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ON THE THEORY OF IONIZED INTERSTELLAR GAS MOTION

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ABSTRACT

It is pointed out that the model of mutually penetrating ideal charged and neutral gases can be used with particular success to describe the motion of an extremely rarefied plasma. As a simplest instance of application of such a model, the acceleration of totally ionized gases in crossed electric and magnetic fields is considered. It is shown that the general solution of the problem in two limiting cases is either magneto-hydrodynamic or the free charge solution.

Magneto-hydrodynamic equations afford a sufficiently exhaustive way for the description of interstellar gas motions, if the mean free path l of electrons and ions is small as compared with the radius of curvature, $R = PC/He$, of the electron trajectory in a given magnetic field moving with mean thermal velocity. Contrariwise, if $l > R$ new phenomena are originating, which may partly be accounted for by introducing an anisotropic electric conductivity and other additional terms. Much more complicated equations are obtained in this case. However, some phenomena remain unaccounted for in them, namely those connected with the probable temperature difference between electrons and ions, or with ionization and recombination.

The phenomena occurring in extremely rarefied gases can be accounted for as exhaustively as possible, if a system of kinetic equations for electrons, ions and neutral molecules is used for these gases. The equations must be solved together with equations of the electromagnetic field, caused by the electronic and ionic currents and charges. However, the solution of these equations is a rather complicated task.

A much more simple model of a rarefied plasma, in which the phenomena connected with the peculiarities of electronic and ionic motions in the magnetic field at $l \gtrsim R$ are taking place, is the system of mutually penetrating ideal gases of electrons, ions and neutral molecules, where hydrodynamical equations are used for each gas (taking into account the mutual friction of gases). This system of equations must be solved together with the

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electrodynamical equations, like in an ordinary magneto-hydrodynamical case. This model was used successfully for investigations of electro-acoustical waves in a plasma, originating in gaseous discharges [1]. It may also be used for calculating the ionic and electronic beams in a plasma. Such a model was also applied by Schlüter [2] in his deductions of magneto-hydrodynamical equations for a plasma.

The main aim of the present communication is to direct attention to the fact that the model of mutually penetrating ideal gases can be used with particular success to describe the motion of an extremely rarefied plasma. An example of such a plasma is the interstellar ionized medium.

Let us consider the phenomenon of acceleration of totally ionized gases in crossed electric and magnetic fields as a simplest application of a mutually penetrating gas model.

This problem was considered earlier [3] from the point of view of magneto-hydrodynamical equations. If the electric field **E** and the magnetic field **H** are to be assumed as homogeneous and constant (i.e. if the field caused by the plasma is neglected) then for **E** directed along the x-axis and **H** along the z-axis the magneto-hydrodynamical equations may be given the following particular solution

$$U_x = U_x^0 e^{-t/\tau},$$

$$U_y = U_y^0 + \left(U_y^0 + c\frac{E}{H}\right)(e^{-t/\tau} - 1)$$

and

$$U_z = U_z^0, \quad \tau = \frac{\sigma c^2}{\lambda H^2}, \tag{1}$$

where U_x, U_y, U_z—are the velocity components, U_x^0, U_y^0, U_z^0—their initial values, σ the medium density, λ the electric conductivity and c the velocity of light.

From the point of view of the model of mutually penetrating gases this problem may be solved for a totally ionized plasma by means of the following equations:

$$\sigma_+\left\{\frac{\partial \mathbf{U}^+}{\partial t} + (\mathbf{U}^+ . \nabla)\,\mathbf{U}^+\right\} = -\nabla P_+ + \frac{e}{m_+}\sigma_+\left\{\mathbf{E} + \frac{1}{c}[\mathbf{U}^+\mathbf{H}]\right\} - \alpha\sigma_+\sigma_-(\mathbf{U}^+ - \mathbf{U}^-),$$

$$\sigma_-\left\{\frac{\partial \mathbf{U}^-}{\partial t} + (\mathbf{U}^- . \nabla)\,\mathbf{U}^-\right\} = -\nabla P_- - \frac{e}{m_-}\sigma_-\left\{\mathbf{E} + \frac{1}{c}[\mathbf{U}^-\mathbf{H}]\right\} - \alpha\sigma_+\sigma_-(\mathbf{U}^- - \mathbf{U}^+),$$

$$\text{div}\,(\sigma_+\mathbf{U}^+) + \frac{\partial \sigma^+}{\partial t} = 0, \quad \text{div}\,(\sigma_-\mathbf{U}^-) + \frac{\partial \sigma^-}{\partial t} = 0, \quad P_+ = \frac{kT_+}{m_+}\sigma_+, \tag{2}$$

$$P_- = \frac{kT_-}{m_-}\sigma_-, \quad \text{curl}\,\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}e\left\{\frac{\sigma^+}{m_+}\mathbf{U}^+ - \frac{\sigma^-}{m_-}\mathbf{U}^-\right\},$$

$$\text{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad \text{div}\,\mathbf{E} = 4\pi e\left(\frac{\sigma^+}{m_+} - \frac{\sigma^-}{m_-}\right), \quad \text{and} \quad \text{div}\,\mathbf{H} = 0.$$

Taking **E** and **H** as constant and directed correspondingly along the x- and z-axis, as in the preceding problem, we obtain for a particular case, when $\mathbf{U}^+$ and $\mathbf{U}^-$ depend only on time, the following system of equations

$$\left.\begin{aligned} \dot{U}_x^+ &= \frac{e}{m_+}\left(E+\frac{U_y^+}{c}H\right)-\alpha\sigma_-(U_x^+-U_x^-), \\ U_y^+ &= -\frac{e}{m_+}\frac{U_x^+}{c}H-\alpha\sigma_-(U_y^+-U_y^-), \\ \dot{U}_x^- &= -\frac{e}{m_-}\left(E+\frac{U_y^-}{e}H\right)-\alpha\sigma_+(U_x^--U_x^+) \end{aligned}\right\} \quad (3)$$

and

$$\dot{U}_y^- = \frac{e}{m_+}\frac{U_x^-}{c}H-\alpha\sigma_+(U_y^--U_y^+).$$

The solution of the latter system of linear equations is not difficult, but we shall not give it here. We shall only consider two limiting cases.

For a limiting case, when the density of the plasma has sufficiently high values, an expression for the mean velocity $(m_+\mathbf{U}^+ + m_-\mathbf{U}^-)/(m_+ + m_-)$ is obtained, which coincides with (1).

For another limiting case, when the density is infinitesimally small (the terms containing the coefficient α in Eq. (3) may be neglected) a solution of the following type is arrived at:

$$\left.\begin{aligned} U_x^+ &= U_0^+ \sin\Omega_+ t, \quad U_y^+ = U_0^+ \cos\Omega_+ t - c\frac{E}{H}, \\ U_x^- &= U_0^- \sin\Omega_- t, \quad U_y^- = U_0^- \cos\Omega_- t - c\frac{H}{E}, \end{aligned}\right\} \quad (4)$$

where

$$\Omega_+ = \frac{e}{m_+ c}H, \quad \Omega_- = -\frac{e}{m_- c}H.$$

This solution coincides with the one for free electrons in crossed fields.

Thus, the system (2) of equations describes the plasma fairly well, both in the case of high and low densities.

System (2) of equations may be easily generalized, either for the case of incompletely ionized gases, in which the process of ionization and recombination is taking place [1], or in the case of relativistic velocities. The system of relativistic hydrodynamical equations of mutually penetrating gases may be successfully applied not only to the interstellar medium, but may also be used for calculations of various accelerators.

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