

Letter to the Editor

# Low-frequency electrostatic wave in a metallic electron-hole-ion plasma with nanoparticles

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**Abstract.** We investigate the dispersion property of a low-frequency electrostatic wave in a dense metallic electron-hole-ion plasma with nanoparticles. The latter are charged due to the field emission, and hence the metallic nanoparticles/nanotubes can be regarded as charged dust rods surrounded by degenerate electrons and holes, and non-degenerate ions. By using a quantum hydrodynamic model for the electrons and holes, we obtain the electron and hole number density perturbations, while the ion and dust rod number density perturbations follow the classical expressions. A dispersion relation for the low-frequency electrostatic wave in our multi-species dense metallic plasma is derived and analyzed. The possibility of exciting non-thermal electrostatic waves is also discussed.

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Carbon nanotubes [1] were discovered around 18 years ago. Since then there have been a great deal of experimental [2, 3] and theoretical [4, 5, 7–10] investigations of localized collective electronic excitations (plasmon modes) in single- and multi-walled carbon nanotubes (CNTs). The knowledge of dispersion properties of numerous plasmonic modes is essential for determining the parameters that control electronic structures of CNTs.

The CNTs can be metallic or semiconducting depending on their radius and the geometric angles. When CNTs are held in plasmas, one encounters charging of CNTs due to the electric field emission [11, 12]. Henceforth, CNTs in a dense metallic plasma can be regarded as an ensemble of charged nanoparticles surrounded by degenerate electrons and holes, as well as non-degenerate ions. Collective interactions between the plasma and charged nanoparticles can give rise to new wave modes in a multi-species dense metallic plasma.

In this letter, we report on the existence of a low-frequency electrostatic wave and its instability in a metallic plasma. The latter is composed of degenerate electrons

and holes, as well as ions, charged nanoparticles, and immobile neutral atoms. At equilibrium, the quasi-neutrality condition is [13–16]  $q_i n_{i0} + e n_{h0} = e n_{e0} + q_d n_{d0}$ , where  $q_i$  is the ion charge,  $n_{j0}$  is the unperturbed number density of the particle species  $j$  ( $j$  equals  $i$  for ions,  $e$  for electrons,  $h$  for holes, and  $d$  for charged dust CNTs),  $q_d = Z_d e$  ( $-Z_d e$ ) for negative (positive) nanoparticles,  $Z_d$  is the number of charges on nanoparticles, and  $e$  is the magnitude of the electron charge. In the presence of the low-frequency space charge electric field  $-\nabla\phi$ , where  $\phi$  is the self-consistent electrostatic potential, the electron and hole density perturbations,  $n_{e1} (\ll n_{e0})$  and  $n_{h1} (\ll n_{h0})$ , are obtained from the inertialess electron and hole momentum equations, respectively,

$$0 = e n_{e0} \nabla\phi - \nabla p_{e1} + \frac{\hbar^2}{4m_e} \nabla \nabla^2 n_{e1}, \quad (1)$$

and

$$0 = -q_i n_{h0} \nabla\phi - \nabla p_{h1} + \frac{\hbar^2}{4m_h} \nabla \nabla^2 n_{h1}, \quad (2)$$

where  $m_e$  ( $m_h$ ) is the electron (hole) mass, and  $\hbar$  is the Planck constant divided by  $2\pi$ . The second and third terms on the right-hand side of (1) and (2) are associated with the quantum statistical pressure laws [17–19] (e.g.  $p_{e1,h1} = (3n_{e0,h0}/8\pi)^{2/3} (4\pi^2 \hbar^2 / 3m_{e,h}) n_{e1,h1}$  for non-relativistic degenerate electrons and holes) and the quantum Bohm force [17–26] associated with quantum electron/hole tunneling effects due to the finite width of the electron and hole wave functions, respectively.

The electrons and holes are coupled with the ions and charged nanoparticles via the space charge electric field. The ion number density perturbation  $n_{i1} (\ll n_{i0})$  is obtained from the ion continuity and ion momentum equations

$$\frac{\partial n_{i1}}{\partial t} + n_{i0} \nabla \cdot \mathbf{u}_i = 0, \quad (3)$$

and

$$\frac{\partial \mathbf{u}_i}{\partial t} - \frac{q_i}{m_i} \nabla\phi = 0, \quad (4)$$

where  $\mathbf{u}_i$  and  $m_i$  are the ion velocity and the ion mass, respectively.

The nanoparticle number density perturbation  $n_{d1} (\ll n_{d0})$  is determined from the continuity and momentum equations [27]

$$\frac{\partial n_{d1}}{\partial t} + n_{d0} \nabla \cdot \mathbf{u}_d = 0, \quad (5)$$

and

$$\frac{\partial \mathbf{u}_d}{\partial t} + \frac{q_d}{m_d} \nabla\phi = 0, \quad (6)$$

where  $\mathbf{u}_d$  and  $m_d$  are the nanoparticle velocity and the nanoparticle mass, respectively.

The self-consistent potential is given by

$$\phi(\mathbf{r}_s, t) = \int ds' \frac{[q_i n_{i1}(\mathbf{r}'_s, t) + e n_{h1}(\mathbf{r}'_s, t) - e n_{e1}(\mathbf{r}'_s, t) + q_d n_{d1}(\mathbf{r}'_s, t)]}{|\mathbf{r}_s - \mathbf{r}'_s|}, \quad (7)$$

where  $ds' R d\varphi dz'$  and  $R$  is the radius. By using the expansion of the Coulomb potential,  $1/|\mathbf{r}_s - \mathbf{r}'_s|$ , in cylindrical coordinates, one can write [10, 28]

$$\phi(\mathbf{r}_s, t) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk}{(2\pi)^2} G(R, k, m) \int ds' \rho \exp[ik(z - z') + im(\varphi - \varphi')], \tag{8}$$

where  $k$  is the longitudinal (axial) wave number and  $m$  is the discrete azimuthal quantum number. We have denoted  $\rho = q_i n_{i1}(\mathbf{r}'_s, t) + e n_{h1}(\mathbf{r}'_s, t) - e n_{e1}(\mathbf{r}'_s, t) + q_d n_{d1}(\mathbf{r}'_s, t)$  and  $G = 4\pi I_m(kR) K_m(kR)$ , where  $I_m$  and  $K_m$  are the cylindrical Bessel functions of order  $m$ .

We shall apply the Fourier transformation [10]

$$F(\mathbf{r}_s, t) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk}{(2\pi)^2} F(R, k, m, \omega) \int ds' \exp[-i\omega t + ik(z - z') + im(\varphi - \varphi')], \tag{9}$$

where  $F$  is any physical quantity and  $\omega$  is the frequency, on the governing (1)–(6). From (1) and (2), we then have

$$n_{e1} = \frac{4\pi n_{e0} e m_e \phi}{\hbar^2 (k_m^2 + k_{F_e}^2)}, \tag{10}$$

$$n_{h1} = -\frac{4\pi n_{i0} e m_h \phi}{\hbar^2 (k_m^2 + k_{F_h}^2)}, \tag{11}$$

where  $k_{F_e, F_h}^2 = 16\pi^2 (3/8\pi)^{2/3} n_{e0, h0}^{2/3}$ ,  $k_m^2 = k^2 + m^2/R^2$ .

The ion and nanoparticle number density perturbations, deduced from (4)–(6) by using (9), are, respectively,

$$n_{i1} = \frac{q_i n_{i0} k_m^2 \phi}{m_d \omega^2}, \tag{12}$$

$$n_{d1} = \frac{n_{d0} q_d k_m^2 \phi}{m_d \omega^2}. \tag{13}$$

The Fourier transformed potential reads [10]

$$\phi = GR^2 (q_i n_{i1} + e n_{h1} - e n_{e1} + q_d n_{d1}). \tag{14}$$

Eliminating  $n_{j1}$  from (14) by using (10)–(13), we obtain the dispersion relation

$$1 + \frac{GR^2}{\hbar^2} \sum_{\sigma=e,h} \frac{m_{\sigma}^2 \omega_{p\sigma}^2}{(k_m^2 + k_{F\sigma}^2)} - \frac{Gk_m^2 R^2 (\omega_{pi}^2 + \omega_{pd}^2)}{4\pi \omega^2} = 0, \tag{15}$$

where  $\omega_{pe, ph} = (4\pi n_{e0, h0} e^2 / m_{e, h})^{1/2}$  is the electron and hole plasma frequencies,  $\omega_{pi} = (4\pi n_{i0} Z_i^2 e^2 / m_i)^{1/2}$  is the ion plasma frequency,  $Z_i$  is the ion charge state, and  $\omega_{pd} = (4\pi n_{d0} Z_d^2 e^2 / m_d)^{1/2}$  is the nanoparticle plasma frequency. The wave frequency, deduced from (15), is

$$\omega = \frac{\sqrt{G} k_m R \omega_{pi} (1 + Z_d^2 m_i n_{d0} / Z_i^2 m_d n_{i0})^{1/2}}{2\sqrt{\pi} (1 + H)^{1/2}}, \tag{16}$$

where  $H = (GR^2 / \hbar^2) \sum_{\sigma=e,h} m_{\sigma}^2 \omega_{p\sigma}^2 / (k_m^2 + k_{F\sigma}^2)$ .

The electrostatic wave discussed above can be driven at non-thermal level if the ions have an equilibrium axial drift  $u_0$  against nanoparticles. Here, one should replace  $\omega$  by  $\omega - ku_0$  in (12). The resulting dispersion relation is

$$1 - \frac{Gk_m^2 R^2 \omega_{pi}^2}{4\pi(1+H)(\omega - ku_0)^2} - \frac{Gk_m^2 R^2 \omega_{pd}^2}{4\pi(1+H)\omega^2} = 0. \quad (17)$$

Letting  $\omega = ku_0 + \delta$  in (17), where  $\delta \ll ku_0$ , we obtain the threshold

$$ku_0 = \sqrt{\frac{G}{\pi}} \frac{k_m R \omega_{pd}}{2(1+H)^{1/2}}, \quad (18)$$

and

$$\delta^3 = \frac{\omega_{pi}^2}{2\omega_{pd}^2} k^3 u_0^3. \quad (19)$$

Equation (19) admits an unstable solution

$$\delta = \frac{(1 + i\sqrt{3})}{2^{4/3}} \left( \frac{\omega_{pi}}{\omega_{pd}} \right)^{2/3} ku_0. \quad (20)$$

The growth rate is

$$\gamma = \frac{\sqrt{3}Gk_m R}{2^{7/3} \sqrt{\pi}(1+H)^{1/2}} \left( \frac{Z_i^2 n_{i0} m_d}{Z_d^2 n_{d0} m_i} \right)^{1/3} \omega_{pd}. \quad (21)$$

To summarize, we have investigated the dispersion property of a low-frequency electrostatic wave in a metallic dense plasma composed of degenerate electrons and holes, as well as non-degenerate ions and charged nanoparticles. Physically, the present electrostatic wave is supported by the restoring forces arising from the quantum statistical electron and hole pressures and quantum Bohm forces acting on the electrons and holes, while the inertia comes from the mass of the ions and nanoparticles. Furthermore, we have shown that the presence of an axial equilibrium ion drift (caused, say, by a dc electric field in our dense metallic plasma) against charged nanoparticles can produce a non-thermal electrostatic wave due to a two-stream instability [27, 29, 30]. From the frequency spectrum of the non-thermal electrostatic wave, one can deduce charges on nanoparticles in dense plasmas, such as those in semiconductors [31] and in thin metal films [32] with charged nanoparticles.

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