ANZIAM J. 44(2003), 625–634

OBSERVER-BASED ROBUST H_∞ CONTROL FOR UNCERTAIN TIME-DELAY SYSTEMS

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(Received 5 October, 2000; revised 5 July, 2001)

Abstract

In this paper, we present a method for the construction of a robust observer-based H_∞ controller for an uncertain time-delay system. Cases of both single and multiple delays are considered. The parameter uncertainties are time-varying and norm-bounded. Observer and controller are designed to be such that the uncertain system is stable and a disturbance attenuation is guaranteed, regardless of the uncertainties. It has been shown that the above problem can be solved in terms of two linear matrix inequalities (LMIs). Finally, an illustrative example is given to show the effectiveness of the proposed techniques.

1. Introduction

The dynamic behaviour of many physical processes inherently contains time delays and uncertainties. Since time delays are often the main cause of the instability of control systems, there has been increasing interest in research into robust stabilisation for uncertain time-delay systems (see for example [3, 12, 14, 15]). Recently, by extending state-space H_∞ controller design methods, several authors have proposed H_∞ control methods for linear systems with delay (see for example [5, 10, 11, 13]). Furthermore, since system uncertainties and exogenous disturbance input are unavoidable in modelling, the H_∞ robust control problem has been studied for many years (see for example [4, 6]). Most of these works mentioned above are based on the assumption that the system states available are such that a memoryless state feedback controller can be constructed to stabilise the proposed systems. However, in many cases, it may be impossible to measure all the states of the system. Hence the problem

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of designing an observer-based robust controller for time-delay systems has received some attention in recent years (see for example [2, 3, 16, 17]). However, to the best of the authors' knowledge, the problem of H_∞ control for time-delay systems using observer techniques has not yet been fully investigated.

In this paper, we first consider the problem of designing a robust H_∞ observer for time-delay uncertain systems. We aim to design the linear state observers such that, for all admissible parameter uncertainties, the observation process remains robustly stable and the transfer function from exogenous disturbances to error state output meets the prespecified H_∞-norm upper bound constraint independently of the time delay. The uncertainties are time-varying but allowed to meet a certain structure. They appear in all the matrices of the state-space model.

By introducing a state observer, a memoryless controller is constructed based on the observer states. The proof of our main results shows that the controller can not only stabilise the proposed system but also guarantee a required H_∞ property. A new and simple algebraic parameterised approach is proposed, which enables us to characterise both the existence conditions and the set of expected robust H_∞ observers for time-delay uncertain systems. We show that a desired solution is related to two LMIs which can be solved very efficiently by the algorithms proposed by Boyd *et al.* [1].

2. Problem description and some preliminaries

Consider the time-delay uncertain system of the form

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t-\tau) \\ \qquad\quad + [B + \Delta B(t)]u(t) + D\omega(t), \\ y(t) = C_1 x(t) + D_1\omega(t), \\ z(t) = [C + \Delta C(t)]x(t), \end{cases} \tag{2.1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $y(t) \in R^r$ is the measurable output, $z(t) \in R^s$ is the control output, $\omega(t) \in R^u$ is exogenous disturbance which belongs to $L_2[0, \infty)$, A, A_d, B, C, C_1 and D, D_1 are known real constant matrices of appropriate dimensions. Here τ is a positive integer for the unknown time delay. Also $\Delta A(t)$, $\Delta A_d(t)$, $\Delta B(t)$ and $\Delta C(t)$ are real-valued continuous matrix functions representing the time-varying parameter uncertainties which satisfy the following constraints:

$$\begin{aligned} \Delta A(\cdot) &= H_1F_1(\cdot)E_1, & \Delta A_d(\cdot) &= H_2F_2(\cdot)E_2, \\ \Delta B(\cdot) &= H_3F_3(\cdot)E_3, & \Delta C(\cdot) &= H_4F_4(\cdot)E_4, \end{aligned} \tag{2.2}$$

https://doi.org/10.1017/S1446181100012980 Published online by Cambridge University Press

where H_i and E_i $(i = 1, 2, 3, 4)$ are constant matrices with appropriate dimensions. The properly dimensioned matrices $F_i(\cdot)$ $(i = 1, 2, 3, 4)$ are unknown but norm-bounded as

$$F_i^T(t)F_i \leq I, \quad i = 1, 2, 3, 4.$$

We consider the state observer and the linear memoryless observer state feedback control law given by

$$\dot{\bar{x}}(t) = A\bar{x}(t) + Bu(t) + L\left[y(t) - C_1\bar{x}(t)\right], \tag{2.3}$$

$$u(t) = -K\bar{x}(t), \tag{2.4}$$

where $\bar{x}(t) \in R^n$ is the observer state, L is the observer gain matrix and K is the controller gain matrix. Now we need to design observer (2.3) and controller (2.4) such that the following objectives can be achieved:

(i) the closed-loop system is asymptotically stable;

(ii) under the zero initial state condition and for arbitrary $\omega(t) \in L_2[0, \infty)$, $z(t)$ satisfies $\|z(t)\|_2 \leq \gamma\|\omega(t)\|_2$, where γ is a predefined constant and $\|\cdot\|_2$ denotes the $L_2[0, \infty)$ norm.

If the above-mentioned conditions can be satisfied, then system (2.1) is said to be asymptotically stable with an H_∞-norm bound γ.

Before ending this section, we recall three lemmas which will be used in the proof of our main results.

LEMMA 2.1. *For any $x, y \in R^n$, we have*

$$\pm 2x^Ty \leq x^Tx + y^Ty. \tag{2.5}$$

LEMMA 2.2. *Assume that a matrix F satisfies $F^TF \leq I$, then for any $z, y \in R^n$, we have*

$$2z^TFy \leq z^Tz + y^Ty. \tag{2.6}$$

LEMMA 2.3 ([8]). *Let A, D, E and F be real matrices of appropriate dimensions with $\|F\| \leq 1$. Then for any matrix $P = P^T > 0$ and scalar $\epsilon > 0$ such that $P - \epsilon DD^T > 0$, we have*

$$(A + DFE)^TP^{-1}(A + DFE) \leq A^T\left(P - \epsilon DD^T\right)^{-1}A + \frac{1}{\epsilon}E^TE. \tag{2.7}$$

3. Main results

By introducing observer error $e(t) \triangleq x(t) - \bar{x}(t)$, we get an augmented system given by

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A + \Delta A(t) - BK - \Delta B(t)K & (B + \Delta B(t))K \\ \Delta A(t) - \Delta B(t)K & A - LC_1 + \Delta B(t)K \end{bmatrix} \times \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} A_d + \Delta A_d \\ A_d + \Delta A_d \end{bmatrix} x(t-\tau) + \begin{bmatrix} D \\ D - LD_1 \end{bmatrix} \omega(t). \tag{3.1}$$

Our aim is to derive sufficient conditions for system (2.1) to be a robust stabilisation with an H_∞-norm bound γ.

We consider the following controller gain and observer gain:

$$K = B^T P_c, \qquad L = P_o^{-1} C_1^T, \tag{3.2}$$

where P_c and P_o are the positive-definite matrices defined in the Lyapunov function

$$V[x(t), e(t)] = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}^T \begin{bmatrix} P_c & 0 \\ 0 & P_o \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \int_{t-\tau}^{t} x^T(s) R x(s)\, ds, \tag{3.3}$$

where R is an semi-positive definite matrix.

THEOREM 3.1. *Consider system* (2.1) *and the control parameters given by* (3.2). *For a given constant* $\gamma > \sqrt{2}$, *if the following LMIs have positive-definite solutions* P_c *and* P_o,

$$\begin{bmatrix} A^T P_c + P_c A + N_1 & P_c \\ P_c & -M_1^{-1} \end{bmatrix} < 0, \qquad \begin{bmatrix} A^T P_o + P_o A + N_2 & P_o \\ P_o & -M_2^{-1} \end{bmatrix} < 0, \tag{3.4}$$

then system (2.1) *is asymptotically stable with an* H_∞*-norm bound* γ, *where* M_1, M_2, N_1 *and* N_2 *are defined by*

$$M_1 = H_1 H_1^T + H_2 H_2^T + 2H_3 H_3^T - BB^T + A_d A_d^T + BE_3^T E_3 B^T + \frac{DD^T}{\gamma^2 - 2},$$

$$M_2 = H_1 H_1^T + H_2 H_2^T + H_3 H_3^T + A_d A_d^T,$$

$$N_1 = 2E_1^T E_1 + 2I + E_2^T E_2 + C^T \left(I - \epsilon H_4 H_4^T\right)^{-1} C + \frac{1}{\epsilon} E_4^T E_4,$$

$$N_2 = -2C_1^T C_1 + C_1^T D_1 D_1^T C_1 + P_c^T B \left(I + 2E_3^T E_3\right) B^T P_c,$$

where ϵ *is a positive scalar satisfying* $I - \epsilon H_4 H_4^T > 0$.

https://doi.org/10.1017/S1446181100012980 Published online by Cambridge University Press

PROOF. First, we consider the asymptotic stability of system (2.1). Assume that ω=0. Take the time derivative of the Lyapunov function (3.3) along the trajectory of the augmented system (3.1) and apply Lemmas 2.1 and 2.2, then after a few manipulations, we have

$$\begin{aligned}\dot{V}[x(t), e(t)] \le x^T(t)\big[&(A - BK)^T P_c + P_c(A - BK) + P_c(BB^T + A_d^T A_d \\ &+ H_1 H_1^T + H_2 H_2^T + 2H_3 H_3^T) P_c + K^T E_3^T E_3 K + 2E_1^T E_1 + R\big] x(t) \\ &+ e^T(t)\big[(A - LC_1)^T P_o + P_o(A - LC_1) + P_o(H_1 H_1^T + H_2 H_2^T \\ &+ H_3 H_3^T + A_d A_d^T) P_o + K^T(I + 2E_3^T E_3)K\big] e(t) \\ &+ x^T(t-\tau)(2I + E_2^T E_2 - R) x(t-\tau). \end{aligned} \tag{3.5}$$

In order to simplify the above inequality, we set $R = 2I + E_2^T E_2$ and apply the control parameters (3.2) to (3.5). Equation (3.5) then becomes

$$\dot{V}[x(t), e(t)] \le x^T(t) S_1 x(t) + e^T(t) S_2 e(t), \tag{3.6}$$

where S_1, S_2 are given by

$$\begin{aligned} S_1 &= A^T P_c + P_c A + P_c\big(A_d^T A_d + H_1 H_1^T + H_2 H_2^T \\ &\quad + 2H_3 H_3^T - BB^T + BE_3^T E_3 B^T\big) P_c + 2E_1^T E_1 + R, \\ S_2 &= A^T P_o + P_o A + P_o\big(H_1 H_1^T + H_2 H_2^T + H_3 H_3^T + A_d A_d^T\big) P_o \\ &\quad + P_c B\big(I + 2E_3^T E_3\big) B^T P_c - 2C_1^T C_1. \end{aligned}$$

It is obvious that if S_1 and S_2 are negative-definite then system (2.1) is asymptotically stable.

Now we want to show that if the linear matrix inequalities (3.4) are satisfied, then system (2.1) is asymptotically stable with an H_∞-norm bound γ.

Consider the index

$$J = \int_0^\infty \big[z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t)\big]\, dt. \tag{3.7}$$

From the above-mentioned proof, we know that system (2.1) is asymptotically stable if S_1 and S_2 are negative-definite. So we can conclude that for any nonzero $\omega(t) \in L_2[0, \infty)$, the following equation can be obtained:

$$J = \int_0^\infty \big[z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}[x(t), e(t)]\big]\, dt - P_\infty - Q_\infty, \tag{3.8}$$

where P_∞ and Q_∞ are defined as follows

$$P_\infty = \begin{bmatrix} x(\infty) \\ e(\infty) \end{bmatrix}^T \begin{bmatrix} P_c & 0 \\ 0 & P_o \end{bmatrix} \begin{bmatrix} x(\infty) \\ e(\infty) \end{bmatrix}, \quad Q_\infty = \lim_{t \to \infty} \int_{t-\tau}^{t} x(s)^T R x(s)\, ds.$$

Obviously $0 \leq P_\infty < \infty$ and $0 \leq Q_\infty < \infty$. Equation (3.8) then becomes

$$\begin{aligned} J \leq \int_0^\infty \big[& x^T(t)(C + \Delta C(t))^T (C + \Delta C(t)) x(t) - \gamma^2 \omega^T(t)\omega(t) \\ & + x^T(t) S_1 x(t) + e^T(t) S_2 e(t) + 2x^T(t) P_c D\omega(t) \\ & + 2e^T(t) P_o (D - LD_1)\omega(t) \big] \, dt. \end{aligned}$$

Using Lemma 2.3, we have

$$\begin{aligned} J \leq & - \int_0^\infty \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}^T M \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix} dt \\ & + \int_0^\infty \left\{ e^T(t) \left[S_2 + P_o(DD^T + LD_1 D_1^T L) P_o \right] e(t) \right\} dt, \end{aligned} \qquad (3.9)$$

where M is defined by

$$M = \begin{bmatrix} -\left[S_1 + C^T (I - \epsilon H_4 H_4^T)^{-1} C + \frac{1}{\epsilon} E_4^T E_4 \right] & -P_c D \\ -D^T P_c & (\gamma^2 - 2) I \end{bmatrix}$$

and ϵ is a positive scalar satisfying $I - \epsilon H_4 H_4^T > 0$. Hence if

$$\begin{cases} M > 0, \\ S_2 + P_o \left(DD^T + LD_1 D_1^T L^T \right) P_o < 0 \end{cases} \qquad (3.10)$$

are satisfied, then $J \leq 0$ holds. Therefore $\|z(t)\|_2 \leq \gamma \|\omega(t)\|_2$ is proved.

According to [7], $M > 0$ is equivalent to

$$\begin{cases} \gamma^2 - 2 > 0, \\ I - \epsilon H_4 H_4^T > 0, \\ S_1 + C^T \left(I - \epsilon H_4 H_4^T \right)^{-1} C + \dfrac{1}{\epsilon} E_4^T E_4 + \dfrac{1}{\gamma^2 - 2} P_c D D^T P_c < 0. \end{cases} \qquad (3.11)$$

Hence Theorem 3.1 can be obtained from (3.10) and (3.11) using Schur complements [1]. Thus we complete the proof.

Next we consider a multi-delay uncertain system of the form

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t) t] x(t) + \sum_{i=1}^{l} [A_i + \Delta A_i(t)] x(t - h_i) \\ \qquad\quad + [B + \Delta B(t)] u(t) + D\omega(t), \\ y(t) = C_1 x + D_1 \omega(t), \\ z(t) = [C + \Delta C(t)] x(t), \end{cases} \qquad (3.12)$$

https://doi.org/10.1017/S1446181100012980 Published online by Cambridge University Press

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $y(t) \in R^r$ is the measurable output, $z(t) \in R^s$ is the control output, $\omega(t) \in R^u$ is exogenous disturbance which belongs to $L_2[0,\infty)$, A, A_i, B, C, C_1 and D, D_1 are known real constant matrices of appropriate dimensions. Here h_i $(i = 1, 2, \ldots, l)$ are positive integers for the time delay. Also $\Delta A(t)$, $\Delta A_i(t)$, $\Delta B(t)$ and $\Delta C(t)$ are real-valued continuous matrix functions representing the time-varying parameter uncertainties which satisfy the following constraints:

$$\Delta A(\cdot) = U_1 T_1(\cdot) V_1, \qquad \Delta B(\cdot) = U_2 T_2(\cdot) V_2,$$
$$\Delta A_i(\cdot) = H_i F_i(\cdot) E_i, \qquad \Delta C(\cdot) = U_3 T_3(\cdot) V_3,$$

where U_i, V_i, H_i and E_i are known constant matrices with appropriate dimensions. Properly dimensioned matrices T_1, T_2, T_3 and F_i are time-varying unknown but norm-bounded as

$$T_i^T(t) T_i(t) \le I, \quad i = 1, 2, 3; \qquad F_i^T(t) F_i(t) \le I, \quad i = 1, 2, \ldots, l.$$

Using the same state observer and observer state feedback control law as (2.3) and (2.4), we have the following theorem.

THEOREM 3.2. *Consider system* (3.12) *and the control parameters given by* (3.2). *For a given constant* $\gamma > \sqrt{2}$, *if the following LMIs have positive-definite solutions* P_c *and* P_o,

$$\begin{bmatrix} A^T P_c + P_c A + N_1 & P_c \\ P_c & -M_1^{-1} \end{bmatrix} < 0, \quad \begin{bmatrix} A^T P_o + P_o A + N_2 & P_o \\ P_o & -M_2^{-1} \end{bmatrix} < 0, \tag{3.13}$$

then system (3.12) *is asymptotically stable with an* H_∞*-norm bound* γ, *where* M_1, M_2, N_1 *and* N_2 *are defined by*

$$M_1 = U_1 U_1^T + 2 U_2 U_2^T + \sum_{i=1}^{l} A_i A_i^T + \sum_{i=1}^{l} H_i H^T + B(2 V_2^T V_2 - I) B^T,$$
$$M_2 = U_1 U_1^T + 2 U_2 U_2^T + \sum_{i=1}^{l} A_i A_i^T + \sum_{i=1}^{l} H_i H_i^T + D D^T,$$
$$N_1 = 2 V_1^T V_1 + 2 \sum_{i=1}^{l} E_i^T E_i + \frac{1}{\epsilon} V_3^T V_3 + C^T (I - \epsilon U_4 U_4^T)^{-1} C + 2I,$$
$$N_2 = P_c^T B (I + V_2^T V_2) B^T P_c + C_1^T (-2I + D_1 D_1^T) C_1,$$

where ϵ *is a positive scalar satisfying* $I - \epsilon U_4 U_4^T > 0$.

https://doi.org/10.1017/S1446181100012980 Published online by Cambridge University Press

PROOF. the proof can be carried out essentially following the same lines as were used for Theorem 3.1, except we need to choose a Lyapunov function of the form

$$V[x(t), e(t)] = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}^T \begin{bmatrix} P_c & 0 \\ 0 & P_o \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \sum_{i=1}^{l} \int_{t-h_i}^{t} \begin{bmatrix} x(s) \\ e(s) \end{bmatrix}^T \begin{bmatrix} 2I + 2E_i^T E_i & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(s) \\ e(s) \end{bmatrix} ds.$$

REMARK. Note that Theorems 3.1 and 3.2 offer sufficient conditions for the existence of the expected H_∞ robust observer design method for single and multi-delay uncertain systems due to Lyapunov theory. The result may be conservative mainly due to the introduction of the lemmas. However, the conservativeness in Theorems 3.1 and 3.2 can be reduced by the design of parameter ϵ. A delay-dependent algorithm (see [9], for example) is expected to be developed in order to reduce the relevant conservativeness.

4. Example

A numerical example is provided below to illustrate our main results. Assume that the parameters of a multi-delay uncertain system (3.12) are given by

$$A = \begin{bmatrix} 6 & 2 \\ 2 & 5 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1 & 0.2 \\ 0.1 & 0.4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.3 & 0.4 \\ 0.85 & 0.1 \end{bmatrix},$$

$$\Delta A(t) = \begin{bmatrix} 0.06\cos t & 0 \\ 0.02\cos t & 0 \end{bmatrix}, \quad \Delta A_1(t) = \begin{bmatrix} 0 & 0.1\cos t \\ 0 & 0.05\cos t \end{bmatrix},$$

$$\Delta A_2(t) = \begin{bmatrix} 0.01\sin(t) & 0 \\ 0.04\sin(t) & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix}, \quad \Delta B(t) = \begin{bmatrix} 0.02\sin t \\ 0.01\sin t \end{bmatrix},$$

$$C = [1.2, 2], \quad \Delta C(t) = [0.05\sin t, 0.04\sin t],$$

$$C_1 = [1, 2.3], \quad D = \begin{bmatrix} 0.6 \\ 0.5 \end{bmatrix}, \quad D_1 = [1, 1.2].$$

According to (2.2), decompose the uncertainties $\Delta A(t)$, $\Delta A_d(t)$, $\Delta B(t)$ and $\Delta C(t)$. We then have

$$T_i(t) = \begin{cases} \cos t & i = 1, \\ \sin t & i = 2, 3, \end{cases} \quad U_1 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$U_3 = [0.5, 0.4], \quad V_1 = [0.2, 0], \quad V_2 = V_3 = 0.1,$$

$$H_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.05 \\ 0.2 \end{bmatrix}, \quad E_1 = [0, 0.5], \quad E_2 = [0.2, 0].$$

Let $\gamma = 2.5$ and $\epsilon = 1$. By Theorem 3.2, solve the LMIs (3.13). We have

$$P_c = \begin{bmatrix} 0.7276 & -0.2192 \\ -0.2192 & 0.2598 \end{bmatrix} \quad \text{and} \quad P_o = \begin{bmatrix} 0.7835 & -0.1135 \\ -0.1135 & 0.6065 \end{bmatrix}.$$

It is obvious that P_c and P_o are positive-definite. By Theorem 3.2, system (2.1) is asymptotically stable with an H_∞-norm bound γ.

5. Conclusion

In this paper, we have proposed a method to obtain the robust H_∞ observer for linear systems with time delays and uncertainties. We have obtained sufficient conditions for the existence of the observer and controller by solving two LMIs. The controller guarantees not only the asymptotic stability of the closed-loop system but also the H_∞-norm bound.

Acknowledgement

The authors would like to thank the referees for their constructive comments and suggestions which have improved the presentation.

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