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BOOK REVIEWS

MATSUZAKI, K. and TANIGUCHI, M. *Hyperbolic manifolds and Kleinian groups* (Oxford Mathematical Monographs, Clarendon Press, 1998), ix+253pp., 0 19 850062 9, £60.

The elaboration of the theory of Kleinian groups and hyperbolic manifolds began at the end of the last century. At that time the chief work was done by Poincaré, Fricke and Klein. Since then this theory has developed rather rapidly. In particular, since the eighties through Thurston's work the theory has become central to modern mathematics.

The book under review presents some of the basic contours of the modern theory of hyperbolic 3-manifolds and Kleinian groups. It is a revised and enlarged translation of the authors' Japanese book published in 1993. The book is suitable for use as an advanced level graduate text, although suitable exercises are unfortunately not provided. Also, we remark that the list of references is far too short, and as a result of this one frequently finds that results are not entirely accurately accredited.

The book consists of eight chapters, including a zero-th chapter which is purported to give a brief summary of the theory of hyperbolic surfaces and Fuchsian groups. However, since this chapter seems not to fit well with the overall scheme of the book, many readers might prefer to omit it.

Chapters 1 and 2 are introductory, including basic notions and results such as hyperbolic manifolds and Kleinian groups (with some well known examples), limit sets, Jørgensen's inequality, Margulis's lemma and some combination theorems.

The following three chapters are concerned with aspects of rigidity for finitely generated Kleinian groups. Chapter 3 focuses on geometrically finite Kleinian groups, leading to Marsden's isomorphism theorem, Mostow's rigidity theorem and Ahlfors's zero measure theorem. The last of these states that the limit set of a geometrically finite Kleinian group is either the complete boundary at infinity or else it is of zero 2-dimensional Lebesgue measure (the book does not mention the stronger result of Sullivan and Tukia that the Hausdorff dimension of these limit sets is strictly less than 2). Chapter 4 gives the proof due to Sullivan and Bers of the celebrated Ahlfors finiteness theorem. It reviews some of the results concerning the Bers conjecture that b-groups are boundary groups of a Bers-slice. Chapter 5 begins with a review of some well known results concerning 2-dimensional spherical measure and limit sets of finitely generated groups. In particular, a proof of the Sullivan rigidity theorem is given. The chapter finishes with Sullivan's result that structural stability implies hyperbolicity. Chapter 6 reviews some recent developments concerning infinite ends (also called Jørgensen ends) of hyperbolic 3-manifolds. It gives a discussion of measured laminations, the Thurston compactification and the double limit theorem. Finally, Thurston's proof is provided of the affirmative answer to the Ahlfors conjecture for geometrically tame Kleinian groups. Chapter 7 gives a crash course on the differences between algebraic and geometric convergence of sequences of Kleinian groups. Then it is shown that every finitely generated Kleinian group is conditionally stable.

Summarising, despite the few weak points mentioned above, the book represents a valuable attempt to stabilise and summarise some of the main streams in the current research on hyperbolic manifolds and Kleinian groups. Any researcher in this area will certainly appreciate it as a useful source of background material.

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