LUNAR LIBRATION TABLES AND DETERMINATION OF CRATER COORDINATES

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1. Introduction

The study of lunar rotation has attracted considerable interest with the advent of the epoch of exploration of the Solar system by space technology. A series of works on an investigation of the lunar gravitational field carried out with the help of artificial lunar satellites have greatly advanced our possibility for that study. The problem concerning the landing on the lunar surface of spacecraft, and the creation of durable lunar bases, impose heavy demands on the accuracy of theoretical description of orbital and rotational motion of the Moon.

The development of the observational technology with the help of radio- and laser ranging (LLR) provides at the present time measurements of the distance to a given point on the Moon with an accuracy of about 2 cm, probably improved in the future to about 5 mm (Banerdt, 1995). By using differential VLBI measurement with extragalactic radio sources angularly near the Moon, it should be possible to obtain routine estimates of angular position of the beacon to 0.1 mas from each observation (Baudry, 1995). Therefore, combining VLBI and LLR techniques will provide a means of achieving new objectives, and that calls for the development of the theories adequate to an accuracy for observations.

In this connection the efforts are carried out in many scientific institutes to improve the theoretical and computing basis of orbital and rotational motion of the Moon and the planets. The results of these investigations were, in particular, realized via the creation of numerical and analytical theories of Lunar Physical Libration (LPhL). There are, for instance, the numerical libration theories of Cappalo *et al.*, (1981), Eroshkin (1985), Shiryaev (1985), the high-precision semianalytical LPhL tables of

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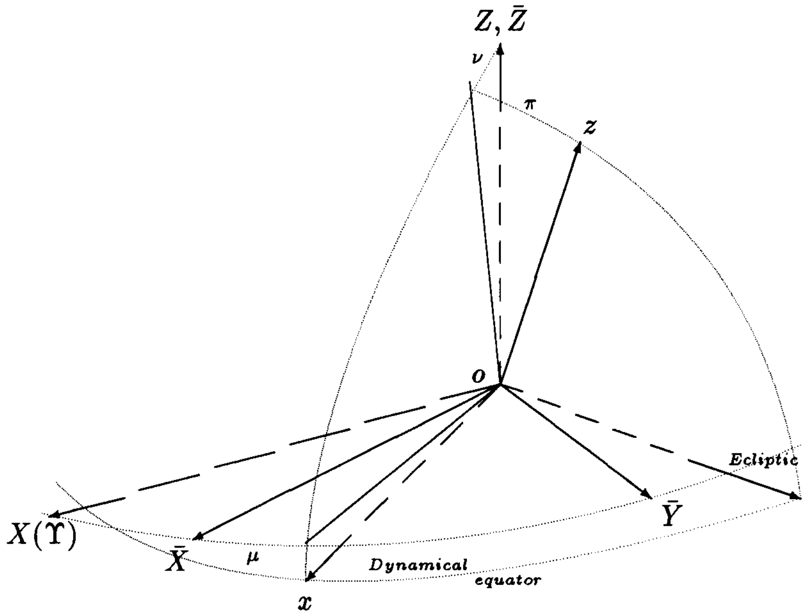

Figure 1. Selenocentrical coordinate system. $\bar{X}\bar{Y}\bar{Z}$ is the uniformly rotating ecliptical coordinate system; the angle $Xo\bar{X}$ is equal to the mean lunar longitude $\bar{L}$; xyz is the dynamical coordinate system, x is directed towards the minimal moment of inertia A, and z towards the maximal moment C; μ, ν, π are libration angles.

Eckhardt (1981a, 1981b), the analytical tables of Migus (1980) and Moons (1982a, 1982b, 1984a, 1984b), and of Petrova (1996). Although the analytical solutions are less accurate than numerical solutions, they have some advantage (see Eckhardt, 1981a).

An essential new knowledge in the shape, gravity and internal structure of the Moon will be derived from data by the US Clementine mission (Banerdt, 1995; Smith *et al.*, 1995; Zuber *et al.*, 1995). The global approach to the exploration of the Moon (Morrison *et al.*, 1995) will make it possible to resolve many problems – in particular, new more accurate models of the lunar gravitational field will be constructed. In this sense the LPhL tables are growing in practical importance.

2. Characteristics of Analytical Libration Tables Constructed for Libration Angles μ, ν, π

The analytical tables of Petrova (1996) are constructed for variables of physical libration μ, ν, π. These variables are the angles defining the position of the Dynamical System of Coordinates (DSC) determined by the lunar principal axes of inertia in the uniformly rotating ecliptical coordinate system. The rate of the rotation is equal to the mean velocity of lunar orbital motion. The description of physical libration by the (μ, ν, π) angles is just as convenient for solution of the libration equation as they are for practical use of the tables in selenodesy.

The solution for libration angles is obtained in the form of Poisson series for the analytical dependence of libration on time and the parameters of lunar gravitational field (dynamical parameters) to be provided. The form of Poisson series as denoted in the formulae are well known:

$$\sum_{r=1}^{\infty} COEF_r \cdot \prod_{i=1}^{9} E_i^{m_{ri}} \cdot \frac{\sin}{\cos}(k_{r1}l + k_{r2}l' + k_{r3}F + k_{r4}D) .$$

A new technique lies in the use of employing analytical parameters E_i which are the differences between values of dynamical parameters of any arbitrary model of selenopotential and those of the dynamical model LURE2 (King *et al.*, 1975). The LURE2 is a standard, more-or-less accurate and widely used model at the present time. The analytical parameters are computed by the following formulae:

$$E_1 = -\frac{\beta \cdot 10^4 - 6.3126}{10^{-2}} \qquad E_2 = -\frac{(\gamma \cdot 10^4 - 2.2737) - (\beta \cdot 10^4 - 6.3126)}{3 \cdot 10^{-2}}$$

$$h = \frac{0.392}{(C/M\rho^2)} \qquad E_3 = \frac{hC_{30} \cdot 10^5 + 1.044}{-2} \qquad E_4 = \frac{hC_{31} \cdot 10^5 - 2.86}{3}$$

$$E_5 = \frac{hC_{32} \cdot 10^5 - 0.482}{0.5} \qquad E_6 = \frac{hC_{33} \cdot 10^5 - 0.27}{0.3} \qquad E_7 = \frac{hS_{31} \cdot 10^5 - 0.88}{1.0}$$

$$E_8 = \frac{hS_{32} \cdot 10^5 - 0.171}{0.2} \qquad E_9 = \frac{hS_{33} \cdot 10^5 + 0.114}{-0.2}$$

A mention has already been made of the convenience of using these tables when choosing the most favorable dynamical model for investigation of dynamical effects, for adjustment of numerical solutions, etc. If the LURE2-model meets the requirements of the user of the tables, only the harmonics whose power indexes are equal to zero ($\prod_{i=1}^{9} E_i^{m_{ri}} = 1$) need be used. In this case the tables do not demand any recalculation, they are identical to the semianalytical "solution 500" of Eckhardt.

An influence on physical libration of variations of dynamical parameters may be easily calculated from the terms of the libration series that have $\prod_{i=1}^{9} E_i^{m_{ri}} \neq 1$. So for the case that a correction to one or all dynamical parameters is required, it will suffice to calculate only the corresponding analytical parameter by the formula and to examine, for instance, its contribution to the amplitude of trigonometrical terms of interest in the series. Let us suppose our interest is to know how much would the value of the constant displacement in τ be sensitive to changes of parameter S_{33}. Take its value from the model of Ferrari *et al.* (1980).
Compute the values $h = \frac{0.392}{0.3905}$ and $E_9 = \frac{-0.033 \cdot h + 0.114}{-0.2} = -0.4043\ldots$.

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Choose from the table for τ in harmonic (0 0 0 0) the terms with $E_9 \neq 0$:

$$\Delta\tau = E_9 \cdot (307.822 - 2.069 - 4.040)'' = -122''.003.$$

Thus the variation of only one parameter (S_{33}) according to the model of Ferrari *et al.* (1980) results in changes of $\sim 122''$ the value of the constant displacement in τ by comparison with LURE2: if $\tau_0^{LURE2} = 214''.352$ then $\tau_0^{Ferrari}(\Delta S_{33}) = 92''.349$.

Such estimates may be carried out by hand without invoking all terms of the libration tables. For such estimates to be made by use of Migus' or Moons' tables it is necessary to carry out the computation of all terms in the zero-harmonic for both models and then to compute the differences between the values obtained.

3. Reduction of Dynamical Coordinates of Lunar Craters to the Observed Ones with the Angles μ, ν, π

Selenocentric reference networks made up in the form of catalogues are based on the coordinate system rigidly bound with the lunar body. The DSC is that system. Coordinates of craters and of other objects on the lunar surface referred to DSC are in some cases called "true coordinates" (Khabibullin, 1990). But the DSC is inaccessible for observation. Consequently a problem ensues to reduce the dynamical coordinates on "visible place", that is, to the observed coordinates.

The observations of craters are carried out in so-called Selenocentrical Geodirected System of Coordinate (SGSC) as in Figure 2. Space orientation of this system is determined by visible rotation of the Moon described by Cassini's rules (see e.g. Khabibullin, 1988). Obviously the position of the SGSC is varied in the lunar body. Therefore allowance must be made of lunar rotation for observable $(U_o V_o W_o)$ and computed $(U_c V_c W_c)$ coordinates of craters to be compared. Consequently the quality of the selenocentric reference networks depends on the study of the LPhL.

The relation between dynamical and observed coordinates is performed by a reduction matrix. Elements of the matrix are functions of libration angles, lunar ecliptical and equatorial coordinates, precession and nutation parameters. Expression of that matrix is known by traditional variables of libration τ, ρ, $I\sigma$ (Kislyuk, 1988). When using (μ, ν, π) angles the transformation of dynamical coordinates $(x,\ y,\ z)$ into visible ones (U_o, V_o, W_o) will be handled in the following way:

$$\begin{pmatrix} W_c \\ V_c \\ U_c \end{pmatrix} = \mathbf{d}_3 \cdot \mathbf{d}_2 \cdot \mathbf{d}_1 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

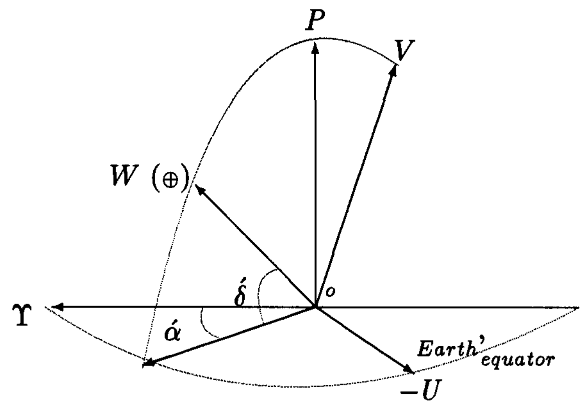

Figure 2. Selenocentric Geodirected System of Coordinates (UVW): o is the center of lunar disc, UV is the pictorial plane; axes V directed to the celestial pole P passes on the intersection of declination circle with the UV-plane, V is in opposition to increasing of right ascension; axis W perpendicular to the UV-plane coincides with the direction to the observer.

where matrix $\mathbf{d}_1$ is expressed by a product of three rotation matrices:

$$\mathbf{d}_1 = \mathbf{m}_Z(-\bar{L} - \mu) \cdot \mathbf{m}_Y(-\nu) \cdot \mathbf{m}_x(\pi),$$

where $\mathbf{m}_r(a)$ is the rotation matrix around the axis r, the deflection angle a is positive if rotation is executed in anti-clockwise direction. $\mathbf{d}_1$ defines the transformation of DSC to the mean ecliptic of the epoch of observation. The angles μ, ν, π are computed from libration tables. The value $\bar{L}$ is the mean ecliptical longitude of the Moon.

The matrix $\mathbf{d}_2$ defines the transformation from the mean ecliptical coordinate system of the epoch of observation to the true Earth equator with the corresponding computation of precession and nutation as recommended in the Astronomical Annual.

And finally, the matrix

$$\mathbf{d}_3 = \mathbf{m}_Y(\acute{\delta}) \cdot \mathbf{m}_Z(180^o + \acute{\alpha})$$

performs the transformation of the computed equatorial coordinates of crater to the SGSC. Here $\acute{\delta}$, $\acute{\alpha}$ are topocentric equatorial coordinates of the Moon.

Let us note the difference of the described transformation from the traditional one. When working with (μ, ν, π)-angles there is no need to use the values of the mean longitude of the node Ω and of the mean inclination I of the lunar equator to ecliptic.

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