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LORENTZIAN GEOMETRY AND PHYSICS IN KASPAROV'S THEORY

KOEN VAN DEN DUNGEN

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We study two geometric themes, Lorentzian geometry and gauge theory, from the perspective of Connes' noncommutative geometry [3] and (the unbounded version of) Kasparov's KK-theory [2, 7]. Lorentzian geometry is the mathematical framework underlying Einstein's description of gravity. The geometric formulation of a gauge theory (in terms of principal bundles) offers a classical description for the interactions between particles. The underlying motivation is the hope that this noncommutative approach may lead to a unified description of gauge theories coupled with gravity on a Lorentzian manifold.

The main objects in noncommutative geometry are spectral triples, which encompass and generalise Riemannian spin manifolds. A spectral triple defines a class in K-homology, via which one can access the topology of the (noncommutative) manifold. In this thesis we present two possible definitions for 'Lorentzian spectral triples', which offer noncommutative generalisations of Lorentzian manifolds as well. We will prove that both definitions preserve the link with analytic K-homology. We will describe under which conditions Lorentzian (or pseudo-Riemannian) manifolds satisfy these definitions. Another main example is the harmonic oscillator, which, in particular, shows that our framework allows us to deal with more than just metrics of indefinite signature.

In the context of noncommutative geometry, the description of a gauge theory can be obtained from so-called almost-commutative manifolds. While the usual approach yields by default a topologically trivial gauge theory (in the sense that the corresponding principal fibre bundle is globally trivial), we show in this thesis that the framework can be adapted, using the internal unbounded Kasparov product, to allow for globally nontrivial gauge theories as well.

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Finally, we combine the two themes of Lorentzian geometry and gauge theory, and we define Krein spectral triples, which generalise spectral triples from Hilbert spaces to Krein spaces. We use this definition to construct almost-commutative Lorentzian manifolds. Furthermore, we propose a Lorentzian alternative for the fermionic action, which allows us to derive (the fermionic part of) the Lagrangian of a gauge theory. We show that our alternative action recovers exactly the correct physical Lagrangian.

The results of this thesis are published in [1, 4–6].

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KOEN VAN DEN DUNGEN, Scuola Internazionale Superiore di Studi Avanzati, SISSA, via Bonomea 265, 34136 Trieste, Italy
e-mail: kdungen@sissa.it