Irregular Red-Giant Variable Stars

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1. Introduction

Aperiodic intrinsic variability becomes increasingly evident for a greater number of red giant and supergiant stars as observations become more technically refined and extended over a longer time. All red variables show an erratic temporal behavior to some degree. Even Miras, with their temporal changes in maximum brightness and irregular fluctuations in the cycle length, are not perfect clocks.

Apart from the Miras, the variable M, S, and C stars are either semiregulars (classified SRa, SRb and SRc) or irregulars (classified Lb and Lc) (e.g., F. Querci, 1986, p. 54). Their location among the various types of pulsational variables in the H-R diagram is shown on Fig. 1 on a Mbol/log Te diagram (Becker, 1987). As commented by Becker, evolutionary tracks of 1, 7, and 15 Mo models show that a red star may become: (1) an Lc or SRc type variable for the massive stars (M > 10Mo); (2) a Mira, SRa, SRb, Lb, or Lc and SRc for the intermediate-mass stars (10 Mo > M > 2.25 Mo); (3) a Mira, Sra, Srb or Lb variable for the low-mass stars ($M_{\perp} < 2.25$ Mo).

As presented by Bessell <u>et al.</u> (1988a,b) and Bessell (this Colloquium), an Australian group has undertaken a wide ranging program of theoretical and observational investigations of the spectra of red giants in order to obtain physical parameters - in particular, accurate temperatures - of Miras, semiregulars, and irregulars. Through IR color-color diagrams, they demonstrate that the semiregulars bridge the gap between the non-variable MO-M6 giants in the solar neighborhood and the Miras; this is due mainly to a temperature difference, the Miras being cooler than the SR variables. In addition to the luminosity and the total mass, the temperature therefore appears to be an important parameter in discriminating between the various pulsational behaviors of the PRG stars.

In fact, the primary question is to understand why a red giant is observed to be quasi-regular (Mira), or semiregular or fully irregular. Its intrinsic physical parameters, i.e., its location in the H-R diagram, play a sensitive role in the type of variability. This is demonstrated by the recent physical explanation by Buchler and Goupil (1988) involving a chaotic attractor which will be presented in Part 4, together with a discussion on other mechanisms such as multiperiodicity or randomness. Part 2 describes light curves of irregular variables, and Part 3 discusses radial velocity variations in some absorption lines and temporal changes in characteristic emission-line profiles well suited for depicting the presence of shock waves linked to the radial pulsation of these stars.

In this review we omit binaries and RCB stars as they are presented in other talks in this Colloquium.

2. Light curves

2.1 The variability types and their light curves

The initial classification of semiregulars and irregulars was based on their visual light curves. Generally speaking, the SR variables are characterized by a form of periodicity hidden by more marked irregular brightness fluctuations; the irregulars are characterized by a pronounced disordered variability. The following examples of light curves have been known for many years (cf. the review by Jacchia (1933) from which several of our examples are drawn). Today amateur astronomers, for example from the AAVSO group, endeavour to observe regularly some typical semiregulars and irregulars.

SRa variables are giants which present Mira-like behavior such as relatively constant periods and spectroscopic similarites. However. they have smaller light-curve amplitudes ($\langle 2.5 mag \rangle$ and strong variations from one cycle to another in the amplitude and the shape of Nevertheless, they have an appreciable range in the light curve. periodicity (35 < P (days) < 1200). Fig. 2 and 3 illustrate typical behavior of the light curves of SRa stars. For a time, S Aql shows double maxima; over other cycles, these are smoothed and the main minimum becomes broader. The carbon star RS Cyg likewise shows double maxima, as do some other Miras (Fig. 3); however, the total amplitude of the light curve is significantly larger in the Miras. Another example, the SRa variable V Hya, observed over 80 years (Mayall, 1965), shows unexpected deep minima (Fig. 4). These might reveal a binary nature of the star, which would be consistent with the transformation of V Hya to a bipolar nebula (Tsuji et al. 1988).

SRb semiregulars show quite individual light curves (Fig. 5) and poorly expressed periodicity, preventing the predictions of the epochs

of minimum and maximum brightness. Periodic oscillations temporarily alternate with slow irregular variations or even with a constancy of the brightness (Fig. 6). These SRb variable stars are giants with mean periods of 20 to 2300 days.

SRc type variables are supergiants with an SRb behavior. Visual amplitudes are about one magnitude or less, as in μ Cep and α Ori, though a few stars present amplitudes up to 4 magnitudes, as in S Per and VX Sgr (Fig. 7). The number of such large amplitude variables is very limited, and if we suppose that the supergiants pass through both the small and large amplitude phase, the latter must be short. The SRc variables have periods of 30 to several thousand days.

Lb variables are irregularly variable giants without any trace of periodicity or with an occasional very weak periodicity (Fig. 8). W CMi has an irregular light curve, but during some intervals of time its light curve suggests a regular light period and during other periods constant brightness (Krempec, 1973). As the time interval between successive maxima may be long, a number of stars firstly classified Lb have been shifted to the semiregular types after longer time-series of observations were obtained. An example is VY Leo, classified Lb in the GCVS, whereas Maran et al. (1980) demonstrate it is a SRa variable with $P\sim1$ year, using a satellite for a long series of uninterrupted observations.

Lc irregulars are supergiants such as VY CMa (Fig. 9). Amplitude variations are generally small (< 1.5 mag) and the time interval between two consecutive maxima is between several hundred and several thousand days. Such features imply that several years of observations are needed to decide whether a supergiant belongs to he SRc class or to the Lc class.

2.2 The various time-scales in light variations

Evidences of short-term, intermediate-term, and long-term light variations in the semiregular and irregular variables are stressed by the following examples.

(a) Short time-scale light variations are observed in the SRb stars R Crt (M8 II). Rapid variations with a time scale of about 1 hour in the DDO magnitudes and colors (over TiO bands and Ca I lines) have been detected by Livi and Bergmann (1982). Strong and rapid oscillations have also been observed in the U-B index between phase 0.80 and 0.90 (Bouchet, 1984; his Fig. 2) on the C SRb star, TW Hor; they are supposedly related to the rapid variations in the UV Fe II V1 emission lines (Bouchet <u>et al.</u>, 1983) (see Part 3).

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(b) Intermediate-term initially are depicted in the supergiant α Ori. As observed initially by Stebbins (1931), they are of several hundred days, superimposed on a main curve with cyclic variation of period about 5.8 years (Fig. 10).

(c) Long-term variations are observed in the supergiant SRc variables in which the light maxima decrease about 2 mag in one century (Maeder, 1980).

3. Spectroscopic observations

A Mira-like pulsating mechanism in semiregulars is indicated by some signatures.

3.1 S-shaped absorption-line radial-velocity curves

Regular temporal observations of lines in the SR and the irregular variables to yield radial velocities as a function of phase are still rarely available. However, S-shaped radial velocity curves are recognized in the following examples:

(a) through the CN absorption lines over the region from 6100 to 6700 A observed by Sanford (1950) in the SR carbon stars RR Her (SRb), T CnC (SRa), and V Hya (SRa) (Fig. 11) (M. Querci, 1986);

(b) through IR CO lines in the M SRa variable X Oph (Fig. 12) (Hinkle <u>et al.</u>, 1984);

(c) through IR atomic lines observed by Goldberg (1979) in the M supergiant SRc, α Ori (Fig. 13).

The curves have much lower amplitude, $\langle 10 \text{ km/s}$, in the SR variables (it is 6 km/s in α Ori) than in the Miras, in which it is up to 30 km/s, indicating less available energy and/or larger damping.

The S-shaped curves are the signature of a shock-pulsation motion associated with a <u>radial</u> global stellar pulsation driving acoustic waves. These waves are generated by turbulence at the top of the hydrogen convective zone and turn into shocks dissipating energy and heating the stellar layers as they propagate outward into layers of decreasing density.

3.2 Temporal variations in emission-line profiles

Emission lines also probe the presence of shock fronts in the atmosphere. Some lines are seen at some phases, or rather at some phases in some cycles.

Balmer emission data in the SR and L classes of M-type stars are summarized by Jennings and Dyck (1972), who consulted the literature over about 80 years. Only five M giants and supergiants are quoted as having shown Balmer emission. The presence of hydrogen emission is not likely to be regular from cycle to cycle or within a cycle. A Mira-like phase behavior is noted from observations by McLaughin (1946) in the supergiant μ Cep in which H β , H γ and H δ emissions appear strongest just before maximum light, disappear as the star fades, and appear in absorption at minimum light. Sanford (1950) reports such behavior also in the SR carbon stars RR Her and V Hya. in which H α emission is observed from 0.25 period before to 0.25 period after the maximum light. However, H α in emission is not observed by this author in T CnC, which we have mentioned presents an S-shaped curve in the CN absorption lines. As for the C star WZ cas (SRa), a large temporal coverage shows episodic H α or H β emission around the light maximum.

For some stars, such as α Ori, the hydrogen lines appear only in absorption and are never reported in emission. This might indicate that shocks are not strong enough in this star to excite observable hydrogen emission. In other stars, no hydrogen is detected either in emission or in absorption during even a long time internal -- an example being the C star TW Hor (Querci and Querci, 1985). That no hydrogen absorption is seen must mean that the chromospheric optical depth at line-center is small (Avrett and Johnson, 1984) or that the shock occurs only in very shallow layers.

On the other hand, ultraviolet Fe II emission lines around 3200 A and h and k Mg II emission are always present in giants and supergiants. They are variable in strength with time. In α Ori, where the Fe II lines are particularly well studied (see M. Querci, 1986, for a review of the literature), they are correlated with either an outflow on infall of material. In the C star TW Hor, they have been seen to appear and disappear on a time scale of 1 day (Fig. 14). This star also shows a particularly stochastic temporal behavior in its IUE spectra (Querci and Querci, 1985).

The Mg II flux in M giants is only slightly variable with time. In α Ori (Dupree et al., 1987) the Mg II h emission line is indicative of an outflow velocity at the epoch where the Fe II lines also indicate outflow from this star.

In the SR and L giants and supergiants the k line displays an asymmetry due to circumstellar absorption (e.g., Bohm-Vitense and Querci, 1987). Such CS absorption is particularly important in the irregular carbon star TX Psc (Eriksson <u>et al.</u>, 1985), and it might alter any conclusion concerning the propagation of shocks into the upper atmosphere of these stars.

All the previous line observations are probing shocks that are due to radial pulsation modes. Generally speaking, these shocks appear to dissipate energy in higher layers than in the Miras since the hydrogen emission, so conspicuous around maximum light in Miras, is rarely observed in emission in the SR and L stars, However, even if the shock lacks enough energy or is too heavily damped to excite Balmer emission, it is sufficiently energetic to excite the Mg II and Fe II lines located in the upper atmosphere.

In fact, due to strong radiative damping, shock fronts may form, say, at the chromospheric temperature minimum in agreement with the model chromosphere of low-gravity stars by Schmitz and Ulmschneider (1981). These ideas may be linked to the short-period acoustic heating theory (e.g., see comments and examples in M. Querci, 1986). Indeed, the steep temperature gradients found necessary in chromospheric models to match observations of the carbon star TX Psc (Luttermoser <u>et al.</u>, 1988) are suggestive of shocks at just this level. The recent suggestion of stochastically changing wave periods in the short period range (Cuntz 1987), giving interacting acoustic wave packets and generating a greater shock strength and a larger wave amplitude in the chromosphere, quite well account for stochastically variable lines, such as Fe II in TW Hor (Fig. 14).

Shock fronts progressing outward from the deeper layers, as happens generally for Miras, are obvious at times in irregular variables through hydrogen emission as is seen in some available examples of SR and in the supergiant μ Cep. For example, in μ Cep, this agrees with de Jager (1984), who shows that shock-wave dissipation starts deeper in the photosphere in supergiants which are near the upper luminosity limit in the H-R diagram.

The light curve, i.e. the brightness of the star at the moment of the spectroscopic observations, helps us to predict the line strength. For example, the Mira prototype, o Cet itself, did not show Balmer emission at all when its magnitude at maximum light was faint in June 1983; strong hydrogen emission is known to be linked to the brighter light maxima.

Though a basic common heating mechanism is likely at work in the atmospheres of Miras, semiregular and irregular giants and supergiants, a high shock efficiency occurs at a different atmospheric level from one star to another and changes with time in a given star.

Outflowing gas seen in the Fe II lines of, for example, α Ori and β Peg (Boesgaard, 1981) is also evidence of mass loss in SR and irregular stars. Models such as these by Cuntz (1987) or by Bowen (1988, and this Colloquium), which specifically apply to Miras, represent progress in

the understanding of the pulsation mechanism associated with mass loss, which influences the course of the evolution of these stars.

4. Mechanisms of non-periodic phenomena

Since the IAU Colloquium on Non-Periodic Phenomena in Variable Stars in 1968, the physical explanations for irregular behavior of varying degree in semiregulars and irregulars have smoothly advanced without being fully satisfactory, if we except a recently proposed mechanism. Let us summarize our knowledge on the question, but refer the reader to IAU Colloquium 111 on Pulsating Stars (1988).

According to Whitney (1984), a semiregular may be either: (a) multiperiodic, i.e., showing superimposed periods or beats which are the interaction of simultaneous modes of oscillation; or (b) truly irregular, i.e., explained either by randomness or by chaos.

To detect the presence and significance of periods in stars, periodograms are calculated. Various methods are available. The difficulty with the astrophysical data is that they are unevenly spaced in time and contain large amount of random noise, as noted by Horne and Baliunas (1986). These authors present an extremely valuable technique to predict periodicities. As an example, Karovska (1987) applied this technique, among others, to observations over 60 years of the SRc variable α Ori finding a multiperiodicity, of which a 1-year period is attributable to the fundamental mode of pulsation. Other examples of multiperiodicity in semiregulars are given in the reviews by Wood (1975) and by F. Querci (1986, p. 59). We shall not debate in this review the period ratios Po/P₁ found in the literature. Theoretical values of such ratios and theoretical pulsation modes in SR and supergiants are discussed in Fox and Wood (1982).

As stressed and illustrated by Detre (1968), irregular stellar variability may be the observable effect of random succession of transitory events. Regarding chaos, Whitney (1984) defines it as changes that are not simply the summation of many small changes, but reflect a collective and cooperative behavior. The chaotic behavior is governed by <u>non-linear</u> equations. It is due to intrinsic forces and generally happens when the amplitude of the motion exceeds a critical value.

Discriminating between these various explanations of the aperiodic fluctuations of a variable has not always been fully satisfactory. An example is μ Cep, the irregular oscillation of which has been studied since 1848. The interpretation of its irregular behavior has shifted from random disturbances to a more satisfactory multiperiodicity (Fig.

15) as reviewed by Whitney (1984). However, this author remarks that the good fit over a sample of the light curve "does not prove that μ Cep has multi-periodic, because random or chaotic behavior can <u>imitate</u> multi-periodic behavior roughened by observational errors".

totally satisfactory example might be the Another not superimposition of sinusoidal variations (4 and 5 periods) to synthetize a light curve for α Ori (Fig. 10), though Karovska (1987) suggests a binary explanation to the excess of brightness around 1980-81 and In fact, Perdang (1985) emphasizes that interpreting stellar 1985-86. variability depends on the analytical tools used. The non-linear oscillation of a model may be strictly periodic or multi-periodic, but a refined analysis of the same model may establish that the oscillation is a stochastic (i.e. chaotic) motion. Today, it appears that chaos is a remarkably common process.

A decade ago, explaining disordered variability necessitated extrinsic stochastic mechanisms, as stressed by Perdang (1984), such as irregular surface features, spots, convective cells, or atmospheric veiling, that would have a larger effect on the light variation than have pulsational effects. It is now demonstrated that irregular aperiodic time behavior of a star may arise <u>spontaneously</u> as a result of the non-linear structure of the stellar hydrodynamics (Perdang, 1985).

In the spirit of non-linear (chaotic) dynamics, Buchler and Goupil (1988) propose a mechanism for the irregular variability shown by radially pulsating stars that are red giants. This mechanism is able to generate regularly modulated or erratic oscillations with period variations as well as intermittent oscillations (almost ceasing and restarting) in addition to steady oscillations. It is based on a mathematical model which involves the <u>nonlinear interaction</u> between the fundamental pulsation mode and one or more overtones in the <u>special</u> situation that the stellar model is near <u>dynamical instability</u> in the H-R diagram.

The reason for the dynamical instability is that the adiabatic index $\gamma = (\delta \ln P/\delta \ln T)$ becomes small ($\gamma \langle 4/3 \rangle$) over the convective partial ionization regions (the hydrogen and first helium ionization zones in which the driving of the oscillation occurs) which becomes very extended (e.g., Tuchman <u>et al.</u> 1978). So, in the H-R diagram -- that is, on a luminosity/Te-plane -- a line can be found representing the boundary of the region of the dynamical instability for each stellar mass. Near this line the frequency of the fundamental mode, as well as its growth rate, become small with respect to the frequencies of the other modes (Fox and Wood, 1982).

Buchler and Goupil find a sufficient-dimensional dynamic to give rise to irregular behavior in considering the nonlinear interaction of such a dynamically marginally stable fundamental mode with the first overtone. Involving other overtones will only increase the complexity of the temporal behavior.

In consequence, the location of a red star in the H-R diagram -that is, its physical properties such as luminosity, mass, temperature, density, chemical compositon -- sensitively determines the richness of its pulsational behavior. Our present limitation in applying a hydrodynamic mechanism to real stars is the observational situation, which has considerable difficulty determining clearly the actual number of dominant pulsation modes in these stars. Standard hydrodynamic techniques (fractal dimension test etc.) have to be applied. However, high quality photometric data spread over a long time are needed.

Let us note that other mechanisms involving an underlying chaotic dynamic to explain the arhythmic oscillations of the irregular stars have also been proposed, for example by Perdang and Blacher (1982), Perdang (1984), Regev and Buchler (1985), and Buchler and Kovacs (1987).

Nonradial oscillations might also have some role to play. Exploratory models for late-type giants and supergiants were made by Ando (1976). Spectroscopic evidence for nonradial behavior in the irregular red stars is still lacking. Maybe the presence of large-scale convective motions at the surface of supergiants such as α Ori might be a proof, if some irrefutable observation confirms their existence.

Finally, examples of stars undergoing a transition from irregular to regular pulsations should be given by the carbon stars observed with IRAS by Willems and de Jong (1988, and references therein), though the evolutionary scenarios are still very controversial (see papers by Kleinmann, Kwok, and Jura, this Colloquium).

5. Conclusion

The paucity of observations of semiregulars and irregulars, either photometric or spectroscopic, is evident. In particular, currently available photometric data are of insufficient accuracy for testing of pulsational modes and shock-wave travel and dissipation. Sustained observations, perhaps best performed from satellites for long series of uninterrupted data, are needed.

Consequently, we plead for more and better quality observations on these stars to test the theoretical results and contribute to the understanding of their irregular variability in the context of their evolution in the H-R diagram.

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