simple functors (e.g., finite, polynomial, those that preserve certain colimits) where we can see how iterating the functor can produce the accessible domain. Then I describe accessible domains that result from more involved specifications (e.g., ordinals and segments of the cumulative hierarchies associated with CZF, IZF, and ZF) by relying heavily on *algebraic set theory* [2]. I end with a discussion of some of the methodological features of category theory in particular that helped characterize accessible domains.

W. Buchholz, S. Feferman, W. Pohlers, and W. Sieg, ***Iterated Inductive Definitions and Subsystems of Analysis: Recent Proof-Theoretical Studies***, Springer-Verlag, Berlin Heidelberg, 1981.

A. Joyal and I. Moerdijk, ***Algebraic Set Theory***, London Mathematical Society Lecture Notes Series, vol. 220, Cambridge University Press, Cambridge, UK, 1995.

Abstract prepared by Patrick Walsh.
E-mail: barnicle@math.ucla.edu
URL: https://escholarship.org/uc/item/6t02q9s4

Gianluca Basso, ***Compact Metrizable Structures via Projective Fraïssé Theory With an Application to the Study of Fences***, Università di Torino, Italy, and Université de Lausanne, Switzerland, 2020. Supervised by Riccardo Camerlo and Jacques Duparc. MSC: Primary 03E15. Secondary 54F50, 54F65. Keywords: Compact metrizable spaces, topological structures, projective Fraïssé limits.

Abstract

In this dissertation we explore projective Fraïssé theory and its applications, as well as limitations, to the study of compact metrizable spaces. The goal of projective Fraïssé theory is to approximate spaces via classes of finite structures and glean topological or dynamical properties of a space by relating them to combinatorial features of the associated class of structures.

Using the framework of *compact metrizable structures*, we establish general results which expand and help contextualize previous works in the field. Many proofs in the domain of projective Fraïssé theory are carried out in a context dependent fashion and have thus far eluded clean generalizations. A reason is to be found in the lack of a clear understanding of which spaces are amenable to be studied via projective Fraïssé limits. We give both positive and negative results in this direction.

We isolate a combinatorial condition which entails a correspondence between finite structures and regular quasi-partitions of compact metrizable spaces. This correspondence greatly aids the combinatorial-topological translation. We apply this machinery to study a class of one-dimensional compact metrizable spaces, which we call *smooth fences*. To this end, we isolate a class of finite structures—finite partial orders whose Hasse diagram is a forest—whose projective Fraïssé limit approximates a distinctive smooth fence with remarkable properties. We call it the *Fraïssé fence* and characterize it topologically by carefully exploiting the bridge between the combinatorial and topological worlds. We explore homogeneity and universality features of the Fraïssé fence and the properties of its spaces of endpoints, and provide some results on the dynamics of its group of homeomorphisms.

The dissertation is partially based on [1, 2].

[1] G. Basso and R. Camerlo, *Arcs, hypercubes, and graphs as quotients of projective Fraïssé limits*, ***Mathematica Slovaca***, vol. 67 (2017), no. 6, pp. 1281–1294.

[2] G. Basso and R. Camerlo, *Fences, their endpoints, and projective Fraïssé theory*, 2020. Preprint arXiv:2001.05338

Abstract prepared by Gianluca Basso.
E-mail: gianluca.basso@protonmail.com
URL: http://people.unil.ch/gianlucabasso

Marta Fiori Carones, ***Filling cages. Reverse mathematics and combinatorial principles***, University of Udine, Italy. 2020. Supervised by Alberto Marcone. MSC: 03B30, 03F35, 05C20, 05C55, 05C62, 06A07. Keywords: Reverse mathematics, graph theory, order theory.

Abstract

In the thesis some combinatorial statements are analysed from the reverse mathematics point of view. Reverse mathematics is a research program, which dates back to the Seventies, interested in finding the exact strength, measured in terms of set-existence axioms, of theorems from ordinary non set-theoretic mathematics.

After a brief introduction to the subject, an on-line (incremental) algorithm to transitively reorient infinite pseudo-transitive oriented graphs is defined. This implies that a theorem of Ghouila–Houri is provable in RCA_0 and hence is computably true.

Interval graphs and interval orders are the common theme of the second part of the thesis. A chapter is devoted to analyse the relative strength of different characterisations of countable interval graphs and to study the interplay between countable interval graphs and countable interval orders. In this context the theme of unique orderability of interval graphs arises, which is studied in the following chapter. The last chapter about interval orders inspects the strength of some statements involving the dimension of countable interval orders.

The third part is devoted to the analysis of two theorems proved by Rival and Sands in 1980. The first principle states that each infinite graph contains an infinite subgraph such that each vertex of the graph is adjacent either to none, or to one or to infinitely many vertices of the subgraph. This statement, restricted to countable graphs, is proved to be equivalent to ACA_0 and hence to be stronger than Ramsey's theorem for pairs, despite the similarity of the two principles. The second theorem proved by Rival and Sands states that each infinite partial order with finite width contains an infinite chain such that each point of the poset is comparable either to none or to infinitely many points of the chain. For each $k \geq 3$, the latter principle restricted to countable poset of width k is proved to be equivalent to ADS. Some complementary results are presented in the thesis.

Abstract prepared by Marta Fiori Carones.
E-mail: marta.fioricarones@outlook.it

Tomasz Cieśla, ***Measurable combinatorics and orbit equivalence relations***, McGill University, Canada, 2020. Supervised by Marcin Sabok. MSC: 03E15. Keywords: Borel reducibility, equidecomposability, lifting probability measures.

Abstract

A Borel action of a Polish group G on a Polish space X determines an equivalence relation $\mathsf{E} \subset X \times X$ whose classes are orbits of the action. Such a relation is called an *orbit equivalence relation*. Given two orbit equivalence relations, $\mathsf{E} \subset X \times X$ and $\mathsf{F} \subset Y \times Y$, we say that E is *Borel reducible* to F if there exists a Borel function $g\colon X \to Y$ such that $\forall x, y \in X \ x\mathsf{E}y \iff g(x)\mathsf{F}g(y)$. Informally speaking, this means that F is at least as complicated as E. An orbit equivalence relation E is called *complete* (or *universal*) if all orbit equivalence relations are Borel reducible to E. In recent years many relations naturally arising in mathematics were proved to be complete, such as the isometry relation of Polish metric spaces (Gao, Kechris, and Clemens), the isometry relation of separable C^*-algebras (Sabok), and the homeomorphism relation of compact Polish spaces (Zielinski). The latter result was strengthened by Chang and Gao, who proved that the homeomorphism relation of continua (i.e., connected compact Polish spaces) is complete. They asked whether the homeomorphism