text and there are over 700 exercises, many of which extend the theory by means of a series of simple steps. This is one of the strengths of the book and the conscientious reader will benefit greatly, especially as solutions are supplied to the odd-numbered exercises. To derive maximal pleasure, the reader should perhaps have leanings towards applied mathematics, particularly fluid mechanics.

ADAM C. McBRIDE

FISCHER, P. and SMITH, W. R. (eds.) *Chaos, fractals and dynamics* (Lecture Notes in Pure and Applied Mathematics 98, Marcel Dekker, 1985), viii+261 pp., $71.50.

The paper by Hirsch in this volume begins with a dictionary definition of *chàos* as "a disordered state of collection; a confused mixture". Thus the title of this volume rather aptly describes its contents. It contains eighteen papers, all more-or-less linked to the theory of dynamical systems, but tenuously linked to each other, except where an author has contributed to more than one paper.

The papers themselves were delivered at, or arose from, two conferences held at Guelph, Canada, in March 1981 and in March 1983. Since it has taken over two years to get the proceedings into print, and since the book costs so much, one might have expected a much better product. Yet it is replete with grammatical and typographical errors, some of which really obscure the sense. The index is inadequate, which is a serious flaw for papers of which some exhibit a plethora of picturesque but non-standard terms (see below). And finally the editing has left text with figure numbers that too often do not correspond with the figures themselves.

By and large, the better papers in this collection are the ones with the less trendy titles. W. F. Langford's "Unfoldings of Degenerate Bifurcations" is an interesting if somewhat condensed account of degeneracy in the context of the Hopf bifurcation theorem, with a beautiful numerical example. He is one of the few authors to attempt to relate his work to that of others in this volume. There is an equally enjoyable balance between theorems and numerical results in a paper by Chow and Green on singular delay-differential equations. But for those whose taste for deep-looking theorems is left unsatisfied by those two papers, Ikegami's account of electrical networks may be the answer. He is generalising the theory of the van der Pol relaxation oscillator, I think. One of the editors of this volume, P. Fischer, has a paper on Feigenbaum's functional equation; the solutions he discusses are only differentiable almost everywhere, but the structure of their periodic points is the same as that of smooth solutions.

Two papers which can be read with pleasure and benefit by all are those by Rössler ("Example of an Axiom A ODE") and by Hirsch ("The Chaos of Dynamical Systems"). Actually Hirsch makes his most precise statements about systems which are definitely *not* chaotic, and both authors draw attention to our lack of precision about the nature of chaos in the Lorenz attractor, which Hirsch refers to as "something of a scandal". The Lorenz equations themselves (or, rather, a generalisation of them) feature in another paper, by Alexander, Brindley and Moroz. By introducing aspects of spatial inhomogeneity they show that energy tends to be fed into modes of short wavelength, and conclude that the value of these generalised equations is therefore limited.

Apart from an article by O. Gurel, about which the less said the better, the remaining papers in this collection are the work of two authors, plus coauthors. One group, by B. B. Mandelbrot, consists of papers III to VII of a series entitled "On the Dynamics of Iterated Maps", though the subset in this book is more-or-less self-contained. These papers are of course beautifully illustrated, but the illustrations, and the computations underlying them, are also used as the basis of a number of conjectures, such as a conjecture on the scaling laws underlying the approximately self-similar structure of the Mandelbrot set (here modestly referred to as the "M-set") of a map. This author's conjectures have in the past proved quite fruitful, and no doubt there is substantial material for much further work in each of these short papers.

The last group of papers, five in all, bear the name of R. H. Abraham and four coauthors. They share with Mandelbrot's papers a propensity for coining descriptive terms. In Mandelbrot's case the terms are precisely defined and introduced relatively sparingly, but in Abraham's work the plethora of such terms riddles his papers like a disease. Let the conclusion of the last of Abraham's papers, coauthored by K. A. Scott and quoted here in full, serve as an illustration:

> "The occurrence of toral and bagel chaostrophes in forced van der Pol systems is established. It remains to draw the surrounding homoclinic tangles, in the wonderful style of Hayashi, by actual simulation instead of fantasy. But that is very difficult. The ubiquitous coincidence of two bifurcation events, called here the captive balloon catastrophe, also suggests further work, in search of a poppyseed bifurcation?"

In the search for a poppyseed bifurcation one is hardly aided by the index, which directs us to exactly this passage. The authors of these papers should have made much greater effort to link their work, interesting as it undoubtedly is, with the work going on in the fields of bifurcation and chaos in the rest of the world, but their self-indulgent use of such language is a serious obstacle.

Like all conference proceedings, this volume contains papers covering a considerable range of quality. But it is late, rather badly produced and unbelievably expensive and so, more than most such volumes, it raises the whole question of the merits of conference reports. Would the better papers here not have found their way into reasonably reputable periodicals in the normal way, and would we really have missed the poorer ones?

DOUGLAS C. HEGGIE

Cohn, P. M., *Free rings and their relations* (London Mathematical Society Monograph No. 19, Academic Press, 2nd ed. 1985), £66.50.

This book is a substantially extended and up-dated version of its first edition. The principal theme of the book is the work of the author, G. M. Bergmann and others upon free associative rings and algebras. Of the abundance of riches in the actual text it is only possible to give some impression without a review of inordinate length.

The text begins at Chapter 0 with some preliminaries which foreshadow the general level and direction of the text. Thus, for example, there appear Hermite rings, the matrix definition of a module, groups and rings of fractions, skew polynomial rings and free associative rings. Chapter 1 introduces the main classes of rings of the book, these being rings with conditions of freeness on their left or right ideals considered as modules. More specifically a free ideal ring, or fir for short, is a ring all of whose left ideals are free of unique rank. If this condition is assumed only for all m-generator ($m \leqq n$) left ideals or for all finitely generated left ideals one obtains an n-fir or a semifir respectively; by this classification an integral domain is precisely a 1-fir. It is shown that a ring is a semifir if and only if it is weakly semihereditary and projective-free. These notions of freeness are used throughout the subsequent chapters.

A generalisation of the Euclidean algorithm, called the weak algorithm, is used to characterise certain classes of firs. It is shown that a free associative algebra over a (commutative) field is a two-sided fir. The Hilbert series of a filtered ring is defined and for a filtered ring with a weak algorithm a formula of J. Lewin on the ranks of modules is obtained, this formula being the analogue of Schreier's formula for groups. Various generalisations of (commutative) unique factorisation domains and of the primary decomposition of Noetherian rings are considered. It is shown that over a $2n$-fir with left and right ascending chain conditions on left and right n-generator ideals every full $n \times n$ matrix admits a complete factorisation into atomic factors and any two such complete factorisations are isomorphic. An integral domain is shown to be rigid if and only if it is a 2-fir and a local ring. Rings, 2-firs in particular, having modules with a distributive submodule lattice, are examined. For matrices over an n-fir several notions of rank are investigated, the most important of which is the inner rank. The Sylvester domains of W. Dicks and E. Sontag are analysed, the name stemming from the fact that over a semifir a law akin to Sylvester's law of nullity is satisfied. A Sylvester domain is shown to have weak global dimension at most 2 and to be projective-free. Issues of commutativity and of centralizers in firs are raised. The centre of an atomic 2-fir is shown to be a Krull domain and every Krull domain is shown to occur as the centre of some principal ideal domain. Bergmann's theorem, which