ON APPROXIMATE VALUES OF THE FORCE OF MORTALITY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The following extension of formula (21) on page 25 of the *Text-Book*, Part II, may perhaps be of some interest.

Writing $\frac{l_{-1}-l_{+1}}{l_0}=\mathrm{A}$, $\frac{l_{-2}-l_{+2}}{l_0}=\mathrm{B}$, $\frac{l_{-3}-l_{+3}}{l_0}=\mathrm{C}$, and so on; we get the successive approximations to the value of μ, as follows:

(1) $\frac{\mathrm{A}}{2}$

(2) $\frac{2}{3}\mathrm{A}-\frac{1}{3.4}\mathrm{B}$

(3) $\frac{3}{4}\mathrm{A}-\frac{3}{4.5}\mathrm{B}+\frac{1}{4.5.6}2\mathrm{C}$

(4) $\frac{4}{5}\mathrm{A}-\frac{6}{5.6}\mathrm{B}+\frac{4}{5.6.7}2\mathrm{C}-\frac{1}{5.6.7.8}2.3\mathrm{D}$

(5) $\frac{5}{6}\mathrm{A}-\frac{10}{6.7}\mathrm{B}+\frac{10}{6.7.8}2\mathrm{C}-\frac{5}{6.7.8.9}2.3\mathrm{D}+\frac{1}{6.7.8.9.10}\times 2.3.4\mathrm{E}.$

The first and second are those given in the *Text-Book* under formulas (22) and (21) respectively; and are correct to $\Delta^2 l_0$ and $\Delta^4 l_0$ respectively; the third and fourth were worked out by expressing μ, A, B, &c., in terms of the finite differences of l_0, and are correct to $\Delta^6 l_0$ and $\Delta^8 l_0$ respectively; the fifth was inferred by analogy and found to be correct to $\Delta^{10} l_0$. It will be noticed that the numerators of the coefficients of B, 2C, 2.3D, &c., are figurate numbers of the second, third, fourth, &c., orders. On making use of the third approximation instead of the second, the values of μ_4 and μ_5 derived from the table on page 494 of the *Text-Book*, Part II, were found to be ·01208 and ·01148, instead of ·01379 and ·01142 respectively, while from μ_6 to μ_9 inclusive there was no change. The second approximation is useful for accurately drawing tangents to curves of not higher than the fourth degree, and the further approximations could similarly be used for the curves of higher degrees.

I am, Sir,
Your obedient servant,
H. N. SHEPPARD.

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London, W.,
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[From the symmetrical form of the coefficients, we may assume that the general, or nth, approximation would be

$$\frac{n}{n+1}\mathrm{A}-\frac{\frac{n.\overline{n-1}}{\lfloor 2}}{\overline{n+1}.\overline{n+2}}\mathrm{B}+\frac{\frac{n.\overline{n-1}.\overline{n-2}}{\lfloor 3}}{\frac{\overline{n+1}.\overline{n+2}.\overline{n+3}}{\lfloor 2}}\mathrm{C}-\&\mathrm{c}.,$$

and that this would be a correct approximation to the value of μ to $\Delta^{2n} l_0$.—ED. *J.I.A.*]