

## JOINT DISCUSSION

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### 7. THE CEPHEID DISTANCE SCALE AND GALACTIC KINEMATIC PARAMETERS

HAROLD WEAVER

Cepheid variables have long been a primary source of knowledge of galactic distances. It is therefore important to examine critically the accuracy of the cepheid distance scale to determine whether it is adequate for present needs.

Proper motions resulting from the solar motion provide a fundamental determination of the distance scale, according to the well-known formula:

$$\text{est } r = S_{\odot} \sin \Delta / k\mu_v.$$

Photometric distances follow from this  $r$  in the usual way:

$$5 \log r - 5 = y = m_{pg} - \chi \text{ C.E.} - M_{pg}.$$

We adopt 4.0 as the value of  $\chi = A_{pg}/\text{C.E.}$  in the  $P, V$  system.

We assume that both the  $P-L$  and  $P-C$  relations (period—intrinsic colour at maximum) are linear in  $\log P$ . The present state of knowledge does not allow us to take account of such factors as shape of light-curve. Thus we write:

$$M_{pg} = -(0.28 + \Delta M) - 1.74 \log P,$$

where the numerical coefficients are Shapley's [1] and  $\Delta M$  the correction to his zero-point. Also:

$$\text{C.E.} = (P - V)_{\text{obs}} - \chi(a + b \log P).$$

Then

$$y = m_{pg} - \chi(P - V)_{\text{obs}} - (\chi a - 0.28) - (\chi b - 1.74) \log P + \Delta M. \quad (1)$$

If we take  $y$  as fixed by the proper motions, via the est  $r$  above, and if we can decide on a  $P-C$  relation, that is, on  $a$  and  $b$ , we can determine  $\Delta M$ , the zero-point correction. The dependence of this on the adopted  $P-C$  relation is crucial. For example, if two observers adopt two  $P-C$  relations differing only by a constant, thus:

$$(P - C)_2 = (P - C)_1 - q, \quad \text{then } \Delta M_2 = \Delta M_1 + \chi q.$$

Many  $P-C$  relations have been proposed, as illustrated in Fig. 11. To derive the  $\Delta M$  appropriate to each, we employ the following additional observational data:

1. Accurate  $\mu_v$  components tabulated by Blaauw and Morgan [2] for eighteen population I cepheids with light-curves of all forms;
2. the solar motion adopted by Blaauw and Morgan [2] (21 km/sec towards  $\alpha = 270^\circ$ ,  $\delta = +30^\circ$ );
3. median apparent magnitudes on the  $P$  system observed photo-electrically by Eggen, Gascoigne and Burr [3] for seventeen of the stars;
4. photo-electric colours  $(P - V)_{\text{obs}}$  at maximum light observed by Eggen, Gascoigne and Burr [3] and independently by Kron and Svolopoulos [4].

With these data we derived the  $P-L$  zero-point corrections listed in Table 1.

# LUMINOSITY OF CEPHEIDS

Table 1. *Zero-point correction to the Shapley P-L relation*

<i>P-C</i> relation	Derived $\Delta M$
Eggen [5]	-1 <sup>m</sup> .45
Kron and Svolopoulos [4]	-1.74
Code [6]	-1.76
Stibbs [7]	-1.89
Gascoigne and Eggen [8]	-2.13
Walraven, Muller and Oosterhoff [9]	-2.15
Schmidt [10]	-2.53

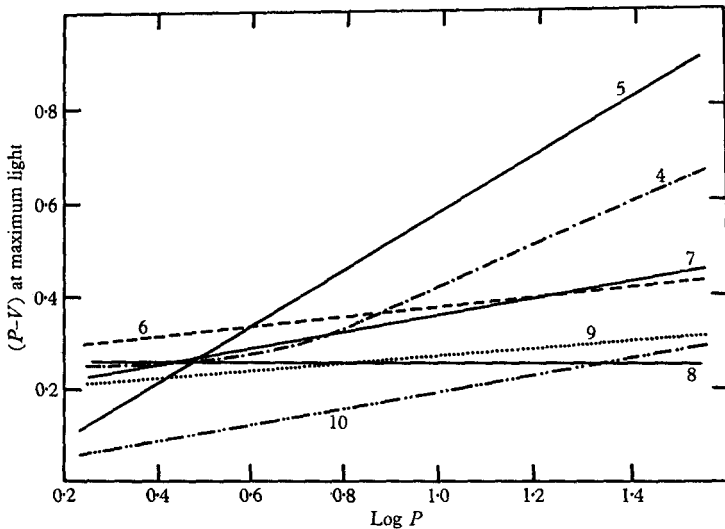


Fig. 11. Some of the period-intrinsic colour relations suggested for galactic cepheids. The numbers refer to references at the end of this paper.

Stibbs' published correction was  $-1.4$ , Gascoigne and Eggen's  $-1.7$ , Walraven, Muller and Oosterhoff's also  $-1.7$ .

We can make an approximate check of some of our results by considering the mean extinction as a function of  $\log P$ . In particular if in equation (1) the coefficients  $b$  and  $1.74$  of  $\log P$  are consistent, the mean extinction will be independent of  $\log P$ . This test strongly rejects Eggen's [5]  $P-C$  relation, but provides no choice between those of Code [6] and Gascoigne and Eggen [8]. With the latter we find a mean extinction of  $1.27$  mag/kpc, with the former  $0.55$  mag/kpc. Each mean refers to the same 55 cepheids. A group of B stars with photo-electrically observed colours and a spatial distribution similar to that of the cepheids yields the value  $0.92$  mag/kpc. These values should be identical except for statistical fluctuation, and if we accept the assertion that we know the intrinsic colours and distance scale of the B stars, we must adjust the cepheid  $P-C$  relations accordingly. Gascoigne and Eggen's intrinsic colour then becomes  $0.34$  for all periods, and Code's  $0.25 + 0.11 \log P$ . The zero-point correction in each case is  $-1^m.80 \pm 0.40$ .

Intrinsic colours have been observed at maximum light for cepheids in four clusters and two double stars [11]. Scatter is present; the mean colour at maximum light for these six stars is  $0.36$  on the  $(P-V)$  system, agreeing closely with the  $0.34$  found above. The four cluster values for the  $P-L$  zero-point correction range from  $-0.8$  to  $-1.7$ , with a mean of  $-1^m.2 \pm 0.2$ . The agreement of this figure with the proper motion values is less satisfactory. The cluster value is uncertain to the extent that the position of the zero-age main sequence is uncertain.

## JOINT DISCUSSION

### REFERENCES

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## 8. DISCUSSION

DR S. A. ZHEVAKIN described his theory of pulsating variables:

This theory [1-7] is based on the idea that stellar pulsations are self-oscillations, continuously maintained from internal sources of energy by means of the valve action of a critical zone of doubly-ionized helium. The theory can be applied to long-period variables of the RR Her type [8], to the  $\beta$  CMA variables, to semi-regular variables like AG Cyg and RS Cnc, and to the long-period  $\alpha$  Cet stars. For all these types of stars the theory

1. explains the cause of the oscillations;
2. predicts the correct amplitudes;
3. predicts the correct relations between variations of radial velocity and of brightness;
4. gives the correct asymmetry of the radial-velocity variations;
5. explains why pulsations can occur only in giants and super-giants, and only in those which occupy a definite region in the Hertzsprung-Russell diagram;
6. in the case of the classical cepheids, leads to a  $P-L$  relation with zero-point according to Baade and dispersion of about 1<sup>m</sup>0.

We shall designate as the envelope the outer part of the star; its oscillations are not quasi-adiabatic. The envelope consists of a zone of critical double ionization of helium, which we take as the layer with  $\gamma_1 - 1 = (d \ln T / d \ln \rho)_{\text{ad}} < 0.26$ , and of the atmosphere above that zone. As the parameter of non-adiabaticity of the oscillations of the helium zone we adopt the ratio  $y_z$  of the non-adiabatic increase of temperature of this zone under compression to the adiabatic increase of temperature:  $y_z = (\delta T / T)_{\text{non ad}} / (\delta T / T)_{\text{ad}}$ . We assume that the non-adiabatic temperature change is derived from the quasi-adiabatic approximation [2].

Computations were made for a series of stellar envelopes, with different values of the effective stellar temperature  $T_{\text{eff}}$ , of the acceleration of gravity in the envelope  $g$ , and with helium content 15-20% by number. These show that while the parameter  $y_z$  varies considerably with  $g$  and  $T_{\text{eff}}$  (it increases with  $g$  and  $T_{\text{eff}}$ ), the ratio of mass of atmosphere  $m_a$  to the mass of the zone of critical ionization  $m_z$  remains almost unchanged,  $m_a/m_z$  varying from 0.88 to 0.93.

The computations have shown further that, if the opacity of the envelope is a prescribed function of density and temperature (determined by the chemical composition), the character of the non-adiabatic oscillations (i.e. the amount of phase shift between the variations of luminosity and of stellar radius, together with the ratio of the amplitudes of these two variations) is essentially determined by the parameter  $y_z$  and the ratio  $m_a/m_z$ . It is almost independent of other parameters.

Fig. 12 demonstrates the variation with  $y_z$  of the phase shift  $\psi$  between the epochs of maximum luminosity and minimum radius (full lines) and of the ratio of the flux leaving the star to the flux entering the ionized zone (dot-dash line). The dashed lines represent