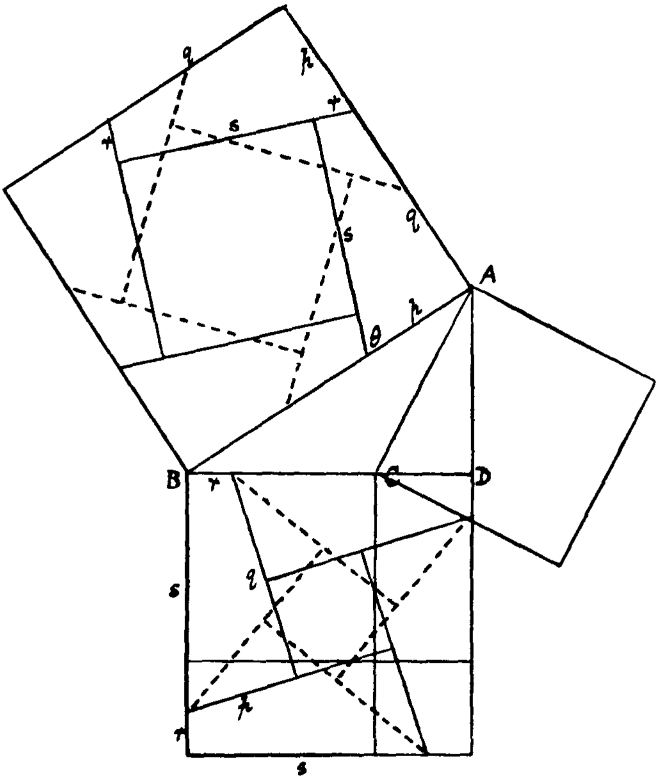
MATHEMATICAL NOTES

A Review of Elementary Mathematics and Science.

A Dissection of the extended form of Pythagoras' Theorem

$$AB^2 = BC^2 + CA^2 + 2BC \cdot CD.$$

The squares on AC, CD are placed centrally in the squares on AB, BD respectively, in such a way that two sets of four equal quadrilaterals are obtained, identical except in orientation. This



can be done in two ways—the dotted lines indicate the second arrangement.

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Let a, b, c denote sides of triangle ABC; h the projection of c on a; p, q, r, s the sides of the required quadrilateral.

Then $\quad q+p=c, \quad s-r=b \quad$ (from AB^2)
$\quad q-p=h-a, \quad s+r=h \quad$ (from BD^2).

These relations determine p, q, r, s and provide simple constructions for drawing the quadrilateral. The second set (dotted lines) is got by changing the third equation into $p-q=h-a$, so that this set has the same sides as the previous set but in different order.

When $a=h$ we get $p=q$ and the two sets degenerate into one and give Perigal's dissection of Pythagoras' theorem (Mackay's Euclid, 1897, p. 93). When $h=b$ the quadrilaterals become triangles ($r=0$). When $h<b$ r becomes negative; the construction still holds if the sign convention is admitted.

The construction of the quadrilateral may also be approached trigonometrically. Taking θ for the angle between sides s, r we find

$$b \cos \theta + h \sin \theta = c$$

which determines two values of θ when $b^2+h^2>c^2$ and one when $b^2+h^2=c^2$. Circles could be drawn centrally in AB^2, BD^2 with radii $\frac{1}{2}b$, $\frac{1}{2}(h-a)$ and tangents to them making θ with AB, etc.

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A new form for the sum of a trigonometric series.

1. The usual text book formula for $\Sigma \equiv \sum_{t=0}^{n-1} [\gamma^t \sin(\alpha + t\beta)]$ is

$$\Sigma = [\sin\alpha - \gamma \sin(\alpha-\beta) - \gamma^n \sin(\alpha + n\beta) + \gamma^{n+1} \sin(\alpha + \overline{n-1}\,\beta)] / R_0$$

where $\quad R_0 \equiv 1 - 2\gamma\cos\beta + \gamma^2 = (1-\gamma\cos\beta)^2 + \gamma^2 \sin^2\beta.$

Now $\quad \sin\alpha - \gamma\sin(\alpha-\beta) = \sin\alpha\,(1-\gamma\cos\beta) + \cos\alpha\,(\gamma\sin\beta)$

$$= \sqrt{R_0}\,\sin(\alpha+\theta)$$

where $\quad \theta = \tan^{-1}[\gamma\sin\beta / (1-\gamma\cos\beta)].$

Similarly $\quad \gamma^n \sin(\alpha + n\beta) - \gamma^{n+1}\sin(\alpha + \overline{n-1}\beta)$

$$= \gamma^n \sqrt{R_0}\,\sin(\alpha + \theta + n\beta).$$

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