elements in such a statement. First, t may go on *decreasing or increasing.* Secondly, in cases where the boundary crosses itself, it is not possible for the current point P to move steadily round the boundary and *always* leave the *area* on left or right.

For example, in fig. 20

$$\frac{1}{2}\int_{t_1}^{t_2}\left(x\frac{dy}{dt}-y\frac{dx}{dt}\right)dt = \text{Area } AP_1P_2 \dots P_5A$$

$$= \text{area* of space (2)} - \text{area* of space (1)},$$

space (2) being to *left* of current point, while space (1) is to *right* of current point.

In fig. 21

$$\text{Integral} = \text{Area} AP_1P_2 \dots P_5A$$

$$= \text{twice area* of space (1)} + \text{area* of space (2)},$$

space (2) not including the shaded portion.

In fig. 22

$$\text{Integral} = \text{Area } AP_1P_2 \dots P_{12}A$$

$$= \text{area* of shaded space} + \text{twice area* of space (4)}$$

$$- \text{sum of areas* of spaces (1), (2), (3)}.$$

It is worth noting that, using double integrals, we have

$$\iint dx\,dy = \frac{1}{2}\int\left(x\frac{dy}{dt}-y\frac{dx}{dt}\right)dt,$$

the simplest case of Stokes's Theorem.

* "Area" being here neither positive nor negative.

On Commutative Matrices.

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