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Steady two-dimensional viscous flow in a jet and in a wake

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In both problems discussed in this thesis the flow is steady, two-dimensional, viscous, incompressible and symmetrical about the x^*-axis. (Asterisks distinguish physical quantities from their non-dimensional counterparts.) The fluid is infinite in extent and the origin O is a singular point. Asymptotic solutions of the Navier-Stokes equations are studied for distances from O which are large in some sense.

The flow in the first problem is due solely to a point source of x^*-momentum at O. In the far field the vorticity is concentrated near the positive x^*-axis; outside this "inner" region the flow is potential. The problem is transformed to the ζ-plane, where $\zeta^3 = (\xi+i\eta)^3 = x + iy$. Asymptotic expansions $\sum_{n=0}^{\infty} \xi^{1-n} f_n(\eta)$ and $\sum_{n=0}^{\infty} \xi^{1-n} F_n(\eta/\xi)$ are proposed for the stream function ψ in the two flow regions. The potential terms are easily determined; f_n can be found in closed analytic form. The first three terms of each expansion are obtained in precise form; f_2 is actually found numerically. Certain arbitrary constants are determined by matching the expansions. Failure to match a solution obtained for $\xi^{-2} f_3(\eta)$ with outer terms is averted by inserting a term $O\left(\xi^{-2}\log\xi\right)$ in the inner expansion. Eigenfunction problems are solved for the inner and outer regions. The first inner eigensolution $\xi^{-2}\left(f_0-2\eta f_0'\right)$ is related to an arbitrary origin shift. The outer eigensolutions correspond to mass

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multipoles at O , the first describing nett production of mass in the finite part of the plane. The inner and outer eigenvalues are completely determined as two infinite discrete sets. The eigenfunctions for both regions are obtained completely. A thorough analysis of the structure of the asymptotic expansions results, so that the description of the far flow field is essentially complete.

The second investigation is related to the merging of uniform shear flows at the trailing edge of a semi-infinite flat plate. The change in boundary condition at the trailing edge accounts for the singularity at O . A self-consistent, though incomplete, asymptotic analysis of the far flow field was made by Hakkinen and O'Neil [1]. In an asymptotic analysis of the problem now linearized with respect to the uniform shear, we obtain expansions for each of three regions:

(i) a wake;

(ii) a region of inviscid flow; and

(iii) an upstream boundary layer on the plate.

Eigenfunction problems associated with (i) and (ii) are solved completely. On the wake centre line, the asymptotic results are in agreement with asymptotic expansions of an exact solution found by Stewartson [2]. To complete Hakkinen and O'Neil's asymptotic analysis of the non-linearized case, the eigenvalue problems for regions (i) and (ii) are studied. The eigenvalue problem for the inviscid region is identical with that for both the linearized problem and the jet problem. The first eigenfunction and eigenvalue (again associated with an arbitrary origin shift) are found for the wake problem. An asymptotic formula is obtained for large eigenvalues which, together with the linearized results, suggests that these form an infinite discrete set. The results further suggest how the expansions of Hakkinen and O'Neil must be modified. The problem becomes more meaningful physically after we discuss how it is related to the broader problem of uniform flow over a finite flat plate.

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