and $p_x = \frac{D_{x+1}}{D_x} \cdot \frac{1}{v}$, or $p_x = \frac{D_{x+1}}{D_x}(1+r)$, as may be found most convenient.

2. Should it be preferred to exhibit a table of decrements of the usual form, it is very easily deduced by the formula $D_x = l_x v^x \therefore l_x = D_x \cdot \frac{1}{v^x}$, or $D_x(1+r)^x$.

I remain, Sir,
Your most obedient Servant,
H. A. S.

Aberdeen, 4th August, 1856.

ON FORMULÆ FOR USING TABLES OF LOGARITHMS.

To the Editor of the Assurance Magazine.

SIR,—I understand that you are about to publish some tables of logarithms to twelve places of decimals, and I have thought that it might be interesting to you to have your attention called to the following formulæ for using the tables:—

Suppose $\log.(x \pm h) = \log. x \pm y$, then

$$\log. y = \log.\left(\frac{mh}{x}\right) \mp \frac{1}{2}\left(\frac{mh}{x}\right) \text{ nearly} \quad . \quad . \quad . \quad (1)$$

$$\log. h = \log.(xMy) \pm \frac{1}{2} y \text{ nearly} \quad . \quad . \quad . \quad (2)$$

The first is obtained by taking the logarithm of y in the equation

$$\pm y = \log.(x \pm h) - \log. x$$

$$= \pm \frac{mh}{x} - \frac{m}{2}\frac{h^2}{x^2} \pm \frac{m}{3}\frac{h^3}{x^3} - \frac{m}{4}\frac{h^4}{x^4} \pm \ldots\ldots$$

$$= \pm \frac{mh}{x}\left\{1 \mp \frac{h}{2x} + \frac{h^2}{3x^2} \mp \frac{h^3}{4x^3} + \ldots\ldots\right\}$$

and the complete expression for $\log. y$ is

$$\log. y = \log.\left(\frac{mh}{x}\right) \mp \frac{1}{2}\left(\frac{mh}{x}\right)\left\{1 \mp \frac{5h}{12x} \pm \frac{h^2}{4x^2} \mp \frac{251h^3}{1440x^3} \pm \frac{19h^4}{144x^4} \mp \ldots\right\}$$

I found this formula for myself, and am not aware that it has yet been published in print.

The second formula is due to Legendre, who establishes it nearly as follows:—Suppose $\log. X = \log. x \pm y$, $X = x \pm h$, whence $X = x\varepsilon^{\pm My}$;

$$\therefore \pm h = X - x = x(\varepsilon^{\pm My} - 1) = \pm x\varepsilon^{\pm \frac{1}{2}My}(\varepsilon^{\frac{1}{2}My} - \varepsilon^{-\frac{1}{2}My}).$$

Expanding the term in parentheses, we have

$$h = (xMy)\varepsilon^{\pm\frac{1}{2}My}\left(1 + \frac{1}{6}\frac{M^2y^2}{4} + \frac{1}{120}\frac{M^4y^4}{16} + \ldots\ldots\right);$$

and, finally, taking the logarithm

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$$\log. h = \log.(x\mathrm{M}y) \pm \frac{1}{2}y + \frac{\mathrm{M}y^2}{24}\left(1 - \frac{\mathrm{M}^2y^2}{120}\right) \text{ nearly,}$$

M and m are the modulus and its reciprocal. The values of their common logarithms, as computed by Mr. Maynard, are

$$\text{Log.}\, m = 9{\cdot}63778\ 43113\ 00536\ 78912\ 29674\ 98645$$
$$\text{Log.}\, \mathrm{M}_1 = 0{\cdot}36221\ 56886\ 99463\ 21087\ 70325\ 01355$$

Before proceeding to exemplify the use of the foregoing formulæ, it will be convenient to mention a method, due (I believe) to Burckhardt,* of splitting numbers into factors for logarithmic purposes.

Take some square number, a^2, a little greater than the given number x, and let the approximate square root of the remainder be b, so that $x = a^2 - b^2$ nearly, $= (a+b)(a-b)$ nearly. Here we have two factors at once, and by a little management a and b may be found, so that both $(a+b)$ and $(a-b)$ shall be capable of further resolution; or we may diminish $(a+b)$ and increase $(a-b)$, or *vice versâ*, by a small quantity c, with the assistance of the formulæ

$$(a+b-c)(a-b+c) = (a^2-b^2) + 2bc - c^2$$
$$(a+b+c)(a-b-c) = (a^2-b^2) - 2bc + c^2$$

As an example capable of easy verification, I take the logarithm of ($\pi = 3{\cdot}14159\ 26535\ 90$)

```
      3·14 15 92 65 35 90(1·7725
      1
27)214
   189                           1 63 60(128
347)2515                         1
    2429                     22) 63
3542) 8692                       44
      7084                   248)1960
35445)160865                     1984
      177225                     +24
     -16360
```

Whence

$$\pi = \overline{1{\cdot}7725}|^2 - \overline{0{\cdot}0128}|^2 + 0{\cdot}00000\ 02435\ 90$$
$$= 1{\cdot}7853 \times 1{\cdot}7597 + 0{\cdot}00000\ 02435\ 90$$

Now, applying Formula (1),

log. 1·7853	=0·25171 12049 84	h	=0·00000 02435 90
log. 1·7597	=0·24543 86340 36	log. h	=3·38665 95
log. x	=0·49714 98390 20	ar. co. log. x	=9·50285 02
y	=0·00000 00336 74	log. m	=9·63778 43
log. π	=0·49714 98726 94	log. y	=2·52729 40

* Burckhardt, *Table des Diviseurs pour tous les Nombres depuis* 1 *à* 3,036,000. Paris, 1817. 4to. (Introduction.)

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This value is correct even to the last figure. As y has no significant digit in the seventh place, it was unnecessary to introduce the second correction. If we *had* required this, we should have found y approximately, and subtracted half this approximate value of y from the log. $\left(\frac{mh}{x}\right)$.

As an example of Formula (2)—Required the number corresponding to the logarithm $m=0{\cdot}43429\ 44819\ 03$.

By the seven-figure table we find, for an approximate value,

$$
\begin{array}{l}
\quad 2{\cdot}71\ 82\ 82(1{\cdot}649 \\
\quad 1 \\
26)171 \\
\quad 156 \\
324)1582 \\
\quad\ 1296 \\
3289)28682 \\
\qquad 29601 \\
\qquad 919=\overline{30}|^2 \text{ nearly.}
\end{array}
$$

$$
\begin{array}{rl}
\therefore & 2{\cdot}718282 \\
 & \quad +19 \\
 & 2{\cdot}718301=\overline{1{\cdot}649}|^2-\overline{0{\cdot}030}|^2 \\
 & \qquad\qquad =1{\cdot}679\times 1{\cdot}619
\end{array}
$$

$$
\begin{array}{lrl}
\log. & 1{\cdot}679 & =0{\cdot}22505\ 06961\ 38 \\
,, & 1{\cdot}619 & =0{\cdot}20924\ 68487\ 53 \\
,, & x & =0{\cdot}43429\ 75448\ 91 \\
,, & (x-h) & =0{\cdot}43429\ 44819\ 03 \\
 & y & = \qquad\quad 30629\ 88 \\
 & & \\
 & h & = \quad -1\ 91715\ 4 \\
 & x & =2{\cdot}71830\ 1 \\
\text{antilog.} & m & =2{\cdot}71828\ 18284\ 6
\end{array}
$$

$$
\begin{array}{lrl}
\log. & x & =0{\cdot}43429\ 75 \\
,, & M & =0{\cdot}36221\ 57 \\
,, & y & =4{\cdot}48614\ 53 \\
 & & \quad 5{\cdot}28265\ 85 \\
 & -\frac{1}{2}y & = \qquad -\ 15 \\
\log. & h & =5{\cdot}28265\ 70
\end{array}
$$

which again is right, even to the last figure.

I have given you these methods, because I do not find them mentioned in Vega, or in any other work upon logarithms, and I think them less troublesome than the use of second differences, especially for the inverse process of finding a number from its logarithm. Legendre's formula makes the antilogarithmic canon a superfluity, and the difference column is not required for either formula.

I possess a copy of Briggs (14 figg.), containing the 101st chiliad, as well as the usual 30 chiliads; and I have a French table containing the 102nd chiliad to 11 figures, besides some odd logarithms to 19 figures in an Italian collection and in Legendre. If you should desire to collate any of these they are much at your service.

I am, Sir,
Your obedient Servant,

13, *Brompton Row.* C. W. MERRIFIELD.

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