A TIME-DEPENDENT EXTENSION TO BROUWER'S METHOD FOR ORBITAL ELEMENTS CORRECTION

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1. Introduction

One of the most popular methods for orbital elements correction, by means of O-C calculus, is based on Brouwer's method [2], which is very well adapted for integrating short periods of time. We propose a general method to integrate over long intervals of time, when we have good observations, based upon a time-dependent functional relation between the derivatives of all elements with respect to all variations in the initial elements. First, we verify the truth of the unrestricted hypothesis by means of the proposed analytical method and a numerical derivation.In [6], we have incorporated a correction frame model jointly with this general method and, then, we have constructed the residual function which is minimized by the least squares method. But, as we are going to see later, there are correlations between the parameters frame correction model and the initial elements involved in the adjustment. They are obtained here, as well as an expression for the equinox correction from these frame parameters. Finally, by means of a simply weighted method, with the observations from MPC's magnetic tape, in FK4 system, we obtain an estimation for these frame parameters and an equinox correction which is in great accord with the adopted value (see [3]).

2. Analytical model for differential perturbation calculation

Let σ_i^o $(1 \leq i \leq n)$ be the initial orbital elements for a body in the Solar System in elliptic motion and let $\triangle\sigma_i^o$ be the desired correction at the same time. The variation induced in right ascension is, at first order,

$$\alpha(\sigma_i^o + \triangle\sigma_i^o; t) = \alpha(\sigma_i^o; t) + \sum_{m=1}^{6} \left.\frac{\partial \alpha}{\partial \sigma_m^o}\right|_t \triangle\sigma_m^o$$

S. Ferraz-Mello et al. (eds.), Dynamics, Ephemerides and Astrometry of the Solar System, 199–202.

and analogously for declination. We assert that, the generally held hypothesis about these partial derivatives with respect to the initial elements, in the sense that is $\frac{\partial \sigma_k}{\partial \sigma_j^o}(t) = \delta_{kj}, \forall t$, is not true. We denote with $\sigma_{k,j}^o(t) = \frac{\partial \sigma_k}{\partial \sigma_j^o}(t)$ this time-dependence from perturbed elements. These partial derivatives follow by means of a combined solution for the planetary Lagrange equations and their partial derivatives with respect to the initial elements:

$$\frac{d}{dt}\sigma_j = \sum_{k=1}^{6} L_{j,k}\frac{\partial \Re}{\partial \sigma_k}$$

$$\frac{d}{dt}\left\{\frac{\partial \sigma_k}{\partial \sigma_j^o}\right\} = \sum_{i=1}^{6}\sum_{m=1}^{6}\left\{\frac{\partial L_{k,m}}{\partial \sigma_i}\frac{\partial \Re}{\partial \sigma_m} + L_{k,m}\frac{\partial^2 \Re}{\partial \sigma_i \partial \sigma_m}\right\}\frac{\partial \sigma_i}{\partial \sigma_j^o} \tag{1}$$

where L is the Lagrange matrix, $\Re$ is the perturbation function and all indices considered range between 1 and 6. To carry out the integration of this system, we need to consider the first and second derivatives of $\Re$ with respect to all σ^o's, which are computed by means of the chain rule through rectangular ecliptic coordinates, following Simon [8]:

$$\frac{\partial \Re}{\partial \sigma_i \partial \sigma_m} = \vec{V}_i \cdot (\partial^2 \Re) \cdot \vec{V}_m + \vec{V}_{i,m} \cdot \partial \vec{\Re}.$$

$\vec{V}$ is the rectangular ecliptic position vector, $\vec{V}_i$ its partial derivative with respect to the element σ_i and $\partial \Re$ and $\partial \Re^2$ the gradient and second order matrix of the perturbation function in ecliptic cartesian coordinates. The expressions for these $\Re$ function derivatives are computed through the orbital vector plane position: for the first order one, see [5] and, for the second order one, see [7]. Taking into account these analytic expressions, we have selected the minor planet Ceres and we have integrated the equations (1) backwards from 1991 to 1906. Taking the initial elements from the I.T.A. publication [4] and making use of the planetary theory VSOP 87 [1], we obtained these partial derivatives (see table 1). On the other hand, we have numerically computed these values by means of the expressions $\sigma_{k,j}^o \simeq \frac{\sigma_k(\ldots,\sigma_j^o+\Delta\sigma_j,\ldots)-\sigma_k}{\Delta \sigma_j}$ (see table (2)). The agreement between the analytical computation and the numerical estimation confirms our proposal about the time-dependence of $\sigma_{k,j}^o$.

3. Errors in initial elements from catalog errors

To relate the errors involved in the initial elements from catalog errors, we consider $\vec{h}$, orthogonal to the orbital plane, and $\vec{p}$, in the perihelion direction, determining the orbital plane. The superscript "o" is used to denote their dependence on initial elements. Let $\vec{X}^o$ be whatever of these vectors, in rectangular ecliptic coordinates, and let $\vec{X} = \vec{X}^o + \triangle\vec{X}^o$ be

TABLE 1. Analytical computation

$\sigma^o_{k,j}$	a	e	i	Ω	ω	M
a^o	1.29915	-0.06408	-0.01294	-0.06643	-1.01247	63.91748
e^o	-0.01659	0.99336	0.00208	0.03862	-0.05611	0.42464
i^o	-0.00080	-0.00272	1.00081	-0.00233	0.09184	0.07861
Ω^o	0.00280	0.00000	-0.00050	0.99748	-0.05017	0.06743
ω^o	0.00281	0.00043	-0.00022	0.00041	0.95057	0.06454
M^o	0.00304	0.00023	-0.00005	-0.00078	-0.03126	0.94964

TABLE 2. Numerical estimation

$\sigma^o_{k,j}$	a	e	i	Ω	ω	M
a^o	1.29907	-0.06408	-0.01294	-0.06641	-1.01262	63.91452
e^o	-0.01636	0.99337	0.00205	0.03924	-0.05609	0.42131
i^o	-0.00047	-0.00273	1.00079	-0.00220	0.09170	0.07791
Ω^o	0.00281	0.00000	-0.00050	0.99750	-0.05017	0.06734
ω^o	0.00281	0.00043	-0.00022	0.00041	0.95057	0.06455
M^o	0.00304	0.00023	-0.00005	-0.00078	-0.03126	0.94964

its perturbed expression. To relate $\vec{X}^o$ to $\vec{X}$, we can apply the rotations $R_x(-\epsilon)$ for equatorial coordinates,R for the correction model and finally $R_x(\epsilon)$ to retrieve the incremented vector. ϵ is the ecliptic obliquity. The subscript in the R-matrix means rotation around the signaled axis and the rotation R is given by:

$$R = \begin{bmatrix} 1 & -\Delta\xi & -\Delta\eta \\ \Delta\xi & 1 & -\Delta\epsilon \\ \Delta\eta & \Delta\epsilon & 1 \end{bmatrix}$$

Finally, we apply this relation to $\vec{h}$ and $\vec{p}$ and we obtain the system of functional relations:

$$\begin{bmatrix} \Delta\Omega \\ \Delta\omega \\ \Delta i \end{bmatrix} = \begin{bmatrix} \cos\epsilon + \cos\Omega\cot i\sin\epsilon & \sin\epsilon - \cos\Omega\cot i\cos\epsilon & -\sin\Omega\cot i \\ -\csc i\cos\Omega\sin\epsilon & \csc i\cos\Omega\cos\epsilon & \csc i\sin\Omega \\ \sin\Omega\sin\epsilon & -\sin\Omega\cos\epsilon & \cos\Omega \end{bmatrix} \begin{bmatrix} \Delta\xi \\ \Delta\eta \\ \Delta\epsilon \end{bmatrix} \tag{2}$$

4. The equinox correction induced by the rotation model

A local lifting $\Delta\delta_E = \Delta\eta$ in the equinox E implies a correction in their right ascension, in the following sense: $\Delta\alpha_E = \left(\frac{\partial\alpha}{\partial\lambda}\Delta\lambda + \frac{\partial\alpha}{\partial\beta}\Delta\beta\right)_E$. So, we compute the values $\Delta\lambda_E$ and $\Delta\beta_E$ by means of the equations:

$$R_x(-\epsilon)R_z(-\Delta\lambda_E)\vec{e}_1 = \begin{bmatrix} 1 \\ \Delta a \\ \Delta\delta_E \end{bmatrix} \qquad R_x(-\epsilon)R_y(-\Delta\beta_E)\vec{e}_1 = \begin{bmatrix} 1 \\ \Delta b \\ \Delta\delta_E \end{bmatrix}$$

where $\vec{e}_1$ is the unit vector pointing toward the equinox. Then, we obtain the values $\triangle a = \triangle\delta_E \cot\epsilon$; $\triangle b = \triangle\delta_E \tan\epsilon$; $\triangle\lambda_E = \frac{\triangle\delta_E}{\sin\epsilon}$ and $\triangle\beta_E = -\frac{\triangle\delta_E}{\cos\epsilon}$. From $\left.\frac{\partial\alpha}{\partial\lambda}\right|_E = \cos\epsilon$ and $\left.\frac{\partial\alpha}{\partial\beta}\right|_E = -\sin\epsilon$, we obtain the relation $\triangle\alpha_E = \triangle\delta_E(\cot\epsilon + \tan\epsilon)$, implying the true correction for the equinox:

$$\triangle E = \triangle\xi - \triangle\eta(\tan\epsilon + \cot\epsilon) \tag{3}$$

5. Numerical Results

We have selected the four first minor planets, because a great number of observations in the FK4 System are provided for the MPC's magnetic tape. After solving the system (1), we obtain the σ_i^o values and then we apply the inverse relation (2), obtaining the following contributions for each minor planet and for each one of $\triangle\xi$,$\triangle\eta$, $\triangle\epsilon$.

TABLE 3. Amounts in parameter frame correction

$\triangle\xi$	1".765	1".312	1".202	1".980
$\triangle\eta$	0".543	0".328	0".084	0".619
$\triangle\epsilon$	0".217	-0".357	-0".190	0".376

Weighing these results with respect to the number of observations of each minor planet, we obtain the following results:

$$\triangle\xi = 1".541 \quad \triangle\eta = 0".384 \quad \triangle\epsilon = -0".02$$

and applying (3) we obtain the correction $\triangle E = 0^s.0325$ that agrees with the adopted value.

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