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I have analytically calculated the evolution of the three components of velocity dispersion of the stars in a galactic disk when these are scattered by massive gas clouds, in a generalization of Spitzer & Schwarzschild's (1953) calculation. The principal assumptions made are: (i) The stellar orbits obey the epicyclic approximation. (ii) The gas clouds are massive, long-lived and on circular orbits. (iii) The typical star-cloud encounter time is short compared to the orbital time. (iv) The evolution due to encounters is treated as a diffusion process.

The effect on the stars of encounters with clouds is then given by the standard expressions for the diffusion coefficients $\langle \Delta V \rangle$ and $\langle (\Delta V)^2 \rangle$ (Chandrasekhar 1960). These are used to derive expressions for the rates of change of the epicyclic energies $E_e = \frac{1}{2}(u^2 + \beta^2 v^2)$ and $E_z = \frac{1}{2}(w^2 + v^2 z^2)$, where $\beta = 2\Omega/\kappa$ and Ω, κ, v are the frequencies of circular motion and of horizontal and vertical epicyclic oscillations. The expressions are then averaged over the epicyclic phases and over the stellar distribution function, which is assumed to be always approximately isothermal. Expressed in terms of the independent velocity dispersions σ_u and σ_w , the results are

$$d\sigma_u^2/dt \approx 2G^2 N_C M_C^2 \ln \Lambda K(\alpha, \beta) / (\sigma_u (h_s^2 + h_c^2))^{\frac{1}{2}} \quad (1)$$

$$d\sigma_w^2/dt \approx 2G^2 N_C M_C^2 \ln \Lambda L(\alpha, \beta) / (\sigma_u (h_s^2 + h_c^2))^{\frac{1}{2}} \quad (2)$$

where $\alpha = \sigma_w/\sigma_u$, N_C and M_C are the number of clouds per unit area and their mass, h_s and h_c are the Gaussian scale-heights of the stars and gas, and $K(\alpha, \beta)$ and $L(\alpha, \beta)$ are dimensionless integrals over the epicyclic phases.

The evolution derived from these equations is in two distinct stages:

(i) Transient Relaxation: The shape of the velocity ellipsoid relaxes to a steady final state with (see also Fig. 1)

$$\sigma_u : \sigma_v : \sigma_w = 1 : 1/\beta : \alpha_s(\beta) \quad (3)$$

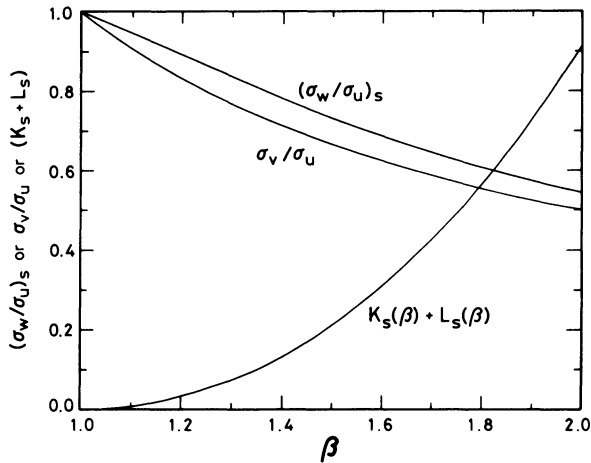


Figure 1. Dependence of velocity-dispersion ratios σ_w/σ_u and σ_v/σ_u and heating rate (as given by $K_s(\beta) + L_s(\beta)$) on rotation-curve shape in steady-heating phase.

(ii) Steady Heating: The total velocity dispersion σ increases steadily on a longer timescale. For $h_s \gtrsim h_c$:

$$d\sigma^2/dt \propto N_c M_c^2 v/\sigma^2 \quad (4)$$

Comparison with Observations: (i) Equation (3) predicts $\sigma_u > \sigma_w > \sigma_v$, whereas $\sigma_u > \sigma_v > \sigma_w$ is observed (e.g. $\sigma_u : \sigma_v : \sigma_w = 1 : 0.59 : 0.52$ according to Wielen (1974)). (ii) Equation (4) predicts $\sigma \sim t^{1/4}$ for $N_c M_c^2$ constant, compared to the observed dependence $t^{1/3}$ or $t^{1/2}$ (Wielen 1974), but this is not conclusive since N_c was probably larger in the past. (iii) Using observational determinations for cloud masses and number densities, I compute $\sigma \approx (10-40) \text{ km s}^{-1}$ for the oldest disk stars compared to $\sigma \approx (60-80) \text{ km s}^{-1}$ observed (Wielen 1974). (iv) If one assumes $N_c \propto \mu_D$, $M_c = \text{constant}$, the disk scaleheight is predicted to vary as $h_D \propto \mu_D^{-1/5}$, where μ_D is the disk surface density. This is consistent with the approximately constant scaleheights observed by van der Kruit & Searle (1981) in edge-on galaxies.

In conclusion, (iv) is consistent with the observations, (ii) and (iii) may be, but (i) (the axial ratios of the velocity ellipsoid) is discrepant. This may either mean that some of the assumptions of the theory must be modified, or that a different physical mechanism, such as scattering by spiral density waves or by massive black holes in the galactic halo, dominates the heating. A fuller account of this work appears in Lacey (1984).

REFERENCES

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 Spitzer, L. and Schwarzschild, M. 1953, Astrophys. J. 118, p. 306.
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DISCUSSION

L. Blitz: You use a different value for the velocity dispersion of giant molecular clouds than Villumsen?

Lacey: I assume that the velocity dispersion of the clouds can be neglected compared to that of the stars. Since the observed value is only 3-4 km/s in one coordinate, I think that this is a better approximation to the true situation than that of Villumsen, who assumes a much larger value for the cloud velocity dispersion.

J.V. Villumsen: I do not remember my exact value, but it was taken equal for stars and molecular clouds, and I took $Q = 1.1$.

Blitz, C.A. Norman: So your dispersion would be about 15-20 km/s.

J.H. Oort: I wonder how uncertain the observed value of 60-80 km/s for the velocity dispersion of the oldest disk stars is. These stars have been strongly selected according to proper motion, and the "observed value" may well be too high.

Lacey: This value comes from the oldest-age group in the McCormick stars, ranked in age by Wielen according to their Ca II emission. It is higher than the average for the old disk as a whole.

Oort: It might be good to look into that again sometime.

J.P. Ostriker: I think the disagreement between prediction and observation is really very severe, because for 50 km/s dispersion you need more massive molecular clouds than most people find.

Lacey: The value 40 km/s for the dispersion of the oldest disk stars corresponds to a mass-weighted mean mass $M_c = 10^6 M_\odot$ and a surface density $N_c M_c = 5 M_\odot \text{pc}^{-2}$, as is found by Sanders, Scoville and Solomon (1983) for instance. To obtain a dispersion of 60-80 km/s one would require the past average value of $N_c M_c^2$ to have been (5-15) times its present value.

Ostriker: OK - and you should really compare in your model the observed and predicted values of σ^4 , because that is how the diffusion coefficient goes, and then you'd see a big difference between the predicted and observed diffusion coefficient (Laughter)



At dinner, clockwise around table: Mrs. Mayor, ?, Schmidt, Ewine van Dishoeck, De Zeeuw, Lacey, Mayor

LZ