

“This book dismantles the final, most daunting barriers to learning about moduli of higher dimensional varieties, from the point of view of the Minimal Model Program. The first chapter draws the reader in with a compelling history; a discussion of the main ideas; a visitor’s trail through the subject, complete with guardrails around the most dangerous traps; and a rundown of the issues that one must overcome. The text that follows is the outcome of Kollár’s monumental three-decades-long effort, with the final stones laid just in the last few years.”

— *Dan Abramovich, Brown University*

“This is a fantastic book from János Kollár, one of the godfathers of the compact moduli theory of higher dimensional varieties. The book contains the definition of the moduli functor, the prerequisites required for the definition, and also the proof of the existence of the projective coarse moduli space. This is a stunning achievement, completing the story of 35 years of research. I expect this to become the main reference book, and also the principal place to learn about the theory for graduate students and others interested.”

— *Zsolt Patakfalvi, EPFL*

“This excellent book provides a wealth of examples and technical details for those studying birational geometry and moduli spaces. It completely addresses several state-of-the-art topics in the field, including different stability notions, K -flatness, and subtleties in defining families of stable pairs over an arbitrary base. It will be an essential resource for both those first learning the subject and experts as it moves through history and examples before settling many of the (previously unknown) technicalities needed to define the correct moduli functor.”

— *Kristin DeVleming, University of Massachusetts Amherst*

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Families of Varieties of General Type

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