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E-varieties and E-pseudovarieties of regular semigroups

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In this thesis, analogues of certain universal algebra results are established for classes of regular semigroups.

In [2], Hall introduced a concept of variety for regular semigroups, which he defined as follows. An *e-variety* is a class of regular semigroups closed under the formation of homomorphic images, *regular* subsemigroups and direct products. An analogue of the notion of free object (termed *bifree object*) was developed for classes of orthodox semigroups by Kaďourek and Szendrei [3], and generalized to classes of regular semigroups by Yeh [6]. In Chapter 3, we establish that analogues of several universal algebra theorems concerning free objects in varieties carry over to bifree objects in e-varieties. For example, we show that bifree objects (when they exist) in the e-variety generated by a single regular semigroup, S, can be embedded in powers of S. We prove Birkoff-type theorems for classes of E-solid (alternatively, locally inverse) regular semigroups, and we explore the links between bifree objects, fully invariant congruences and lattices of e-varieties.

An analogue of variety (termed *pseudovariety*) appropriate for *finite* algebras was introduced by Eilenberg and Schützenberger [1]. In Chapter 4, we introduce a notion of pseudovariety (termed *e-pseudovariety*) appropriate for regular semigroups. An e-pseudovariety is defined to be a class of finite regular semigroups closed under the formation of homomorphic images, *regular* subsemigroups and *finite* direct products. After verifying that not every e-pseudovariety consists of the finite members of an e-variety, or of the regular members of a semigroup pseudovariety, we consider questions concerning *identities* and e-pseudovarieties. For example, we show that each e-pseudovariety is ultimately defined by a sequence (and not necessarily defined by a set) of *regular unary semigroup identities*. However, it is also shown that sequences of such identities do not necessarily define e-pseudovarieties. We proceed to show that a class of *E-solid* finite regular semigroups is an e-pseudovariety if and only if it is ultimately defined by a sequence of semigroup identities. (A similar result is given for locally inverse finite regular

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semigroups, although in that case the notion of identity required is more complicated.) Thus we find that in very many respects, the theory of e-pseudovarieties closely reflects the existing theories of e-varieties and pseudovarieties.

The contents of Chapter 2, 3 and 4 are largely contained in [4] and [5].

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