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Approaches to reducing gear mass and their effects on gearing stresses and deformations

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Abstract

This study compares empirical and topology optimization methods for reducing gear body mass. Specimens produced via empirical guidelines and topology optimization were compared to referent full-disc gear, focusing on stresses and deformations. Values were determined numerically (Ansys was used) and the calculation method was verified using ISO 6336. The empirical approach exhibited substantial increases in stress and deformation while topology optimization method had promising outcomes. While decreasing mass, it also diminished tooth root stress on the tensile side by 17.1%.

Keywords: topological optimisation, 3D modelling, gears, mass reduction

1. Introduction

Gear pairs are used to transmit power and movement between shafts. Technologies used for gear design and manufacture are mature and covered by technical standards such as ISO 6336 and AGMA 2001 (ISO Standard 6336:2006, 2006; 2001--D04, 2004). Furthermore, significant improvements in gear performance can be made through optimization of both gear macro- (number of teeth, module, width) and micro-geometry (teeth shape modifications) (Miler & Hoić, 2021). Hence, with multiple papers published on a subject, there is a consensus among the researchers on how to design gear pairs depending on the design aim (Hohn, 2010; Bozca & Fietkau, 2010; Diez-Ibarbia, et al., 2016).

One of the most encountered objectives is reducing the gear pair mass (Yakota, et al., 1998). Reduced gear mass is an especially attractive property when intermittent operation is expected as it decreases the starting moment; during each start, it is necessary to overcome the inertia of the system. It is especially popular in battery electric trucks; (Verbruggen, et al., 2020) combined powertrain topology and the number of gears in the gearbox, decreasing the total-cost-of-ownership for 5.6%. In the context of optimizing gear pairs, conventional advice suggests reducing gear module, increasing the number of teeth, and incorporating positive profile shift values to minimize overall volume (Savsani, et al., 2010; Miler, et al., 2017).

However, existing guidelines primarily focus on gearing dimensions and lack insights into gear body design – the section between the rim and the hub connecting the gear to the shaft. The gear body mass, particularly the mass of the driven gear, significantly impacts inertia in reducers with high transmission ratios. Although larger gear modules enhance the pairs load capacity, they concurrently increase the starting torque, potentially resulting in a design loop. In other words, larger module needed to mitigate the increased stress due to inertia further increases the inertia. In such scenarios, modifications are made to the driven gear body to reduce its mass and, consequently, starting torque. Structural interventions aim to find a balance, between the two. For this reason, careful consideration and adjustment are critical when optimizing gear design in such dynamic conditions. Additionally, reducing the gear body mass will increase its compliance, which has a rather positive effect as it reduces the vibration (Ramadani, et al., 2018).

The reduction of gear mass is influenced by the manufacturing technology used to produce the gear. This paper investigates how reductions in mass affect gear stiffness and load-bearing capacity. Prior studies have highlighted the impact of mass reduction on vibrations and noise levels during gear operation (Hou, et al., 2020), necessitating careful consideration of gear assembly, sequence, and material selection. In this study, two technologies for reducing the driven gear mass were compared to the referent case (gear with full disc), focusing on the stresses occurring at gear teeth flanks and roots as well as their deformations due to decreased stiffnesses of gear rims. The main research question was formulated as follows:

> *How does the reduction of gear mass, achieved through different manufacturing technologies, impact the stresses and deformations in gear pairs?*

A commonly used design solution for reducing gear mass was shown and the results were compared to solutions obtained via topology optimization. A particular transmission case and system operating conditions were selected to evaluate the mass reduction solutions.

2. Gear mass reduction methods

This paper explores the possibilities of reducing gear body mass based on different gear manufacturing technologies and their effects on gear operation. The mass reduction changes the gear stiffness and its load-bearing capacity, while also affecting vibrations and noise levels occurring during gear operation. There are several technologies used in gear body manufacturing (Linke, et al., 2006): welding, casting, forging, particle separation machining, and powder metallurgy.

Welding

In the case of small gear series, welding the gear rim and gear body components provides an avenue for significant mass reduction. By joining gear segments (gear hub, body, and rim) through welding, it is possible to decrease the overall gear mass. However, this construction method requires careful control of the welding process and subsequent treatments to minimize deformations and possible defects inherent to welding. While offering reduced material waste and cost advantages, welded gears may experience increased noise levels at high rotational speeds due to their thin walls.

Casting

Gears of substantial dimensions are commonly manufactured using casting techniques. This allows for the creation of gears with intricate body geometries but potentially lower load-bearing capacities due to materials that are used in the process. While the casting process generally produces gears that require subsequent mechanical processing (grinding), technologies are available that do not require it, offering flexibility for different requirements. During the gear casting, attention must be given to mould design and material flow to ensure minimal porosity and uniform material distribution.

Forging

Forging is employed when a high specific load-bearing capacity is needed. Small series of gears can be press-formed from rolled materials, while large series are typically forged in dies. This process enhances material structure and toughness. Careful design and forging practices are crucial to prevent material accumulation and ensure smooth transitions in the gear profile.

Machining

Gears can be created from steel workpieces using particle separation machining. Initial turning, drilling, or milling operations remove excess material, resulting in a rough gear shape. Subsequent processes involve gear tooth formation, heat treatment, and surface finishing. While this method produces high-quality gears within narrow tolerances, it also generates significant material waste.

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Powder metallurgy

The compacting and sintering of metal powders offer an efficient alternative for producing gears, particularly in industries like automotive manufacturing. This method produces precise gears without the need for extensive post-processing. However, these gears exhibit lower strength and wear resistance, limiting their application in certain contexts.

3. Method

3.1. Referent gear

Two methods used for gear body mass reduction were selected and evaluated - the empirical mass reduction approach through empirical guidelines and reduction through topology optimization. The former set of gears is produced either through machining or a combination of casting and machining, while the latter requires the use of powder metallurgy. In order to compare the gear sets, it was necessary to create a benchmark; hence, both sets of results were compared to the referent gear pair.

Referent gear dimensions were obtained for the test case and included initial and final calculations. In the initial calculation step, gear inertias are not accounted for as their dimensions are not yet known. The initial calculation yields the gear module, providing the gear geometry, and facilitating the inclusion of gear inertias in the final calculation step.

Spur gear pairs considered in this paper are subjected to the following operating conditions: required power of 26 kN, an input rotational velocity n_{EM} = 2850 RPM, and the start-up time t_{su} = 0.9 s. Further, the gear pair transmission ratio is i = 6.5 and the system inertia at the reducer output is 80 Nm^2. Constant torsion direction was assumed and the drive operated in an intermittent regime. Finally, both safety factors, related to tooth flank (surface durability) and tooth root stress were set to 2. The initial gear module was calculated for given conditions. The calculation was done based on the surface flank durability, which is generally a more conservative requirement:

$$m \le \sqrt[3]{\frac{i+1}{i} \cdot \frac{2T_{1\max}}{\lambda \cdot z_1^2 \cdot \sigma_{HP}^2} \cdot K_{H\alpha} \cdot K_{H\beta} \cdot Z_M^2 \cdot Z_H^2 \cdot Z_\varepsilon^2} = 4.814 \text{ mm} \approx 5 \text{ mm} \tag{1}$$

where T_{1max} is the maximum torque at the drive gear, $\lambda = b/m$ is the gear face width factor, $z_1 = 14$ is the drive gear number of teeth, K_{Hi} are load factors for contact stress, sigma is the permissible contact stress, and Z_i are factors related to contact stress.

Assuming that the gears are homogenous discs, it was possible to calculate the moment of inertia of a gear (expressed in Nm^2):

$$GD_i^2 = G \cdot D^2 = \frac{\pi}{4} \cdot \gamma \cdot d_1^2 \cdot b \cdot D^2 = 60.2 \cdot \frac{d_1^4}{2} \cdot b \tag{2}$$

where D = sqrt(d_1/2) is the diameter of inertia of rotating mass and $\gamma = \rho g$ is the specific weight of the gear, assuming that it is made of steel. The effect of inertia outlined in the previous section becomes more evident by observing Equation (2). Finally, it should be noted that the referent driven gear mass was calculated and is equal to 151 kg.

3.2. Gears with reduced body mass

When reducing the gear mass, its hub and rim dimensions were held constant to facilitate the comparison between the results. The shaft hub diameter was taken as 100 mm, while the outer gear hub diameter was 160 mm. The gear rim thickness was selected according to ISO 6336 (ISO Standard 6336:2006, 2006) which states that the ratio between the gear rim thickness and the tooth height h should be greater than or equal to 0.5. The rim thickness will be the main contributing factor to discrepancies between tooth root stresses across different gear body designs.

The first option for reducing the gear mass involves the use of empirical guidelines combined with machining operation. The gear hub was connected to the toothed rim using spokes. The recommended number of spokes is given in empirical guidelines (Oberšmit, 1982), and ranges between 4 and 8. In this

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paper, seven spokes were used, and spoke thickness was taken as equal to the thickness of the rim and the hub. In some cases, spoke thickness can be further reduced to reduce mass. Finally, all the inner radii between the spokes, rim, and gear hub were set to 15 mm. The resulting gear is given in Figure 1 (middle).

The mass of the second gear was reduced via topology optimization, which was carried out using the nTopology software. The gear was partitioned into seven segments to reduce the computational cost of the optimization; number seven was selected to ease the comparison to the gear produced using empirical guidelines. The partition used in the optimization was discretized into approx. 557000 finite elements; total material volume mass volume ratio was 0.3 to ensure that both reduced-mass gears have the same mass. Minimizing the gear body structure compliance (maximum stiffness) was set as the aim of the topology optimization. After the material distribution was obtained in the form of a scalar field, it was used to create topology-optimized gear in Solidworks (Dassault Systèmes, 2020). It should be noted that limited interventions into the structure were needed to ensure that the solution was viable for manufacturing. The resulting gear design is shown in Figure 1 (right).

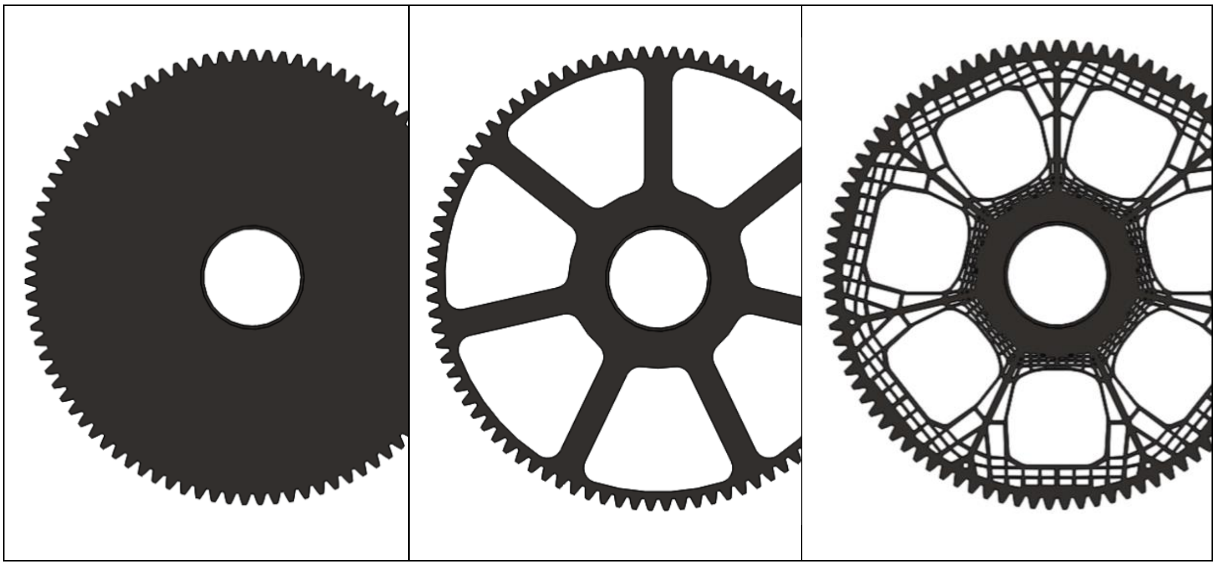

Figure 1. Gear models; a) referent, b) empirical guidelines, c) topology optimization

3.3. Evaluating stresses and deformations

Stress calculations were carried out for gear pairs defined in the previous section. The finite element simulation of gears was carried out as 2D, as such an approach is widely accepted within the field (Čular et al., 2023). In other words, gear pairs are assumed to be 2D bodies with thickness equal to the gear thickness *b*, resulting in uniform stresses along the axial gear dimension. In order to carry out the finite element analysis, the 8-node quadrilateral element was used.

Furthermore, to reduce the computational load, numerical simulations for the referent gear were carried out using the one-seventh of the gear. A referent model was made using Solidworks, while Ansys (Ansys Inc., 2020) was used to carry out the analysis. The resulting model, including both the boundary conditions and active loads, is shown in Figure 2. For both reduced-mass gears, partition sizes had to be increased since the tooth load (normal to the flank) was expected to have a more prominent effect. This was mainly due to the decreased stiffness of the gear body caused by material removal.

The normal load acting on the tooth flank was taken as concentrated force; its magnitude and angle are that of the lowest point of single-tooth contact B, which corresponds with the highest driven gear load. The convergence was confirmed by increasing the mesh density; the difference in tooth root stresses between the analytical and numerical solutions was 6.62% while the flank stresses were within 1.6%. Finally, it should be noted that the results obtained using Ansys were verified through comparison to results obtained via ISO 6336 standard (analytical calculation). Comparisons were done for both the flank and tooth root stresses (see Section 4).

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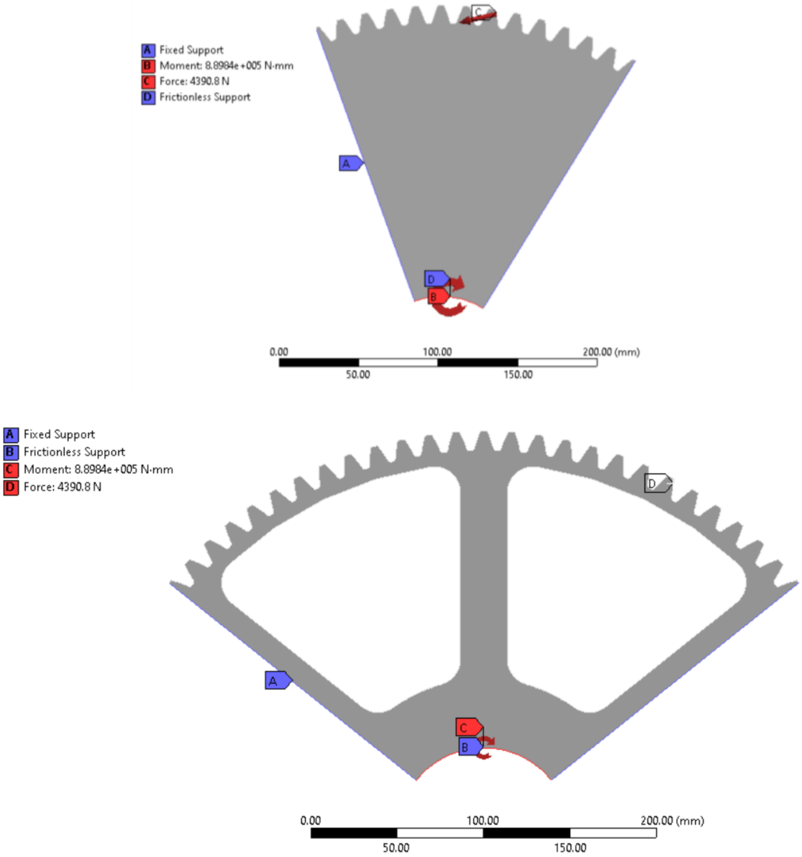

Figure 2. Partitions used for calculation and boundary conditions; referent gear (top) and reduced-mass gears (bottom)

4. Results and discussion

The maximum stress for the referent gear was obtained for the case when only one pair of teeth is in mesh (point B along the pitch line). Since the gear tooth is considered as a cantilever beam, two root stresses must be considered depending on the rotation direction - compression and tension. While the compression is larger in absolute value, fracture generally occurs at the side subjected to tension. For this reason, only the tension side was considered in this paper. The maximum tooth stress value of 23.8 MPa was obtained through FEM, compared to 20.7 MPa by analytical methods. The difference was lower for surface pressure; 309.7 MPa and 302.4 MPa were obtained by FEM and analytical calculation, respectively.

4.1. Gear made using empirical guidelines

The calculation process for reduced-mass gear obtained via empirical approach was also carried out. The tensile side root stress obtained through the finite element method is notably lower than the stress derived analytically. This discrepancy can be attributed to the conservative nature of the analytical approach. Additionally, this difference can be partially attributed to the simplicity of the gear tooth base transition, which is represented as a circular arc. Moreover, according to (Gregov, et al., 2010), substantial variations in stress levels between results obtained from ISO 6336 and through finite element analysis occur when analyzing root stresses in gear tooth rims. Similar disparities in stress levels were identified in a study examining the fatigue cycle in the root of a cylindrical gear with a thin rim (Čular et al., 2023). Comparing the obtained compressive side root stress with the stress in the root of the tooth for the reference gear, a significant increase is evident, emphasizing a greater impact of the load on adjacent teeth.

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4.2. Topology-optimized gear

Stresses occurring in the root of the topology-optimized gear were analyzed next. The compressive side root stress is only 31.1% higher than the stress in the root of the reference gear. On the tensile side, the stress is 17.1% lower than the stress in the root of the reference gear. Stresses on teeth flanks are slightly lower than those in the reference gear. Taking all factors into consideration, this topological optimization method has proven to be successful. A gear with these characteristics can be manufactured through processes such as compacting or powder metallurgy. Adaptations for casting are also possible; however, this would require changes to the lattice structure in certain areas. Lastly, it should be noted that all significant FEM outputs are given in Table 1.

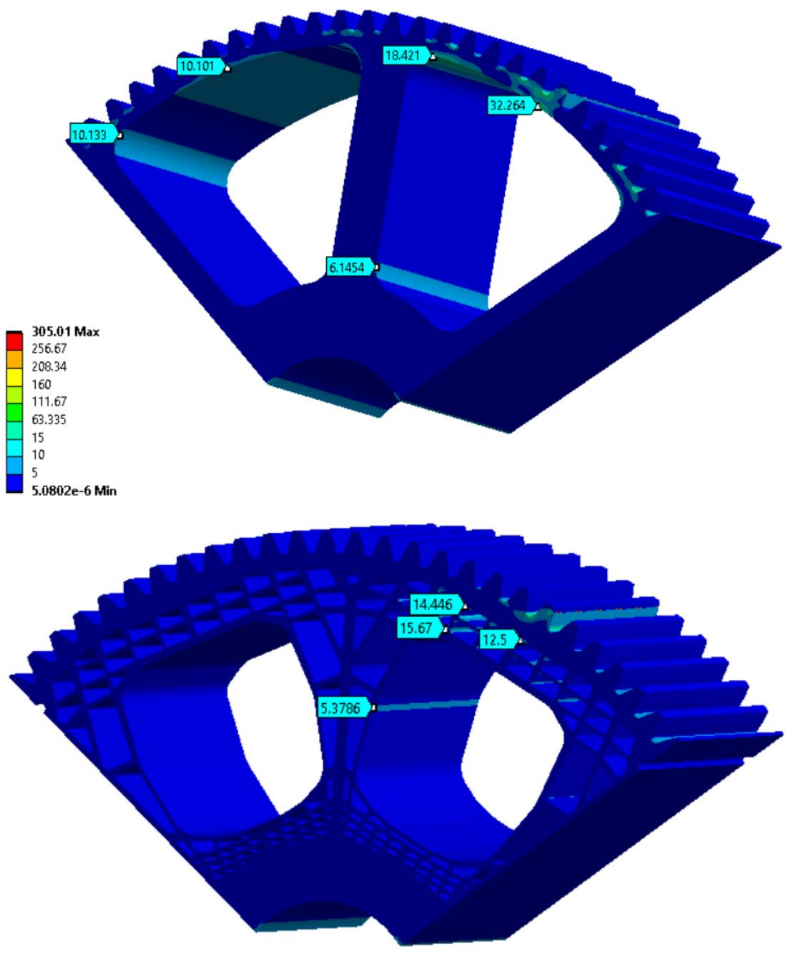

Figure 3. Finite element analysis results for gears with reduced mass (unit: MPa)

4.3. Displacements

In addition to the significant increase in compressive side root stress, a substantial jump, compared to the reference gear, is observed in the total displacements of gears obtained by machining. These displacements have surged by a remarkable factor of 13.66, showcasing pronounced deformations, particularly evident in the teeth located in the sectoral opening where no loading is applied, despite the substantial 27 mm thickness of the remaining ribs. When observing the topology-optimized gears, their displacements were significantly lower compared to machined gears; they were 2.93 times larger than values obtained for full-disc referent gears. Based on the results, it is evident that careful design of rims will be needed to retain the functionality of gearing as deformations can affect the pitch line geometry, possibly even causing

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Table 1. Result comparison (driven gear)

	Referent gear	Machined gear	Topology-optimized gear
Mass	151 kg	60 kg	60 kg
Largest deflection	1.9 μm	26 μm	5.6 μm
Relative value	-	13.66	2.93
Maximum tooth root stress - FEM (analytical value in brackets)	23.8 MPa (20.7 MPa)	70.3 MPa (42.1 MPa)	31.2 MPa
Relative value	-	2.96	1.31
Maximum surface pressure- FEM (analytical value in brackets)	309.7 MPa (302.4 MPa)	305.1 MPa (302.4 MPa)	304.6 MPa
Relative value	-	-1.49%	-1.67%

4.4. Limitations

It should also be added that this paper focused solely on changing the gear topology while different approaches can also be considered when aiming to reduce the gear mass. Possible examples warranting further research are multi-material designs as well as uses of different materials. However, both of the listed approaches, while reducing the gear mass, might introduce additional complications given their relatively unknown fatigue life properties and a steep increase in cost. Hence, they can be considered as viable only in special uses, where limiting the mass is of paramount importance.

Further, while topology-optimized design might significantly reduce the gear mass, it will also affect other aspects of gear operation. It will certainly affect the lubricant flow, as well as the power losses due to lubrication (e.g., oil churning). Further, decreased stiffness of the gear ring will affect the mesh stiffness, which was found to affect the gearing efficiency due to changes in load distribution (Sánchez et al., 2016; Miler & Hoić, 2021).

Finally, the manufacturing topology-optimized structure obtained in this paper would be demanding from the technological viewpoint and was only used to illustrate the method. When aiming to produce more industrially viable solutions, it would be firstly necessary to increase the topology optimization filter diameter. Doing so would yield a structure with fewer lattices, i.e., the gear rim structure would include a larger amount of solid material, reducing the specific rim stiffness (stiffness-to-density ratio). It can hence be reasonably expected that the resultant structure (after increasing the filter diameter) would yield a bit larger deformations, but would also be much easier to produce.

5. Conclusion

In conclusion, this study was focused on the critical aspect of reducing gear mass and its implications on gearing stresses and deformations. The established technologies for gear design and manufacture, standards such as ISO 6336 and AGMA 2001, provide a solid foundation. Notably, optimization efforts in both macro- and micro-geometries were previously studied, widening the basis for researchers and practitioners to build onto when designing gear pairs based on specific aims. The primary objective explored in this paper revolves around minimizing gear pair mass, a particularly attractive prospect in applications with intermittent operation. The reduction strategies traditionally involve adjustments in the gear module, tooth count, and incorporating positive profile shifts. However, the study emphasizes a crucial but often overlooked element—the gear body design, specifically the mass between the rim and the hub.

- The results reveal that empirical methods that require simple design solutions of gear pairs, such as machining, exhibit substantial increases in stress and deformation, especially in the context of reduced gear mass. On the other hand, the topology optimization method showcases promising outcomes, effectively reducing mass and stresses on tooth flanks and the tensile side of the tooth root. Empirical reduced-mass gear design is conservative in analytical predictions due to the simplistic representation of the gear tooth base. Topological optimization presents promising outcomes, with only a 31.1% increase in compressive side root stress and a 17.1% decrease on the tensile side (compared to referent gear). This method proves successful for mass

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reduction while maintaining gear integrity, demonstrating its potential for advanced manufacturing processes like compacting or powder metallurgy.

- The choice between these methods depends on specific system requirements. Particle removal machining is suitable for scenarios without stringent noise requirements, while topological optimization excels in creating high-quality gears with minimal deviations in stress and displacement, albeit at a higher manufacturing cost.

In essence, this research contributes valuable insights into the nuanced interplay between gear mass reduction, manufacturing technologies, and ensuing operational performance. Finally, it can be viewed as an initial research on the trade-offs and considerations crucial for optimizing gear design in diverse engineering applications.

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