Math. Proc. Camb. Phil. Soc. (2007), **143**, 255 © 2007 *Cambridge Philosophical Society*
doi:10.1017/S0305004107000382 *Printed in the United Kingdom*

CORRIGENDUM
The abelianization of the congruence IA-automorphism group of a free group

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(Volume 142 (2007), 239–248)
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Abstract

We fill a gap in the construction of the extended Johnson homomorphism, stated in Section 2 of the paper *The abelianization of the congruence IA-automorphism group of a free group*.

(i) Page 5, lines 7–13: For a $\mathbf{Z}/d\mathbf{Z}$-valued Magnus expansion θ, it is not true if d is even that the image $\mathrm{Im}(\tau^\theta)$ of a crossed homomorphism τ^θ is $(H^* \otimes_z \Lambda^2 H) \otimes_z \mathbf{Z}/d\mathbf{Z}$ in $(H^* \otimes_z H^{\otimes 2}) \otimes_z \mathbf{Z}/d\mathbf{Z}$. If d is odd, it is true. Moreover, if d is even,

$$\mathrm{Im}(\tau^\theta) \subset ((H^* \otimes_z \Lambda^2 H) \otimes_z \mathbf{Z}/d\mathbf{Z}) \oplus \left(\frac{d}{2} \cdot (\mathbf{Z}/d\mathbf{Z})\right)^{\oplus n^2}. \qquad (1)$$

Hence for even d, we have to modify the construction of the extended Johnson homomorphism, also denoted to by τ^θ,

$$\tau^\theta : IA_{n,d} \longrightarrow (H^* \otimes_z \Lambda^2 H) \otimes_z \mathbf{Z}/d\mathbf{Z}$$

by composing a natural projection $\mathrm{Im}(\tau^\theta) \to (H^* \otimes_z \Lambda^2 H) \otimes_z \mathbf{Z}/d\mathbf{Z}$ with a restriction of the crossed homomorphism τ^θ to $IA_{n,d}$. Although we modify the construction of the extended Johnson homomorphism τ^θ, our result

$$IA_{n,d}^{\mathrm{ab}} \simeq ((H^* \otimes_z \Lambda^2 H) \otimes_z \mathbf{Z}/d\mathbf{Z}) \oplus \Gamma(n,d)^{\mathrm{ab}}$$

remains true for any d as an abelian group. Furthermore we see that this isomorphism is an $\mathrm{SL}(n, \mathbf{Z}/d\mathbf{Z})$-equivariant for odd d, and not for even d.

In order to show (1), we use the same notaton as in [**1**]. Set $R := \mathbf{Z}/d\mathbf{Z}$. Define

$$\Gamma_2^d := \langle xyx^{-1}y^{-1}, x^d \,|\, x, y \in F_n \rangle.$$

It is easy to show that for any R-valued Magnus expansion θ, $\Gamma_2^d = \theta^{-1}(1 + \hat{T}_2)$. Then we have

$$\theta_2(\Gamma_2^d) = \begin{cases} \Lambda^2 H_R, & d : \text{odd} \\ \Lambda^2 H_R \oplus \left\langle \frac{d}{2}[x_i]^{\otimes 2} \,|\, 1 \leqslant i \leqslant n \right\rangle, & d : \text{even}, \end{cases} \qquad (2)$$

Denote by U the right-hand side of (2). It is obvious that $U \subset \theta_2(\Gamma_2^d)$. To prove $\theta_2(\Gamma_2^d) \subset U$, it suffices to check that the images of the generators of Γ_2^d are in U. We easily see that $\theta_2(xyx^{-1}y^{-1}) \in \Lambda^2 H_R$. Furthermore,

$$\theta(x^d) \equiv 1 + \frac{d(d-1)}{2}[x]^{\otimes 2}$$

modulo $\hat{T}_3$. Hence if d is odd, $\theta_2(x^d) = 0$, and if d is even,

$$\theta_2(x^d) = \frac{d}{2}[x]^{\otimes 2} \in H_R^{\otimes 2}.$$

By writing $[x] = a_1[x_1] + \cdots + a_n[x_n]$, $a_i \in R$, we have

$$\theta_2(x^d) = \frac{d}{2}\left\{\sum_{i=1}^{n} a_i^2[x_i]^{\otimes 2} + \sum_{i<j} a_i a_j([x_i]\otimes[x_j] - [x_j]\otimes[x_i])\right\} \in U.$$

By the argument stated in [**1**, section 2] we obtain a required crossed homomorphism

$$\tau^\theta : \mathrm{Aut}F_n \longrightarrow \mathrm{Hom}_z\big(H_R, \theta_2\big(\Gamma_2^{\mathrm{d}}\big)\big) = H_R^* \otimes_z \theta_2\big(\Gamma_2^{\mathrm{d}}\big),$$

and (1) is proved.

(ii) Page 6, line 7: $H_2(IA_n, \mathbf{Z})$ and $H_2(IA_{n,d}, \mathbf{Z})$ should read $H_2(IA_{n,d}, \mathbf{Z})$ and $H_2(\Gamma(n,d), \mathbf{Z})$ respectively.

(iii) Page 6, line 13: IA_n^{ab} should read $IA_{n,d}^{\mathrm{ab}}$.

Acknowledgements. The author would like to thank Masatoshi Sato and Nariya Kawazumi for discovering and showing me a precise proof for the existence of an extended Johnson homomorphism using the Magnus expansions.

REFERENCE

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