

THE INTRINSIC SHAPES OF ELLIPTICAL GALAXIES

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ABSTRACT. Distribution functions for the intrinsic shapes of elliptical galaxies are discussed, starting with the simplest and proceeding to the more complex. A variety of competing "proxy" observables, which can in principle be used to recover at least some of information lost in the projection of a galaxy onto the plane of the sky, are considered.

1. INTRODUCTION

A frequently repeated theme in these proceedings has been that elliptical galaxies span a wider range of observed properties, and could in theory span a wider range of physical properties, than has heretofore been appreciated. This increasing complexity is reflected in the growing number of observables, on one hand, and theoretical parameters, on the other, which can be used to describe ellipticals.

As the dimensionality of the space of elliptical galaxies grows, the difficulty in determining the distribution of intrinsic properties from the distribution of observed properties grows too. Not only are larger statistical samples and better data needed, but more sophisticated analyses are required. I shall argue that while there has been marked improvement in the observations, the machinery necessary for interpreting these have not kept pace.

In particular, there is now a considerable body of data on position angle twists and ellipticity variations in ellipticals. But while these data have been used to draw the qualitative conclusion that at least some ellipticals are triaxial, the data have not yet been used to produce a convincing joint distribution function for intrinsic ellipticity and triaxiality.

Just beneath the question of what ellipticals are lies the question of how they formed. We seek a statistical description of ellipticals because we imagine that this will allow us to discriminate among competing models for their formation. Aguilar's contribution to the

present proceedings provides an example of the testable predictions of one particular model. Along the same lines, I suspect that a gradual collapse model for the formation of ellipticals, similar to the one often proposed for the formation of the spheroid of our own Milky Way, is unlikely to produce much triaxiality. As difficult as the determination of the intrinsic shapes of ellipticals may ultimately prove to be, the effort will not go entirely unrewarded.

But if the problem really is such a difficult one, which we might reasonably infer from the fact that it has not yet been treated satisfactorily, we must make every effort to cast it in as simple a form as possible. In the spirit of first approximation, we should be willing to disregard details which we suspect will only marginally influence our results.

There are two strategic questions which I would like to address, one more abstract, the other more practical. The abstract question concerns the choice of "interesting" parameters, and the choice of model distribution functions over those interesting parameters. I will argue that there are both observational and theoretical reasons to prefer some distribution functions over others. The practical question involves the choice of weapons. Since we lose one of the three dimensions of an elliptical galaxy to projection, we must find some other observable as a proxy. The candidate proxies include surface brightnesses, velocity dispersions, rotation velocities and position angle twists. I will consider in turn the relative strengths of each of these.

I will then briefly raise the question of the shape of the mass distribution in elliptical galaxies, which may be very different from the shape of the light distribution, and which requires a different observational approach.

2. DISTRIBUTION FUNCTIONS

2.1. $\phi = \phi(\epsilon)$

If one makes the assumption that elliptical galaxies are axisymmetric, the problem of inverting the observed distribution of apparent ellipticities is then a straightforward one. Nonetheless, there are differences in the distributions derived from different data sets which bear on important physical questions.

Binney and de Vaucouleurs (1981) used Lucy's method to invert the distribution of apparent flattenings in the Second Reference Catalog, and found a peak in the distribution of intrinsic ellipticities at $\epsilon = 0.38$, and a relatively flat distribution from $\epsilon = 0.20$ to $\epsilon = 0$. This plateau, if not an artifact of the inversion technique or the data, might be taken to indicate a process, perhaps a dynamical instability, which favors the formation of perfectly round galaxies. But Benacchio and Galletta (1980), who used data on cluster ellipticals from the work

of Strom and Strom, found an extreme deficiency of very round systems.

A third data set is available, in a doctoral thesis by Djorgovski (1986). Taking the ellipticity at a representative isophote ($r = 20.5$ mag/arcsecond²) yields a histogram of apparent ellipticities which is intermediate between that used by Binney and de Vaucouleurs and that used by Benacchio and Galletta. The paucity of very round systems in the Benacchio and Galletta data may be the result of their use of the maximum ellipticity for each galaxy rather than a mean ellipticity or the ellipticity at a fiducial isophote. I must apologize for not having gone through the exercise of inverting Djorgovski's distribution.

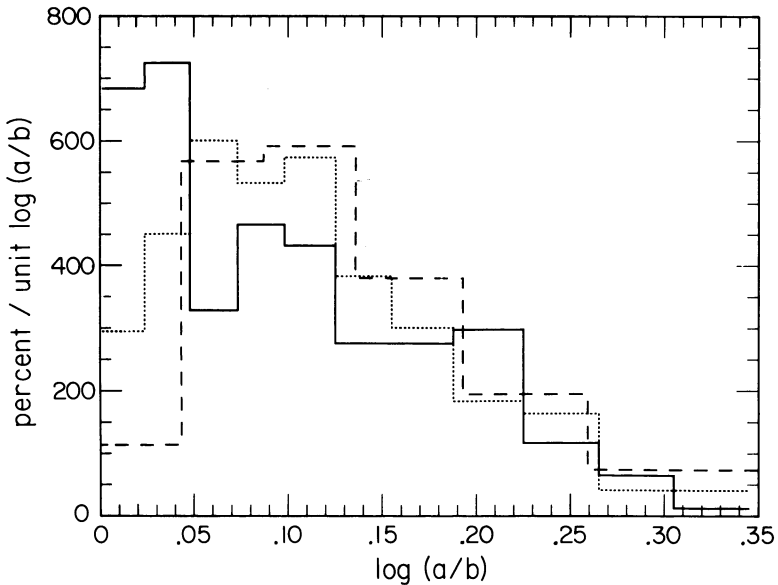


Figure 1. The distribution of observed axial ratios in the samples used by Binney and de Vaucouleurs (dashed line), Benacchio and Galletta (solid line), and Djorgovski (dotted line).

Djorgovski's data includes ellipticities at a wide range of radii. I was struck by the fact that among the systems which were very round at the chosen isophote, some were considerably flatter at other isophotes and others exhibited large position angle twists. Jedrzejewski (1986), who obtained photometry for a sample of southern ellipticals, finds that only 2 out of 49 galaxies have ellipticities everywhere less than 0.1. At the other extreme, de Vaucouleurs (1977) finds that there is an upper limit to the ellipticity of bona fide ellipticals, $\epsilon < 0.55$.

2.2. $\phi = \phi(\epsilon, \tau)$

The next level of complexity drops the assumption of axisymmetry and allows for a third axis intermediate in length between the longest and shortest axes. Such configurations, usually called triaxial, span the full range from oblate to prolate. If we adopt the convention $a > b > c$, and define a triaxiality parameter $\tau \equiv (a-b)/(a-c)$, then $\tau = 0$ for oblate galaxies and $\tau = 1$ for prolate galaxies. The wanted distribution is now a function of two variables, ϵ and τ .

This second variable greatly complicates matters. As a first approximation, some authors make the assumption that all ellipticals have the same value of τ , i.e. that the distribution in τ is a delta function. But while a single value of τ may be easier to deal with, it is easy to imagine physical processes which would give a range of values.

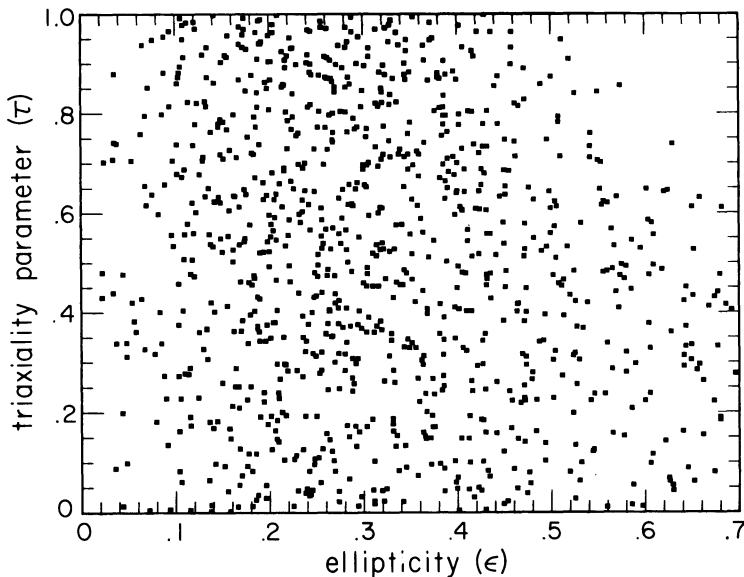


Figure 2. A 1000 point realization of a numerical model for the joint distribution of ϵ and τ .

For the sake of argument, consider a model for the distribution in ϵ and τ in which all three axes are mutually independent, with normal distributions about a common mean value. The distribution for such a numerical model is shown in Figure 2. The τ distribution spans the range from oblate to prolate almost uniformly. The ellipticity distribution peaks at an ellipticity of 0.3, and shows relatively few very round galaxies. The absence of round galaxies is explained by the low probability that all three axes will have the same length. While this model contains no physics, one might suppose that the velocity dispersion tensors in elliptical galaxies preserve some memory of the

shape of the proto-objects from which they formed. Gaussian fluctuations about an average value might apply in such a case.

2.3. $\phi = \phi(\epsilon, \tau, d\epsilon/d\ln r)$

As bad as distribution functions over two variables may seem, we find ourselves driven to a third variable. One of the more convincing arguments for triaxiality comes from the observation of twisting isophotes. Twists occur if a) a galaxy is triaxial and b) its axial ratios vary as a function of radius. The effect results from the fact that the major and minor axes of the projection onto the plane of the sky are not the projections of the intrinsic major and minor axes.

A quantitative treatment of position angle twists therefore demands a model for the radial variations in ellipticity and triaxiality. Since the former are easier to determine, and since we wish to avoid introducing a fourth random variable, it seems reasonable to take τ to be constant throughout a galaxy.

One approach to ellipticity variations would be to assume a constant gradient in ϵ with respect to $\ln r$, assuming further that this gradient varied randomly about some mean value. While such an approach has the advantage of simplicity, it does considerable injustice to what we know about ellipticity variations. A more flexible but more complicated alternative would be to adopt an ellipticity autocorrelation function, from which one could compute the probability of observing a change in ellipticity $\Delta\epsilon$ at distance $\Delta\ln r$ from a randomly chosen point. One might then compute predicted and observed position angle autocorrelation functions. The position angle autocorrelation would need to be studied as a function of observed ellipticity, since the biggest twists occur at the smallest apparent ellipticities.

3. OBSERVATIONS BEARING ON INTRINSIC SHAPES

Several different kinds of observations can be used as proxies for the unobservable third axis, and used to infer the the intrinsic shapes of ellipticals. The following table summarizes the different approaches:

PROXY	ASSUMPTIONS
surface brightness	$\mu = \mu(\epsilon)$
velocity dispersion	$\sigma = \sigma(\epsilon)$
minor axis rotation	shortest axis coincides with J vector
P.A. twists	none; ϵ gradient distribution needed
dust-lanes,	equilibrium configurations?
radio jets	self-gravitating disks?

Surface brightness and velocity dispersion measurements have thus far been inconclusive in helping to determine the intrinsic shapes of ellipticals. The underlying idea in both cases is simple. For an oblate galaxy, the surface brightness and velocity dispersion should be higher when viewed edge-on than when viewed pole-on. For a prolate system, the converse should apply.

Unfortunately, this simple situation is complicated by the possibility that surface brightness and velocity dispersion might be expected to vary with intrinsic axial ratio. Merritt (1982) found that given some latitude in the dependence of these quantities on intrinsic shape, and second, some latitude in the assumed scatter at a given intrinsic shape and luminosity, both oblate and prolate models could reproduce the available data. There has, however, been some improvement in our understanding of the dependence of velocity dispersion and surface brightness on intrinsic luminosity (Dressler *et al.*, 1987), and the time may be ripe for a new investigation of the subject.

Position angle twists, while the most reliable indicator of the existence of triaxiality, are rather difficult to analyze quantitatively. Benacchio and Galletta (1980) interpreted the isophote twists observed in the Strom's data in terms of a model with $\tau \approx 0.5$, the value for maximal triaxiality, and with ϵ varying with radius from zero to a maximum value which had a Gaussian distribution about $\epsilon = 0.38$, with a dispersion of 0.13. They took the conservative view that only position angle twists greater than 10° would be considered significant.

With the advent of CCD's, position angle accuracies of $1-2^\circ$ have become relatively easy to achieve. Leach (1981) obtained data for a sample of 32 galaxies, and interpreted them in terms of a triaxial model with Gaussian distributions in b/a , c/a , $\Delta(b/a)$, and $\Delta(c/a)$. While not unique, his adopted distribution function gives roughly equal numbers of nearly prolate ($2/3 < \tau < 1$) and triaxial ($1/3 < \tau < 2/3$) galaxies, with only half as many nearly oblate ($0 < \tau < 1/3$) ones.

Minor axis rotation offers a tool for studying triaxiality which does not require the presence of axial ratio variations. Binney (1985) has calculated the frequency distribution of the ratio of minor to major axis rotation for $\tau = 0.20$, 0.50, and 0.95. Working with data for only ten galaxies, he found that the extreme prolate hypothesis could be ruled out with considerable confidence. Most of the data used in his study were drawn from work by Gunn and myself, so I have no qualms about saying that much better data could be obtained today. Parallel efforts are underway by Franx and Illingworth and by Jedrzejewski and myself. Thus far, the evidence is that roughly equal numbers of galaxies do and don't show minor axis rotation at the 10 km/s level.

The picture is clouded by the discovery by Davies and Birkinshaw (1986) that NGC 4261, otherwise a relatively undistinguished elliptical, shows considerable minor axis rotation, and almost no major axis

rotation. While their data are not inconsistent with the hypothesis of a nearly prolate object tumbling about its shortest axis, this demands an unlikely orientation to the line of sight. The alternative hypothesis, that the galaxy rotates about its long axis, violates the one explicit assumption in Binney's analysis.

Another observation which bears on intrinsic shapes is the relatively rapid rotation seen in intrinsically faint ellipticals (Davies *et al.*, 1983). They are consistent with models of rotationally flattened oblate systems with isotropic dispersion tensors, and might therefore be thought to be oblate. If intrinsically faint systems are indeed oblate, then they ought not to exhibit position angle twists.

Dust-lane orientations have been frequently been taken as indicators of a galaxy's intrinsic shape (e.g. Hawarden *et al.*, 1981; Bertola, these proceedings). Galaxies with dust-lanes along their minor axes have been called prolate, and those with dust-lanes along their major axes have been called oblate. But in a triaxial system, gas and dust can settle into equilibria about either the longest or the shortest axis.

The "skewed" dust-lane ellipticals are yet more puzzling. While small deviations from orthogonality could result from chance projections of triaxial ellipticals, larger angles are harder to explain. One possible explanation is self-gravity, which Sparke (1986) has invoked to explain the apparent stability of inclined "polar ring" S0s. Another possible explanation is that the skewed dust-lanes are transient phenomena. If they resulted from the disruption of a smaller galaxy, one might expect such systems to have a higher incidence of shells, tails, etc.

The alignments (or rather the misalignments) of the radio and optical axes of radio galaxies have also been used to test the intrinsic shapes of ellipticals. A recent effort on the subject, by Birkinshaw and Davies (1985), shows a frightening degree of misalignment. Heckman *et al.* (1985) have found that radio lobes show better alignment with the kinematic minor axes of the gas in ellipticals than they do with the optical isophotes. One might once again appeal to self-gravity of the gaseous disks, which might precess as a unit. One would then expect to see signatures of such precession in the radio emission from misaligned doubles.

Centaurus A, NGC 5128, deserves special mention, by virtue of the fact that it is both a dust-lane elliptical and a radio double, and by virtue of its proximity, which makes it the best studied of such systems. It shows modest rotation along its major axis, and little or none about its minor axis (Wilkinson *et al.*, 1986).

4. SHAPES OF HALOS

There is considerable evidence that spiral galaxies are embedded dark halos which produce roughly logarithmic potentials. While less overwhelming, there is analogous evidence for ellipticals in their X-ray temperatures and profiles and in the velocities of their globular clusters and dwarf companions. Since the run of mass with distance from the center seems not to follow the run of light with distance from the center, one might think that the shape of the halo and the shape of luminous component could also be different.

It is inherently harder to determine the shapes of potentials than it is to determine the shapes of the luminous matter because the equipotentials of centrally condensed systems are rounder than their equidensity contours. A mass distribution with an ellipticity of 0.3 and a logarithmic potential will produce a potential with an ellipticity of only 0.1. We are therefore looking for subtler effects in potentials than in the mass itself.

Thermal bremsstrahlung from the X-ray coronae of ellipticals should trace the potential perfectly, since the pressure in the X-ray emitting gas cannot be anisotropic. The situation is rendered somewhat less clean by the possibility of ram-pressure distortions of X-ray coronae. I have examined the X-ray isophotes in the papers by Forman *et al.* (1985) and Trinchieri *et al.* (1986), and would only point to one case (NGC 720) where the X-ray isophotes look to be significantly out of round and yet sufficiently symmetric about the central elliptical to believe that one is seeing the shape of the potential.

If one permits discussion of the halos around non-elliptical galaxies, then there is more evidence. Whitmore, McElroy and Schweizer (1987) have observed polar rings around 3 S0 galaxies, measuring velocities both in the rings and in the central S0 components. The velocities in the two nearly orthogonal planes are close enough to permit interesting limits on the shapes of the potentials, with ellipticities of 0.2 or less. One might suspect, however, that such systems form preferentially in systems with rounder halos, in which case they might not give a fair representation of all halos. Moreover a potential with an ellipticity of 0.1 corresponds to a oblate mass distribution with an ellipticity of 0.3, which is just the typical intrinsic ellipticity seen in elliptical galaxies.

Another way to determine the shapes of spiral halos is to look for minor axis rotation in the disks of spirals. The effect is described in a paper by Binney (1978) called "Twisted and Warped Disks as Consequences of Heavy Halos." If halos are triaxial, then the apparent minor axis of a spiral galaxies would not necessarily coincide with the projection of its rotation axis. The effect is greatest in galaxies which are nearly face on, and demands careful measurements of position angles, which are made more difficult by the presence of spiral structure. This latter difficulty might be avoided by looking for the effect in the stellar disks of S0 galaxies.

5. SUMMARY

In preparing this review I was dismayed at how little can be said with any confidence about the intrinsic shapes of elliptical galaxies despite the substantial effort that has gone into studying them. I do not, however, believe that this effort has been wasted. There are promising opportunities to build on these earlier efforts which I suspect will yield the wanted answers.

A. Ellipticity and position angle data now exist for large samples of ellipticals. What is needed is a good statistical treatment of ellipticity variations and a good statistical treatment of position angle variations. One could then try to "predict" the observed position angle variations using the observed ellipticity variations and a variety of assumptions about the triaxiality distribution.

B. Minor axis rotation can be measured both in ordinary ellipticals and in those with dust-lanes. Either we will not see many repeats of the pathology of NGC 4261, and will be able to draw conclusions about triaxiality, or we will be forced to re-examine some cherished notions.

C. X-ray isophotes can give us the shapes of the potentials in which our ellipticals are embedded. Those of us who believe in triaxiality expect some twisting, and depending upon how ellipticals form, might expect some 90° misalignments.

REFERENCES

- Benacchio, L. and Galletta, G. 1980, M.N.R.A.S., **193**, 885.
 Binney, J. 1978, M.N.R.A.S., **183**, 779.
 Binney, J. 1985, M.N.R.A.S., **212**, 767.
 Binney, J. and de Vaucouleurs, G. 1981, M.N.R.A.S., **194**, 679.
 Birkinshaw, M. and Davies, R.L. 1985, Ap.J., **291**, 32.
 Davies, R.L. and Birkinshaw, M. 1986, Ap.J.(Letters), **303**, L45.
 Davies, R.L., Efstathiou, G., Fall, S.M., Illingworth, G., and Schechter, P.L. 1983, Ap.J., **266**, 41.
 de Vaucouleurs, G. 1977, in The Evolution of Galaxies and Stellar Populations, ed. B.M. Tinsley and R.B. Larson (New Haven: Yale University Observatory) p. 43.
 Djorgovski, S. 1986, unpublished Ph.D. thesis, University of California, Berkeley.
 Dressler, A., Lynden-Bell, D., Burstein, D., Davies, R.L., Faber, S.M., Wegner, G., and Terlevich, R. 1987, Ap.J., in press.
 Forman, W., Jones, C., and Tucker, W. 1985, Ap.J., **293**, 102.
 Hawarden, T.G., Elson, R.A.W., Longmore, A.J., Tritton, S.B., and Corwin, H.G. 1981, M.N.R.A.S., **196**, 747.

- Heckman, T.M., Illingworth, G.D., Miley, G.K., and van Breugel, W.J.M. 1985, *Ap.J.*, **299**, 41.
- Jedrzejewski, R.I. 1987, *M.N.R.A.S.*, in press.
- Leach, R. 1981, *Ap.J.*, **284**, 485.
- Merritt, D. 1982, *A.J.*, **87**, 1279.
- Sparke, L.S. 1986, *M.N.R.A.S.*, **219**, 657.
- Trinchieri, G., Fabbiano, G., and Canizares, C. 1986, *Ap.J.*, **310**, 637.
- Whitmore, B.C., McElroy, D.B., and Schweizer, F. 1987, *Ap.J.*, **314**, 439.
- Wilkinson, A., Sharples, R.S., Fosbury, R.A.E., and Wallace, P.T. 1986, *M.N.R.A.S.*, **218**, 297.

DISCUSSION

Aguilar: I want to mention the results of some dissipationless collapse calculations that David Merritt, Martin Duncan and I have made and that may explain the lack of round galaxies. We have run a series of collapses started from spherical, oblate, and triaxial initial conditions. It seems that only cold initial conditions ($2T/W < 0.1$) result in final models with realistic surface density profiles ($r^{1/4}$ -laws) but whenever this happens the models develop an instability associated with the predominance of radial orbits. This instability produces prolate bars out of spherical initial conditions. Non-spherical initial conditions produce prolate and triaxial configurations but never oblate or spherical models. We should point out, however, that we have not yet included rotation in our simulations.

Binney: Sverre Aarseth and I played similar games, though starting from flattened initial conditions. We found that even the modest amount of net angular momentum pushed the final configuration towards oblate axisymmetry. We should not forget that practically all ellipticals do rotate at some level.

Capaccioli: Let me assume that the variations of ellipticity and position-angle do not occur in the same regions of the galaxy, i.e. that they are spatially not correlated. Would this influence the analysis?

Schechter: While I have not done such experiments myself, I would expect a correlation between the location of observed ellipticity variation and position angle variations. There are several galaxies in Djorgovski's sample which exhibit rapid 90 degree position angle twists. These occur near zero ellipticity.

Jarvis: Do you observe from your data, or expect on theoretical grounds, a correlation between the amount of rotation on the minor axis and the strength of the isophotal twisting?

Schechter: One of the strengths of the minor axis rotation test is that it is independent of ellipticity variations. Our sample was selected to avoid galaxies with large twists, which might conceivably introduce a selection effect.

King: At the cost of introducing still another complication, I would like to ask whether there are galaxies with good elliptical isophotes but where the twists are too large to account for with constant axial directions.

Williams: There are some potentially very worrisome systems observed, where there is a significant axis ratio change and position angle twist, while the isophotes are never very round. NGC 584 is an example. It is very difficult to do this by projection effects alone, without resorting to large three-dimensional axis ratio gradients, which will result in large deviations from elliptical isophotes (not observed). We will be forced to non-coaxial models, which I suspect can reproduce both twists and axis ratio changes at any flattening without so seriously distorting the individual isophotes.

Lauer: There are systems of strongly interacting elliptical galaxies that show strong twists; it seems that this is a likely case of galaxies with non-coaxial ellipsoids.

Schechter: I agree.

Porter: Preliminary indications are that brightest cluster ellipticals can show large twists simultaneously with large ellipticities. Whether they are too large to be projection effects, I'm not prepared to state yet.

Gerhard: Two comments. (i) Some years ago I came across an N-body model with *intrinsic* twists of the principal axes, which lasted for ~ 15 dynamical times. While this may be a little too short to say much about the inner parts of ellipticals where dynamical time-scales are short, it may mean that intrinsic twists in the outer parts of these systems are dynamically possible (cf. 1983, *Mon. Not. R. astr. Soc.*, **203**, 198.). (ii) I would like to emphasize the value of gas disks in ellipticals for the deciphering of their intrinsic shapes. In a joint poster paper with Mario Vietri, it is shown that, if the circumstances are favorable, one may determine both the axial ratios from the geometry and the velocity field.

Schechter: Thank you for reminding us of your models with intrinsic twists. Under the intrinsic hypothesis, one might expect greater twisting in the outer parts than in the inner parts. Your method for determining the shapes of spheroidal components looks to be a powerful one. There is, of course, a rear guard which banishes from the class of all ellipticals all objects with any hint of a disk.

Williams: In three dimensions, for ellipsoidal figures, there are two potentially variable functions—the two axis ratios of the figure. The projection of these onto the sky results also in two variable functions: axis ratio changes and position angle twists. You have proposed a statistical analysis which admits only one variable three dimensional function to produce both projected functions. I suspect that you would be able to find projection angles which will allow this, but then you will introduce correlations between projection angles and the one variable function which do not exist in the actual objects. I am afraid this will very much complicate the analysis, but may well be unavoidable.

Schechter: You have considerably more experience in this matter than I do. Nonetheless, I would think one could place limits on such systematic effects through Monte Carlo simulations. The sense of the effect you describe would be due to the inferred distribution of triaxialities closer to maximal triaxiality than is really the case.

Davies: If NGC 4261 is to be oblate/triaxial as is shown to be possible in the posters of Statler and Levison, I think it is surprising that the $\Delta\theta = \theta_{rot} - \theta_{min}$ -histogram that I showed yesterday has no entries with $30 < \Delta\theta < 80$. Do you agree? I believe this oblate-triaxial configuration to be possible but unlikely. If the orbits are populated in this particular way, it appears that we should expect to see the kinematic axes to be non-perpendicular. Observers wishing to test this need to use more than two position angles, at least four, I think.

Schechter: Perhaps galaxies populate either the short axis tubes or their long axis tubes, but not both.

Burstein: On a somewhat different topic, I note that the Strom & Strom and Djorgovski samples include elliptical galaxies with a wide intrinsic range of luminosity, while the RC2 sample, with its Malmquist-bias, will be dominated by high luminosity galaxies. Independent of errors or other kinds of selection effects, a plot of ellipticity vs. radial velocity for the RC2 sample might show if a real difference in these samples exists.

Schechter: I have used only Djorgovski's "sample I", which is magnitude limited and therefore has roughly the same luminosity distribution as the RC2 sample. The Stroms' sample is more nearly volume limited, and as such permits a test for a trend of ellipticity with absolute magnitude.

Valentijn: Responding to your request for more and uniform data on axial ratios and isophotal twists, I can announce that together with A. Lauberts a large two-dimensional photometric project is in progress at ESO. We have scanned all 16000 galaxies present in the ESO-Uppsala catalogue, on both red and blue original sky survey plates. In an automated mode, we are extracting magnitudes, colour gradients, axial ratios, isophotal twists, and various other parameters, as function of radius. Individual plates are calibrated using photo-electric data. As of May 1986, we have determined parameters of 6000 objects, and we envisage a full presentation of the results in the summer of 1988.