers and Cohen, 11.114.069) and the structure of circumstellar envelope can be studied by rotational transition of CO in the radio (Knapp and Morris, 39.112.093).

6.2 Infrared spectra of oxygen-rich stars

While infrared spectra of AGB stars have been observed since the early 1970s, the spectral database has recently been greatly expanded as the result of the *IRAS* survey. The 9.7μm silicate feature was detected in over 2000 stars by the *IRAS Low Resolution Spectrometer (LRS)* and shows a variation in strength from strong emission to strong absorption (Volk and Kwok, 43.112.078). The inferred optical depths in the feature range from 0.1 to > 100, implying a change in mass loss rate of over three orders of magnitude.

While precise locations of late AGB stars on the HR diagram are difficult to determine due to uncertainties in both luminosity and temperature, the distribution of the silicate absorption objects in the *IRAS* colour-colour diagram shows that they lie on a well-defined band (Olnon et al., 37.112.050). In comparison, stars that show the silicate feature in emission (e.g. Mira variables) occupy part of the colour-colour diagram to the left of the absorption objects (Walker and Cohen, 45.113.025; van der Veen and Habing, 45.112.040). This band can be interpreted as an evolutionary sequence with AGB stars evolving from the colour temperatures of >600 K for Mira variables to ≈ 250 K for late AGB stars (Bedijn, 44.112.123; Volk and Kwok, 46.112.024; Kwok, 1990, MNRAS **244**, 179).

6.3 Infrared spectra of carbon-rich stars

Carbon stars are traditionally classified as such based on their photospheric optical spectra. Several thousand objects have been catalogued as the result of objective prism surveys (Stephenson, 11.114.086). However, a separate class of carbon stars, which we will refer to as infrared carbon stars, can be identified based on the presence of the 11.3 μm SiC feature. Unlike visual carbon stars, infrared carbon stars often suffer from extreme circumstellar extinction, and are bright objects in the mid-infrared.

Several hundred stars have been observed to possess the SiC feature based on their *IRAS LRS* (Little-Marenin et al., 43.114.013). They generally have colour temperatures in the range of 300–600 K, suggesting high optical depths in the circumstellar envelope. The observed spectral shapes of the SiC feature and the overall energy distributions of infrared carbon stars can be fitted by radiative transfer models assuming that the circumstellar envelopes are created as a result of mass loss from the stars (Rowan-Robinson and Harris, 33.112.005; Chan and Kwok, 1990, Astron. Astrophys. in press).

While the photospheric continua of visual carbon stars suggest that these stars have very little circumstellar extinction, *IRAS* photometric measurements have revealed that most visual carbon stars have far infrared excesses (Thronson et al., 44.155.770). These unexpected excesses have been interpreted by Willems and de Jong (45.065.070) as due to the remnants of the material lost while the stars were oxygen-rich, and the mass loss process has been interrupted since the stars have become carbon-rich. The observed colours and energy distributions of visual carbon stars can in fact be reproduced with a detached shell model (Chan and Kwok, 46.064.109).

6.4 Radio spectra of oxygen–rich stars

Many late AGB stars show the characteristic double–peaked OH emission profiles, implying an expanding circumstellar envelope. They are referred to historically as OH/IR stars. The OH envelopes can be resolved with radio interferometric techniques, with some showing bipolar geometries (Bowers, 1990, in Cool stars, stellar systems and the sun, ed. G. Wallerstein, ASP Conf. Series vol. 9, p. 417). The number of OH/IR stars have greatly increased as the result of OH surveys of cool *IRAS* sources (Lewis et al., 39.112.024; te Lintel Hekkert, 1990, Ph.D. thesis, University of Leiden).

6.5 Radio spectra of carbon–rich stars

Since CO is the most abundant molecule in the atmosphere of late–type stars, the rotational lines of CO are commonly observed in the circumstellar spectra of AGB stars, both oxygen and carbon–rich (Knapp et al., 31.112.001). However, the lines are generally stronger in carbon–rich objects. The CO lines are usually optically thick and have flattop profiles. The estimated mass loss rates for some infrared carbon stars exceed 10^{-5} $M_{\odot}\mathrm{yr}^{-1}$ (Knapp and Morris, 39.112.093). Several visual carbon stars have also been detected with CO emission with double–peaked profiles. This is consistent with the presence of a molecular shell detached from the photosphere (Olofsson et al., 45.112.057).
Many infrared carbon stars have rich molecular spectra, and in the case of CW Leo ~30 molecular species have been observed (Olofsson, 43.112.086). This includes organic molecules with molecular weights as high as 147 ($HC_{11}N$), suggesting that complex interstellar molecules could have been synthesized in cool star atmospheres. While the detection of complex molecules has so far been restricted to linear molecules, there is no doubt that the other complex molecular species are also present in the circumstellar envelopes of carbon stars, their line strengths only diluted by complexity of their rotational energy structure.

6.6 Conclusions

Infrared and radio observations of AGB stars show that these stars are losing mass. The high mass loss rates observed in OH/IR stars and infrared carbon stars suggest that these stars are close to the end of their AGB evolution. This also implies that there are two sub–branches of the AGB and not every star evolves through a carbon–rich phase (Kwok et al., 49.065.108). It is possible that a star begins on the AGB as a mass–losing oxygen–rich star and the dredge–up of heavy elements from the core gradually enriches the envelope (Iben and Renzini, 34.065.032). As the C/O ratio becomes near unity, silicate grains can no longer form as the result of all oxygen atoms being tied up in CO. Mass loss terminates and the star becomes a visual carbon star. After a period of the order of 10^4 yr mass loss resumes again with the formation of carbon–based grains and the star becomes an infrared carbon star. Mass loss continues until the entire envelope is depleted and the core is exposed. However, for some stars (probably those with higher initial masses) the C/O ratio never reaches unity before the depletion of the envelope. As a result they will remain oxygen–rich throughout the AGB and be observed as OH/IR stars during the last several 10^4 yr of their AGB evolution.

7 SN 1987A

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The detection of SN 1987A in February of 1987 revealed an event that dominated supernova research for the next three years and will continue to influence that and related fields for years to come. The basic situation was summarized by Nomoto in his report to Commission 35 for the last General Assembly. There are by now a substantial number of reviews that summarize the observations and theoretical analysis of SN 1987A. Among these are Dopita (46.125.612), Arnett et al. (1989, Ann. Rev. A. Ap. **27**, 629), Imshennik and Nadëzhin (1989, Sov. Sci. Rev. E; Ap. Sp. Phys. **8**, 1), Hillebrandt and Höflich (1990, preprint), and books that deal with SN 1987A and related topics including The ESO Workshop on SN 1987A (45.012.002), Proceedings of the Fourth George Mason Fall Workshop on Supernova 1987A

(45.012.090), Atmospheric Diagnostics of Stellar Evolution: Chemical Peculiarity, Mass Loss, and Explosion (46.012.067), Supernovae (ed. Petschek, Springer Verlag, 1990), Supernovae (ed. Woosley, Springer Verlag, in press), Supernovae (eds. Wheeler, Piran, and Weinberg, World Scientific, in press).

SN 1987A provided a number of confirmations of long-standing theoretical predictions. The principal one was that such an explosion would produce a strong flux of neutrinos. The data on the time of arrival and energy spectrum of the neutrinos was completely consistent with decades of work on the problem of gravitational collapse and neutron star formation. This observation and associated theory put important limits on the lifetime, charge, and magnetic moment of the electron neutrino. The flux was consistent with current predictions for the collapse of an iron core predicted by evolutionary calculations.

The information content of the neutrino flux was not sufficient to determine the nature of the explosion mechanism. The two popular choices are a prompt explosion due to rebound of the newly formed neutron core, or a delayed explosion due to heating of the infalling matter by the neutrino flux. Colgate (1989, Nature **341**, 489) argued for a different mechanism, a slow push by a "hot bubble", that would give different dynamics than an impulsive explosion with possible effects on the kinematics and mixing of the core material outward.

The principal prediction that follows from the existing theory and the detection of the neutrinos is that SN 1987A produced a neutron star. It is becoming more fascinating by the day that this prediction has not been directly verified. A report of rapid oscillations proved to be a subtle interference from an improperly shielded TV camera. Reports of a flattening of the light curve that could signal the asymptote to the bolometric flux provided by a buried pulsar have not yet been confirmed as of this writing. Although it seems very unlikely, there is a small possibility that SN 1987A made a black hole by producing a proto-neutron star that lingered for 10 seconds while it produced the observed neutrino signal and then collapsed, swallowing the neutrino flux abruptly after the flux was too faint to reveal such a feature. This possibility can only be ruled out by direct detection of the neutron star by some means. As it stands, any neutron star in SN 1987A must be rotating less fast than might have been expected, i.e. of order a few milliseconds, or must have a smaller magnetic field than might have been expected, i.e. of order 10^{12} Gauss, or both.

Another important prediction verified by SN 1987A was of the existence and properties of the gamma ray flux associated with the production and decay of radioactive ^{56}Ni to ^{56}Co and then to ^{56}Fe. Not only were gamma rays and X-rays from SN 1987A observed, and their properties used to study the nature of the explosion and subsequent mixing, but freshly synthesized Ni, Co, and Fe were directly observed, especially in the infrared. The mass of radioactive nickel produced by the explosion, about 0.07 solar mass, was on the low side of expectations, but otherwise very consistent with prior estimates.

Identification of the progenitor and its luminosity allowed an estimate of the helium core mass of the star, about 6 solar masses, and hence of its approximate initial main sequence mass, about 20 solar masses. Knowledge of the progenitor also gave information about the progenitor's radius at the time of the explosion. Subsequent dynamical calculations gave a good account of the light curve and an estimate of the mass at the time of the explosion, about 16 solar masses, implying some mass loss prior to the explosion. Calculations of the light curve accounting for the relatively compact state of the progenitor were consistent with the unexpected light curve shape (Woosley, 46.125.601). Further study has suggested that other similar light curves may have been detected and that such sub-luminous supernovae may be very common (Miller and Branch, 1990, preprint; Gaskell, 1990, preprint). The dynamical calculations also gave estimates of the energy of the explosion, about 10^{51} ergs that were very much in keeping with a priori expectations.

SN 1987A provided another verification of a long-standing prediction when Crotts (46.156.020) discovered the light echo from the supernova as the light from the original blast reflected off wisps of interstellar gas. This discovery gave a new tool to study both the supernova and the nature of the interstellar medium in its vicinity.

As it aged, the supernova developed infrared features consistent with molecular emission, especially CO (Danziger, 1990, in Supernovae, ed. Woosley, in press). At about 500 days after the explosion, SN 1987A began to form dust, a process which has been successfully modeled by Lucy (1990, in Supernovae, ed. Woosley, in press).

SN 1987A was a boon to the development of more quantitative supernova radiative transfer models both in the early atmospheric phase, in the later nebular phase and in its interaction with the circumstellar medium. Fransson and Lundqvist (49.125.145) showed that the fluorescence of the circumstellar nebula provides a diagnostic of both the burst of hard flux from the shock breakout and of the surrounding medium. Especially important were the development and application of non-LTE analyses (Höflich, 49.125.161; Eastman and Kirshner, 1989, Ap. J. **347**, 771; Schmutz et al., 1990, Ap. J. **355**, 255), although LTE analyses (Branch, 44.125.249; Harkness and Wheeler, 49.125.162) proved useful in studying the basic nature of the atmosphere at early times. There are still conflicts in terms of whether the non-LTE analyses can (Höflich) or cannot (Schmutz et al.) account for the strength and profile of the H alpha line. Many of the non-LTE analyses, in particular, are so complex that the nature of individual calculations is still not widely understood in the community, but the stage has been set for significant progress.

These spectral synthesis and atmosphere calculations led to a growing confidence that supernovae will become quantitative distance indicators. Estimates of the distance to the LMC based solely on SN 1987A agree with classical estimates to within about 10 percent, although Schmutz et al. argue that this agreement is model dependent and somewhat fortuitous.

Different techniques are required to analyze the spectra of the nebular phase when the ejecta become optically thin in the continuum, but remain optically thick in the lines, especially hydrogen (Axelrod, 46.125.059; Fransson and Chevalier, 1989, Ap. J. **343**, 323; Swartz et al., 49.125.230; Xu and McCray, 1990, in Supernovae, ed. Woosley, in press). These models require careful attention to the non-thermal effects of the deposition of the gamma rays from the radioactive decay. At late times there is still a quasi-continuum in the optical which may be the effect of the overlap of many lines from the heavy element-rich core (Xu and McCray), but many of the most prominant features in the optical may arise from the metals in the outer envelope, not the core (Swartz et al.) so that it is not yet clear that one can do quantitative nucleosynthesis on SN 1987A based on observed optical spectra and current modeling techniques.

There are a number of lines of evidence that SN 1987A was not spherically symmetric and underwent substantial mixing. Wampler and Richichi (49.125.186) resolved the circumstellar nebula in lines of [O III] at about 300 days after the explosion and Wood and Faulkner (IAUC 4739) gave evidence for bi-polar flow in this nebula from data at 600 days. The early release of X and gamma rays suggests that radioactive matter is mixed into the outer envelope, although this still might be some as yet ill-understood aspect of the gamma ray transport. The gamma ray lines seem to indicate a width that calls for a velocity of order 3000 km/s that exceeds the minimum observed for hydrogen, $\sim$ 2100 km/s, but the width of the gamma ray lines depends somewhat on the placement of the continuum. IR lines of nickel also seem to indicate a velocity of $\sim$ 3000 km/s and other IR metal lines give evidence for inner fragmentation of the ejecta (Spyromilio et al., 1990, MNRAS **242**, 699).

A number of groups are beginning to compute multi-dimensional models to explore the nature of mixing in SN 1987A and related contexts (Arnett et al., 49.125.144; Ebisuzaki et al., 1989, Ap. J. Lett. **344**, L65). These calculations require a finite perturbation of order 10 percent in the density structure, but do produce substantial mixing that agrees with some of the properties deduced for SN 1987A.

Although the light curve of SN 1987A could be reproduced with the condition that the progenitor be a blue supergiant, the question of why it was a blue supergiant emerged as one of the principal open issues. This question has not been adequately answered and the queries have broadened to cover a wide range of fundamental problems concerning massive star evolution.

There is no adequate evolutionary calculation of the progenitor of SN 1987A that is consistent with the observations of the supernova and the assumption that it was a representative massive star in the Large Magellanic Cloud. Calculations that assume Schwarzschild convection for mixing with no other ad hoc assumptions do not predict the star to return to the blue from the Hayashi track. Models that never go to the Hayashi track are not consistent with the UV observations of the supernova that show that the UV flash from the shock outbreak fluoresced from a surrounding slow moving circumstellar nebula that was entirely consistent with the progenitor having been a red supergiant shortly before its explosion, say 10^4 years. Calculations invoking the Ledoux criterion for mixing or various prescriptions for convective overshoot predict that the models should jump from the main sequence to the Hayashi track and back in a short thermal timescale, about 10^4 years. This is adequate for the final jump from the red to the blue state in which the star exploded, but completely fails to account for the large number of stars observed in the spectral range B0 to F0 in the LMC which demand an evolution on a slow core or shell burning nuclear timescale of order 10^6 years.

The most successful models for the latter seem to be basic models with Schwarzschild convection and moderate mass loss (Tuchman and Wheeler, Ap J, in press). These models do not predict a return to the blue from the Hayashi track, but the models are near the conditions for the penetration of the helium core by the convective envelope and some perturbation of the models might allow the red to blue transition. Alternatively, some models with semiconvection or overshoot that make the first blue to red transition too rapidly do have a core helium burning phase in the blue. Whether this "blue loop" can account as well as the basic Schwarzschild models for the overall distribution of the massive stars in the LMC remains to be seen.

The Schwarzschild models predict a small gap near the main sequence after the termination of core hydrogen burning and before the hydrogen shell–burning phase reaches thermal equilibrium. This gap is not seen. There is an interesting suggestion of a break in the rotation at about spectral type B1, just where this gap should be (Rosendhal, 3.065.002). Further study of the photometric and rotational properties of supergiants around B1 would be very useful to better understand the evolution of massive stars in the LMC and of the progenitor of SN 1987A.

8 BINARY AND MILLISECOND RADIO PULSARS AND EVOLUTION OF CLOSE BINARIES

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The vast majority of the presently known about 500 radio pulsars consists of single neutron stars, having pulse periods typically in the range 0.1 to 5 seconds and surface dipole magnetic field strengths B_s in the range $3 \cdot 10^{11} - 3 \cdot 10^{13}$G. Only a small fraction ($\approx$ 3 per cent of all pulsars) are known to be members of binary systems. The relatively small handful of binary pulsars have, however, made an enormous contribution to our understanding of the evolution of neutron stars and X–ray binaries. The binary radio pulsars appear to differ in a number of important characteristics from the bulk of the single radio pulsars, but have many characteristics in common with the millisecond radio pulsars, a new class of ultra–rapid pulsars discovered in 1982 (Backer et al., 32.141.547).

The binary and millisecond pulsars tend to have much shorter pulse periods and much weaker dipole magnetic fields B_s than the bulk of the radio pulsars:

- About half of the known binary radio pulsars (55%) and all millisecond pulsars have $P < 12$ ms, whereas 97% of all pulsars have $P > 30$ ms.
- The majority of the binary pulsars and all of the fourteen known millisecond pulsars have $B_s < 4.10^{10}$ Gauss, whereas 96% of all pulsars have $B_s > 3.10^{11}$ Gauss.

A further characteristic that suggests that the binary and millisecond pulsars are closely related and have a common origin is that about half of all millisecond pulsars are found in binaries, against only 3% of all pulsars.

Already soon after the discovery of the first binary by Hulse and Taylor (13.141.304), a system consisting of two neutron stars whose orbit is decaying by the emission of gravitational waves (Taylor and Weisberg, 1989, Ap.J. **345**, 434), it was recognized that this pulsar may have been spun up to its present rapid rotation rate (59 ms) by the accretion of matter from an evolving companion star, after its magnetic field had partly decayed. This concept of an old defunct pulsar being spun up and thereby rejuvenated by the accretion of matter (Bisnovatyi–Kogan and Komberg, 11.141.313; Smarr and Blandford, 18.141.306), received strong support since 1983 with the discovery of the first millisecond pulsar in a binary system (Boriakoff et al., 34.141.002).

In all but two of these systems the companion itself is a star near the endpoint of its nuclear evolution (a white dwarf), which indicates that in a preceding phase it was a (sub–)giant star which was transferring matter to the neutron star. This made clear the connection between the progenitors of the binary millisecond pulsars and certain subclasses of the bright galactic binary X–ray sources: systems in which the X–ray emission is powered by the accretion of matter onto neutron stars. It is now clear that the binary millisecond pulsars must be the descendants of bright low-mass X-ray binaries such as Sco X-1 and Cyg X-2 (see the review by Kulkarni, 1990, in "Neutron stars and their birth events", ed. W. Kundt, p. 59).

A key discovery, linking the binary millisecond pulsars with the bright low-mass X-ray binaries, was that of Quasi Periodic Oscillations (QPO) in the X-ray emission of the latter systems (cf. the review by Van der Klis, 1989, Ann. Rev. A. Ap. **27**, 517). The one-to-one relationship between QPO-period and X-ray intensity observed in sources such as GX 5-1, can be most easily understood in terms of a model of "gated accretion" onto a rapidly spinning ($P \approx 10$ milliseconds) weakly magnetized ($B_s \approx 10^9$G) neutron star (Alpar and Shaham; see the review by Lamb, 1989, 40.142.003, in "Timing Neutron Stars", eds. H. Ögdman, E.P.J. van den Heuvel, p. 649). Thus the bright low-mass X-ray binaries appear to contain weakly magnetized neutron stars with millisecond spin periods very similar to those observed in the binary millisecond pulsars.

Important new highlights in this field in the last few years were:

- The discovery by Kulkarni (42.126.015) that the white dwarf companion of the binary pulsar PSR 0655+64 has a cooling age of at least 10^9 years (as evidenced by its low surface temperature). As this pulsar has a surface dipole magnetic field strength $\approx 10^{10}$ Gauss and was formed earlier than its white dwarf companion, this provided strong evidence that the magnetic fields of neutron stars do not decay below a certain "bottom" value, which can be as high as 10^{10} Gauss.

  Further evidence for the absence of field decay once a certain "bottom" value has been reached, was derived from the statistical properties of millisecond pulsars in the galaxy (Bhattacharya and Srinivasan, 42.126.041; van den Heuvel et al., 42.067.002). The observed number of millisecond pulsars in the galaxy indicates that their total galactic number must be several tens of thousands (Stokes et al., 42.126.078), which is of the same order as the total number of descendants of their progenitor systems, the low-mass X-ray binaries.

  The fact that all millisecond pulsars produced since the formation of the Galaxy are still observable today therefore implies that their magnetic fields do not decay. (The electromagnetic braking torques due to the $\leq 10^9$ G magnetic fields of millisecond pulsars are so low that, in the absence of magnetic field decay, a millisecond pulsar will remain observable for a Hubble time).

- The discovery of many binary and millisecond pulsars in globular star clusters (Lyne et al., 44.126.057; Lyne et al., 45.126.061), which were already known to be unusually rich in binary X-ray sources. More than 70% of all known millisecond pulsars and about half of all known binary pulsars have been found in globular clusters during the last three years, mostly due to the work of Lyne and co-workers and Kulkarni and co-workers. Many clusters contain more than one such pulsar, for example M15 has three binary and millisecond pulsars and in Terzan 5 two radio pulsars have been detected together with ten more radio sources, all probably pulsars (S. Phinney, private comm.). The total pulsar population of all globular clusters together is estimated to be at least 10^3 and possibly as high as 10^4 (Kulkarni et al., 1990, Ap.J. **356**, 174).

  This large number is in line with the high abundance of X-ray binaries in globular clusters, and is presumably due to a variety of capture processes (two-body tidal collisions and exchange collisions) by which neutron stars in globular clusters can form binary systems with normal cluster stars and "consume" such stars in a mass-accretion process, in which they themselves are being spun up to a rapid rotation rate. The high abundance of millisecond pulsars in globular clusters is further evidence for the absence of magnetic field decay once a "bottom" magnetic field strength has been reached.

- A third major breakthrough, especially important for our understanding of the fate of close low-mass X-ray binaries and the formation of single millisecond pulsars in the galactic disk, has been the discovery of two eclipsing binary radio pulsars with companion stars that are clearly being evaporated by the high-energy pulsar radiation impinging on their surface. The first discovered system PSR 1957+21 (Fruchter et al., 45.126.099) contains a 1.6 millisecond pulsar with a ≤ 0.02 $M_\odot$ companion which is surrounded by a large "halo" of out-flowing wind matter which forms a long comet-like tail behind this star. Optical observations in combination with radio observations and theoretical considerations suggest that the companion star will be completely evaporated within the coming few million years (see e.g. van den Heuvel and van Paradijs, 46.126.041). The possible occurrence of such evaporation of red dwarfs in low–mass neutron star binaries had been predicted already in 1987 by Ruderman and co–workers (Kluzniak et al., 46.126.040; Ruderman et al., 49.065.014) and was beautifully confirmed by the discovery of this system and of a second system of similar type, PSR 1745–24 in the globular cluster Terzan 5 ($P_{orb} = 1.9$ hours, $P_{pulse} = 11$ ms, $M_{compan} \approx 0.1\ M_{\odot}$). It thus seems that the low–mass X–ray binaries in which the companion is a red dwarf star (i.e. with orbital periods $\leq$ 12h) may terminate life as single millisecond pulsars, whereas those in which the companion is an evolved low–mass star ($P_{orb} \geq$ 12h) will terminate as a binary with a low–mass helium white dwarf companion. In globular clusters single millisecond pulsars can also form as result of binary disruption by encounters with field stars.

This entire field is still in rapid progress. What can already be concluded from the above findings is that, apart from the rich information obtained on the final evolution and fate of low–mass X–ray binaries, the most important result is that our thinking about the interior structure and evolution of neutron stars should be completely revised. The large variety in strengths of observed "bottom fields", ranging from 2.10^8 to 10^{11} Gauss (or perhaps as high as 10^{12} G in Her X–1) strongly suggests that – contrary to the beliefs of the past twenty years – the magnetic fields of neutron stars do not decay spontaneously. The observations can be consistently understood in terms of a model in which crustal magnetic fields do decay only as a result of accretion – and in rough proportion to the amount of matter accreted. Statistical considerations suggest that the galactic single pulsar population may for several tens of percents consist of neutron stars that have been recycled in mass–transfer binaries, and as a result underwent various degrees of field decay (Bhattacharya and van den Heuvel, 1991, Phys. Repts., in press). This may have created the earlier impressions that magnetic fields of single neutron stars do decay spontaneously.
For further reading we refer to the reviews by van den Heuvel (1987, 44.126.030; 1989, in "Timing Neutron Stars", eds. H. Ogelmar, E.P.J. van den Heuvel, p. 523), Kulkarni (1990, in "Neutron Stars and their Birth Events", ed. W. Kundt, p. 59), Verbunt (1990, in "Neutron Stars and their Birth Events", ed. W. Kundt, p. 179), Srinivasan (1990, A.Ap.Rev. **1**, 209) and Bhattacharya and van den Heuvel (1991, Phys. Repts., in press).

9 PROGRESSES IN COMPUTATIONAL METHODS OF STELLAR MODELS

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In this summary we shall concentrate on the various numerical approaches used to compute stellar evolutionary sequences of one–dimensional stellar models. It must be emphasized that this discussion concerns the numerical solution of the differential equations and not the physical ingredients (as opacity, nuclear energy rates, etc.).
The problem of solving the four coupled differential equations of stellar structure can be handled by three basic methods (Wilson, 29.065.111). The most commonly used is the Henyey method (Henyey et al., 1964.5520). In this method derivatives are approximated by truncated Taylor expansions and the equations replaced by difference schemes. A detailed description of a difference scheme obtained from the discretisation of the structure equations may be found in Kutter and Sparks (8.065.011). Let us note here that the resulting difference equations may have extra solutions in addition to those of the differential equations. As pointed out by Fryxell et al. (1989, in *Numerical Methods in Astrophysics*, ed. P.R. Woodward, Academic Press, New York), this "numerical noise" may be considered either as *a mathematical problem* or as an indication of the *inability* of the difference equations to represent the physics involved, this latter point of view being often of more practical value. In the last decade various modifications of the Henyey method have been proposed: Wilson (29.065.111) describes a computational method which combines the best features of the "integration" and "Henyey methods" (the U,V fitting method by Schwarzschild,1958.5224, falls within the category of the integration methods). Budge (43.065.010) discusses a fourth order Henyey method with accurate error control for calculating quasi–static, nonrotating stellar models. A general numerical method, based on Henyey method, has

been developed by Nobili and Turolla (46.021.021) for solving systems of ordinary differential equations which exhibit one, or more, critical point. Savonije and Takens (17.065.017) present a generalisation of the Henyey scheme that introduces the mass of the convective core and the density at the outer edge of the convective core boundary as unknowns which have to be solved for simultaneously with the other unknowns. Van der Linden (43.065.015) extends this method for application to convective zones of any form (central, intermediate as well as surface convective zones).

The way discretisation is performed may have drastic consequences on the results obtained. To illustrate this point let us note that standard implicit evolution codes (e. g. Kippenhahn et al., 1967 in *Methods in computational Physics*, eds Alder et al., Academic Press, **7**, 129) use difference schemes with all variables centered at common points in space and time. The use of common centering leads to a numerical instability as time steps become shorter than the thermal diffusion time across individual zones. As explained in Habets (43.065.027), this instability is due to averaging of the energy production ϵ over two adjacent mesh points in the difference form of the equation of energy conservation. If the energy flow L_r is close to zero in two adjacent mesh points n and n+1, the solution of the structure equations (derived with the trapezoidal rule) gives $\epsilon^n \sim \epsilon^{n+1}$. If this solution is adopted, further evolution picks up the "unstable branch" of the differential equations for energy conservation (Sugimoto, 3.065.013). Hence this solution is not realistic physically. Sugimoto (3.065.013) shows that a scheme in which the difference equation for energy flow and the difference equation for energy conservation are off center in opposite directions in the limit of rapid evolution, removes the instability. A similar stable numerical method adapted for the treatment of dynamical problems is presented in Sugimoto et al. (32.065.069). In the difference scheme described by Kutter and Sparks (8.065.011) the state variables (as pressure and temperature) are defined at zone centers and structural variables (as radius, luminosity and velocity) are defined at zone boundaries. Computations by Sparks and Endal (27.065.041) and Gilliland (31.080.003) using staggered structural and state variables do not develop the above instability. A more sophisticated way to overcome this difficulty is described in Habets (43.065.027).

As is well known the calculation of composition changes inside the stars requires the solution of a set of coupled ordinary differential equations called a nuclear-reaction network. Arnett and Truran (2.065.004) used an implicit method solving simultaneously the equations for the various nuclear species; the equations for the changes of chemical composition were linearised. Maeder (33.065.025) has extended this method in order to include also the non-linear effects. In evolutionary computation, the standard procedure is to solve the network separately from the other evolution equations. This "decoupling" of network and structural equations can lead to instabilities in explosive nuclear burning at conditions close to nuclear statistical equilibrium. In Müller's (42.065.001) approach, the system of equations consisting of the equations describing the chemical composition changes and the conservation of energy are solved with the Newton-Raphson techniques, i.e. abundances and temperature are updated together. This guarantees that the strong temperature dependence of the nuclear reaction rates does not lead to instabilities. In van der Linden (43.065.015), the four structure equations are solved simultaneously with a diffusion equation for calculating the change with time of the composition.

The late evolutionary stages present particular difficult problems to handle in evolutionary computations. As is well known, when a massive star enters the final stages (beyond the He-burning phase), the time steps become very short due to the huge amount of energy which escapes from the core through neutrino emissions. As already mentioned such a decrease of the time steps may lead in certain circumstances to instabilities. Moreover some standard assumptions as the instantaneous mixing in the convective zones or the hydrostatic hypothesis (at least during silicon-burning) are no longer valid. Various numerical recipes have been proposed to handle the problem of time-dependent convection both for energy transport and chemical mixing (Unno, 1967.54115, Arnett, 2.125.014, Wood, 11.065.127, Sparks and Endal, 27.065.041, Hollowell et al., 1990, Astrophys. J., **351**, 245). For what concerns the mixing of the chemical elements, most authors seem to apply a diffusion treatment (Weaver et al., 22.065.033, Habets, 43.065.027). The numerical scheme for the solution of the diffusion equation is described in Schatzman and Maeder (29.065.033). A finite difference form of this equation is also given in van der Linden (43.065.015). The introduction of the inertial term in the equation of motion may give rise to some numerical difficulties in certain circumstances. For instance, most inviscid hydrodynamic difference schemes are unable to describe shocks. Indeed, the absence of a viscosity term in the equations does not allow for the dissipation of kinetic energy into heat. The method most often used to reduce this problem is adding an artificial dissipation term into the difference equations as suggested by von Neumann and Richtmyer in 1950 (Richtmyer and Morton, 1967, in *Difference Methods for Initial-Value Problems*, 2nd ed., Interscience, New York). A detailed description of a method, including the artificial viscosity term, for calculating stellar core collapse is discussed in Bowers and Wilson (32.065.078). Recently Noh (1987, J. Comput. Phys., **72**, 78) has pointed at several errors induced by the artificial viscosity. Alternative methods have been proposed, many of them are discussed in the recent workshop *The Numerical Modelling of Nonlinear Stellar Pulsations, Problems and Prospects* (1990, ed. J. R. Buchler, Kluwer Academic Publishers). Marti et al. (1990, Astron. Astrophys., **235**,535) have implemented a Godunov-type method for stellar collapse. They have made a sample of stellar collapse calculations with their code and a standard finite difference scheme which uses the artificial viscosity technique. Differences in the behaviour of the velocity field and the global energetics of the collapse have been found. Fryxell et al. (1989, in *Numerical Methods in Astrophysics*, ed. P.R. Woodward, Academic Press, New York) have studied different numerical tools for investigating the coupled processes of hydrodynamic and nuclear burning. For one-dimensional problems they found that a good second order lagrangian difference scheme, without artificial viscosity term, will provide satisfactory results for a wide range of problems if adequate zoning is used.

The very fast developments of the computing devices and the construction of more sophisticated numerical schemes have enabled to explore new areas in the domain of stellar evolution as, for instance, inclusion of rotation (Black and Bodenheimer, 14.062.010; Pinsonneault et al., 49.080.011; Durisen et al., 50.065.084) or the description of hydrodynamic events in multi-dimensional models (Chevalier and Klein, 21.125.005; Nagasawa et al., 46.065.142; Benz and Thielemann, 49.125.266; Müller et al., 50.064.022). The subject is too vast to be detailed here. The great computing efficiency can also permit now to make fast comparisons of different approaches to physical problems in stellar evolution. The treatment of convection in the outer layers has been reexamined by Pfenniger (1982, unpublished), who uses a mixing length proportional to the density scale height H_ρ instead of the pressure scale height H_p. The choice of H_ρ prevents the well known and unphysical density inversion from occuring (cf. Bisnovatyi-Kogan and Nadyozhin, 7.065.082); this density inversion is the source of many numerical difficulties encountered in the models of red supergiants. Pedersen et al. (1990, Astrophys. J, **352**, 279) studied the effects of modifications to the original mixing length theory on the predicted effective temperatures of cool stars, in Mazzitelli (49.065.081), some algorithms for numerical integration, mainly the shell shift technique, are tested.

At this time, even if a great deal of evolutionary sequences have been published, detailed comparisons between the results obtained by different authors and an investigation of the origin of the differences (especially concerning the final stages of stellar evolution) are difficult due to a lack of sufficient details in the description of the methods used in most publications. Hopefully in the future, thanks to the greater flexibility of the computing devices, more tests and comparisons will be carried out in order to allow the "Star-Makers“ to improve the numerical tools at their disposal.