PART IV

RELATIONS BETWEEN CELESTIAL AND SELENOCENTRIC REFERENCE FRAMES.

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ABSTRACT. The very great accuracy with which the motions of the Moon can now be monitored by laser ranging, differential VLBI and occultation observations, implies that the interpretation of the measurements is conditioned by the choice and the accurate knowledge of a selenocentric, a terrestrial and a celestial frames. Two different types of selenocentric reference frames can be envisioned. The present selenographic frames are discussed but the author proposes that one should introduce a system defined by a purely geometric means. Some consequences of such a choice are discussed. One feature of the future conventional terrestrial frame is very important for Earth-Moon dynamics. Its origin should coincide with the center of mass of the Earth as determined by lunar laser ranging. As far as the quasi-inertial reference systems are concerned, the liaisons between a purely lunar dynamical system, subject to some hardly modelable effects, and purely celestial systems are analysed. The reduction of observations made with various techniques implies the use of different systems, and several problems are stated that should be solved before a unique system for Earth-Moon dynamics might be used.

I. INTRODUCTION

During the past decade, remarkable progress was made in the accuracy of measurement of the position of the Moon. Presently, lunar distances between points on the Earth and on the Moon are measured with lunar laser ranging to better than 10 cm. On the other hand, differential VLBI techniques applied to the determination of angular separation between transponders on the Moon and natural radio-sources may reach the accuracy of a few milliseconds of arc, possibly 0".001 (Slade et al., 1977). Other techniques such as observations of occultations of stars by the lunar limb aim also towards a milli-arc-second precision while projects of lunar sub-satellites such as that of POLO would increase to even a better equivalent precision our knowledge of some dynamical features of the Moon such as its gravitational field or its shape.

When various motions of a celestial body can be monitored to a sub-

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metric precision, it is also necessary that they be related to reference frames that are defined and constructed with a similar accuracy. Otherwise, systematic erroneous trends may be introduced in the expressions of these motions and, consequently, in their interpretation.

Such a problem arises in the study of the motions of the Earth in space. In the last years, a great number of investigations have dealt with the conceptual and practical aspects of reference coordinate systems for Earth dynamics. The latest results of these efforts were presented in 1980 at the IAU colloquium n° 56 in Warsaw (Gaposchkin and Kolaczek, 1981). The study of the motion reduces to the motion of a conventional terrestrial frame attached in some defined way to the planet Earth referred to a conventional celestial frame that materializes in some manner an inertial or quasi-inertial reference system (see Mueller, 1981 and Kovalevsky and Mueller, 1981)

A similar problem is to be considered in the case of the Moon : how to describe correctly the motions of the Moon without introducing spurious effect due to an inadequate choice of reference frames. Of course, the necessity of well-defined reference systems was recognized by all the authors of overall discussions on precise lunar observations using laser ranging or VLBI, such as Calame (1976 and 1977), King et al. (1976), Williams (1977) or Ferrari et al. (1980). All have made the choice of such systems, although this does not usually appear explicitely. Actually, very few papers have been specifically devoted to this subject, or are closely connected with it. Our aim is to discuss the problem of reference frames in the dynamics of the Moon, disconnecting it, as much as feasible, from other physical or mathematical factors entering in the discussion of the observations.

In reducing a positional observation of the Moon, one is led to introduce three reference frames :

1. A selenocentric frame to which positions of points on the Moon are referred .

2. A terrestrial reference frame used to identify the position and the motion of the observation points.

3. A celestial frame to which all the motions of the Earth and the Moon are referred.

Several different choices of frames may be made in each of the three cases. They may differ in the conception of the system as well as in the way that the frame is realized. In order to clarify the discussion and to analyse in detail the differences between frames, we shall consider the successive phases that are needed in the construction of a reference frame (Kovalevsky and Mueller, 1981) :

1. An ideal system : a general statement giving the rationale for the ideal case.

2. A reference system : a definition of a physical structure that is to be used to realize the ideal system.

3. A conventional reference system : a detailed model of the physical structure as well as of the physical environment relevant to the description of the behaviour of this structure.

4. A conventional reference frame, or simply "reference frame" : a set of parameters conventionaly chosen to realize the conventional reference system.

To these four steps, one has to add a further consideration : the accessibility of the reference terms. In other words, how easy is it to refer features that are not part of the conventional reference frame to it, to determine their coordinates ?

Let us consider separately each of the three groups of reference frames mentioned above.

II. SELENOCENTRIC REFERENCE FRAMES

The aim of a selenocentric reference frame is to provide a coordinate system for features on the surface of the Moon and in studying the dynamics of the Earth-Moon system.

1. Ideal system.

Since there is no evidence of observable deformation of the Moon, one may use any coordinate system suitable for a rigid body : a consistent set of coordinates of well defined landmarks or dynamical features. Even if there were periodic tides, the definition can still hold if one states, as for the ideal terrestrial system, that the body has only deformations and no rotations or translations when referred to such a system.

2. The reference system.

The choice of the system is strongly affected by the type of observations that may be used to construct the reference frame and to extend it to other features. Let us describe two approaches that are well fitted to two very different types of observations of the Moon.

a) The classical selenographic systems. This reference system is defined by the directions of the principal axes of inertia. In the case of the Moon, this is not a very advantageous choice : the exact position of the inertial axes of a quasi-spherical object whose gravitational field can be described only by a large number of small terms is very difficult to define. However, it is possible from a theory of the rotation of the Moon, to link dynamically the instantaneous axis of rotation to the principal axis of inertia. Then, observations of lunar features from the Earth can be referred to this system. This choice was, historically, very appealing, since Cassini's laws are good to better than 1" as seen from the Earth. The origin of trirectangular coordinates in such a system should, obviously, be the center of mass of the Moon.

b) The geometrical system. In this case, the system is defined by a consistent set of coordinates of a number of features on the surface of the Moon. In principle, two such features are sufficient and the origin of coordinates is arbitrary and may have no physical significance.

It is clear that, conceptually, the second system is simpler than the first. This is a general property of reference systems when it is possible to have a geometrical and a dynamical definition. In the case of the selenographic system, it is necessary to have a theory of the rotation of the Moon so as to connect the axes of inertia to an observable phenomenon.

3. The conventional reference system.

The conventions adopted for the selenographic reference system derive from the conventions that were set up at a time when no deviation from Cassini's laws was observed. The lunar equator, supposed to coincide with an inertial plane, is defined by the position of the ascending node N of the lunar orbit on the ecliptic and its inclination i. The origin X of the longitudes is also defined in relation to the position of the vernal equinox γ.

$$\overset{\frown}{\gamma N} + \overset{\frown}{NX} = 180° \quad (1)$$

If cassini's laws were strictly exact, this formula would suffice to tie the selenographic coordinate system to the lunar orbit. The modelling that would then be necessary would have been the modelling of the motion of the Moon in a celestial system to which the vernal equinox is also referred. But this expression (1) is only approximate and one has to take into account the theory of the rotation of the Moon, including the physical and free librations. In other terms, the definition of a selenographic system of coordinates on the Moon implies the choice of a dynamical model of the Earth-Moon system, including the elements of the orbit. The masses of the Earth and the Moon, a theory of precession and nutation, the main spherical harmonics of the Earth and of the Moon, a theory of planetary perturbations, etc...

This very complex situation would of course not exist was a purely geometric definition of the system to be adopted. However, we shall see that this would not solve all the problems.

4. The reference frame.

The definition of the selenographic system implies a very complex procedure for its realization. The reason is that the position of lunar features - in practice, craters - have to be somehow referred to some fictitious point in the lunar orbit (such as the node N) as well as to the vernal equinox. This means that, not only is a selenographic reference frame dependent upon a dynamical model of the Earth-Moon system, but also upon the realization of an inertial system to which the motion of the Moon is referred and in which there exists an access to the equinox. The actual procedure is usually divided in two phases. In a first phase, the positions of a few craters (sometimes even a single one : Mösting A) are measured with respect to stars and reduced using a lunar ephemeris. The second phase consists of a densification of the network of primary craters by relative measurements, the reduction of which implies again the use of a lunar ephemeris for the evaluation of the libration parameters.

The necessity to use so many quantities derived independently of the Moon itself inevitably leads to a number of systematic errors. Among these, we may quote the fact that the equinox of the FK 4, to which observations of craters with respect to stars are referred, is not the dynamical equinox defined by the motions of the Earth (Siedelmann et al., 1981) and implied in the definition of the selenographic reference system. To this effect, one should add the fact that these observations are usually not made with FK 4 stars - too sparse in the sky - but with fainter stars belonging to catalogues such as SAO (e.g. Froeschlé, 1977) which introduces its own errors, some of which are systematic. Another source of error is the offset between the center of mass of the Moon whose motion is given by the theory and the center of figure used in the reduction of the positions of the craters. A third reason for systematic biases comes from the fact that the first terms of the development of the lunar potential are very poorly known (see, Mulholland, 1980) while they are strongly correlated in the observations with other dynamical features such as the free librations (Calame, 1977).

All these facts contribute to make a selenographic reference frame very insecure for basing precise observations. Actually, there does not even exist a single conventional reference frame for the Moon. Each catalogue of lunar features uses its own lunar and libration theories and makes its own reference frame as is the case for the "DOD selenodetic control system" (Eigen and Hathaway, 1966) , the "Kiev lunar triangulation system" (Gavrilov and Kisliuk, 1967) etc... For instance, the last is the "Selenocentric control system" (D.L. Meyer, 1980) which differs from earlier frames through the use of a theory of the physical libration deduced from lunar laser observations (Williams, 1977). The accuracy ranges from 200 meters in the central regions to a kilometer in the limb areas.

In parallel, lunar laser ranging and VLBI using ALSEP transponders have also the capability to determine selenocentric coordinates of points of the Moon. Intrinsically, the accuracy is several orders of magnitude better than the observation leading to the reference frames described above. Overall reduction of lunar laser observations give a consistent set of coordinates of four retroreflectors on the Moon (Lunakhod 2, Apollo 11, 14 and 15). Differential VLBI on ALSEP transponders may be used to determine another set of five sites (Apollo 12, 14, 15, 16 and 17). Using photogrammetric determinations of the relative positions of equipment on the same sites, it was possible to unify all these determinations into a single system (King et al., 1976). This system, or a similar one, could be the starting point of a geometric selenocentric reference frame, in accordance to the definition given for a geometric reference system.

It would be impractical to follow in this case the example of conventional terrestrial reference frames as described by Guinot (1981). It is not yet possible to "maintain" a system from the coordinates of several points on the Moon. But it is possible to fix a priori the coordinates of the Apollo 15 retroreflector and add a constraint on another retroreflector such as Apollo 14 to ensure a complete definition of the orientation of the frame. This would be a definition comparable with what was used earlier when the Greenwich mean meridian was used as the origin of the Earth coordinate system, and is sufficient for the present needs.

III. DISCUSSION

What would be the consequences of such a proposal to the main problems of positioning on the Moon ? Clearly, a change in the definition of a reference frame does not remove the main difficulties due to the physics of the problem, the principal of which is the uncertainty about the actual coefficients of the lunar libration. The geometric definition removes completely all of the dynamics from the reference frame. Presently, the same cause could affect directly the reduction through the description of the Earth-Moon system and indirectly through the use made of the reference system. For instance, an error in the position of the vernal equinox may affect the coordinates of lunar features through the use of formula (1) as well as the orbital elements of the Moon that are used to evaluate the position of the Moon. Therefore, there would be a strict delimitation of what appears to be the dynamical description of the Moon and the reference frame itself.

In return, if some of the positional unknowns were removed, one would have to introduce all second order coefficients of the potential, since the $\vec{MX}$ axis would no longer be identified with an axis of inertia and, whenever necessary, the three components of the vector $\vec{MO}$ between the center of mass M of the Moon and the origin O of the reference frame. Since the chosen coordinates of Apollo 15 will be close to those presently found in the selenographic system, it may be inferred that S_{22} may still be taken equal to zero for all dynamical purposes. Therefore, the equations of condition and the covariance matrix will be rather similar to the present situation, when the coordinates of Apollo 15 are taken as unknowns.

The drawback of this is the non-accessibility of the system if the Apollo 15 reflector is not observed or is not a part of the data analysis. This difficulty will have to be overcome by constructing catalogues of positions of secondary features on the Moon, that can be used to materialize the frame whenever necessary.

This is not a simple matter as soon as other features than retro-reflectors are considered. If the result of King's discussion may be considered as satisfactory for ALSEP transponders, the extension to craters is a much more difficult task, since there is not direct accurate link between the position of lunar landers and craters when photo-identification by Apollo 15 could not be made. The "Apollo control system" (Schimerman, 1976) is the only serious attempt to do this. Although it covers only parts of the equatorial belt of the Moon, it provides the coordinates of 330 craters, two laser reflectors (Apollo 15 and Lunakhod II) and two ALSEP transponders (Apollo 16 and 17) in one unique system. The accuracy ranges from 200 to 800 meters and this is compatible with the precision of the existing catalogues of craters. It is therefore possible now, without degradation, to put the existing selenographic catalogues of craters in a geometrical system defined as suggested above. This should remove some of the systematic errors of the present control systems on the Moon, and completely decouple the dynamical aspects of the Earth-Moon system from the definition of a selenocentric reference system, if not from its realization.

IV. TERRESTRIAL REFERENCE FRAMES

The definition of a terrestrial reference frame, the positioning of stations in this frame and, above all, the knowledge of the orientation of this frame in space play a major role in the interpretation of accurate lunar observations, especially lunar laser ranging.

Many papers have recently been written on the problem of the terrestrial reference frame and Earth rotation. So, this topic will not be rediscussed here. However, it must be noted that, presently, there exist as many terrestrial frames as there are techniques to determine the Earth rotation. The terrestrial frame used in Earth-Moon system studies using lunar laser is essentially defined by the vector (center of mass of the Earth - McDonald Observatory) and will probably remain such even when more stations produce continuous observations. It is absolutely necessary to link this vector to the future conventional terrestrial system in such a way that the results obtained by lunar laser ranging be strictly compatible with others. This is particularly important if the long term consistency in the results for Earth rotation that may be expected from lunar laser ranging is to be used, other techniques such as satellite laser or radio observations being used for short term studies.

The relative positions of lunar laser stations with respect to other points defining the conventional terrestrial reference frame may be obtained from colocation experiments. Errors in this determination may have important consequences :

- An error in longitude would essentially be cancelled out by a rotation of the celestial reference frame and would not be too serious.

- An error in latitude would introduce a spurious rotation of the system that would affect the theoretical position of the Moon.

- An error in the position of the Earth center of mass would introduce erroneous terms in the dynamical behaviour of the Moon, affect S_{10} and C_{10} terms of the Earth model and introduce fictitious periodic longitude terms.

As a consequence, the determination of the Earth rotation and polar motion using lunar laser observations would not be comparable to the observations made with other techniques and particularly satellite laser ranging.

For all the determinations of the rotation of the Earth and of the polar motion by dynamical methods, it is essential that the terrestrial reference systems used have a common origin and that this origin be the center of gravity (see Kovalevsky and Mueller, 1981). Since this point is model dependent, it is necessary that it should be defined by the most stable dynamical system. This is why in the definition of the future conventional terrestrial frame, lunar laser ranging stations and observations should play the major role in fixing the center of mass of the Earth and that other systems such as those produced by satellite ranging or VLBI should be constrained to keep the position of the center of mass of the Earth as determined by lunar laser ranging.

In return, it seems that there is no major drawback in constraining the latitude and the longitude origins of the terrestrial system used by lunar laser ranging to the results obtained by other techniques and in particular VLBI.

V. QUASI-INERTIAL REFERENCE FRAMES

The observations of the positions of the Moon in space use different quasi-inertial reference frames, depending upon the technique of observation.

In the case of lunar laser ranging, a dynamical reference system is defined in such a way that the equations of motion of the Moon have no rotational terms. However, such a definition is not easy to handle in practice because of the existence of the secular acceleration due to the tidal interaction with the Earth. This effect is difficult to model (see Lyttleton, 1980 or Mulholland and Calame, 1981) and its magnitude is still empirically determined from observations. This is a source of indetermination in the practical realization of a fixed origin of the reference frame. This situation is still more complicated if one does not rule out the possibility of variation of the constant of gravitation (Van Flandern, 1975 and 1981).

In any case, whatever is the care with which the dynamical lunar reference frame is defined, an error in the equinox position can occur in many ways. It has been stated (Mulholland, 1975) that such an error would be artificially absorbed by an apparent displacement of the principal axis of inertia of the Moon. This is true but is still to be avoided because of the indeterminacy it introduces in two of the three reference frames used in the reduction of lunar laser observations. In any case, too many loosely defined effects enter in the longitude of the Moon to be sure that they are all fully removed. Consequently, some residual rotations may remain in the reference frame. Furthermore, this system is not accessible by any other type of observation and cannot be used to refer to other celestial bodies, so that its use may introduce some inconsistencies between the dynamical description of the solar system and the Earth-Moon system.

This system is also difficult to tie to the systems used in other types of observations of the Moon, for instance the occultations of stars (see Froeschlé and Meyer, 1981). In this case, the reference frame is the conventional celestial frame (FK 4). Although the results for the acceleration of the Moon by both techniques are comparable (see Mulholland, 1980 or Lambeck, 1980), this is not sufficient to insure the equivalence of both reference frames.

It is therefore necessary that, at some point, these reference frames be modified in such a way that they could be identified to a single quasi-inertial reference frame that could be universally used. It is clear now that this will be a system deduced from VLBI observations of extragalactic radio-sources. While the extension of the VLBI system to stars used in occultation observation will be done using HIPPARCOS observations (Kovalevsky, 1981), the connection between the lunar dynamical and the VLBI systems implies specific experiments : to link artificial sources deposited on the Moon or on board of lunar orbiter to extragalactic radio-sources belonging to the VLBI reference frame. The reduction of ALSEP transmitters to quasar VLBI observations could show the way to this connection but will not suffice to tie both systems due to its present lack of precision.

VI. CONCLUSIONS

From the discussion made above, one may attempt to describe what would be an ideal situation as far as the three reference frames are concerned.

1. A selenocentric reference frame defined by the coordinates of one feature and constraints on a second one. This will free this reference frame from any dynamical model.

2. A terrestrial system built in such a way that the origin is the center of mass as determined by lunar laser observations.

3. A celestial system, based on the fixed positions of extragalactic radio-sources provided that a liaison is made with the dynamical reference system defined by the motion of the Moon or that the motion of the Moon can be monitored in this system.

No doubt that this situation is not practicable presently. Several important results have to be obtained and new experiments should be set up for this.

1. In order to link the Moon to VLBI system, an X band transmitter should be placed on the Moon and allowed to transmit routinely during many years. The same goal could also be achieved if the Lunar Polar mission (POLO) is decided and launched with an onboard transmitter. Then, the analysis of the orbit of the subsatellite would allow to connect it to the center of mass of the Moon and to extragalactic radio-sources using differential VLBI techniques. This would link the Moon to the VLBI quasi-inertial reference frame.

2. The new terrestrial system will be defined after the main MERIT campaign. It is essential that its definition refers to the center of mass of the Earth as sensed by lunar laser techniques.

3. We have noted, on several occasions, that observations involving positions of lunar features in relation with reference frames are very heavily dependent upon the theory of libration of the Moon, which is very sensitive to the values of the low order harmonics of the lunar potential. To improve their knowledge is an urgent need. Project POLO is an ideal program to determine the lunar potential and also to connect the Moon to extragalactic sources. Undoubtedly, a positive decision on POLO would contribute very significantly to the definition of reference frames used in the dynamics of the Earth-Moon system.

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DISCUSSION

Anderle : Do you think it is necessary to establish a study group for this problem ?

Kovalevsky : We have enough already. I do not believe that we can have a good geometric selenocentric reference frame before there is a really good theory of libration. The launch of a POLO satellite would be a very good step for this.

Kolaczek : Can you compare the two accuracies of the two types of lunar coordinate systems, geometric and dynamical ?

Kovalevsky : In the Apollo Control System, the errors are 200-800 meters. I cannot say for the dynamical systems, but perhaps Dr. King can tell us.

King : Assuming that one site is fixed to establish the origin, the system of laser retroreflectors is no worse that a few meters, perhaps better. The relative accuracy of the ALSEP locations in this system is of the order of ten meters. The Apollo photogrammetric system is tied to the ALSEP system with an error that is, in some cases, about 10-20 meters, but it is adequate for cartographers, and in that sense the system could be defined now. We need to know what accuracy is required from such a system for the analysis of occultations.