A CASE FOR ALFVEN WAVE HEATING

F. CALIFANO, C. CHIUDERI and G. EINAUDI
Department of Astronomy and Space Science, University of Florence
Largo Enrico Fermi, 5
50125 Florence, Italy

ABSTRACT:

The resistive dissipation of Alfvén waves in magnetically structured media is examined within the framework of an analytically solvable model in plane geometry. A new class of rapidly oscillations solutions is found, for which the role of resistivity extends to the whole system.

1. INTRODUCTION

Alfvén waves are the inevitable result of the response of a magnetized plasma to relatively slow perturbations. By travelling along field lines Alfvén waves transport energy away from the source region. They might therefore constitute an ideal heating mechanism, if their energy could be damped at the appropriate rate where heat is needed. The dissipation of Alfvén waves is tied to the formation of small spatial scales across the ambient field. In homogeneous fields the only transverse spatial scale is given by the wavelength in the direction normal to the field. To achieve an efficient dissipation, the ratio between perpendicular and parallel wavelengths has to be unacceptably small. Perfectly homogeneous systems, on the other hand, are not likely to be found in nature.

If the system is non-uniform, the propagation and dissipation properties of Alfvén waves may change dramatically, as a large number of studied has shown in recent years (Sedlacek 1971, Tataronis and Grossman 1973, Kappraff and Tataronis 1977, Hasegawa and Uberoi 1982, Heyvaerts and Priest 1983, Dewar and Davies 1984, Pao and Kerner 1985, Mok and Einaudi 1985, Lee and Roberts 1986, Einaudi and Mok 1987, Davila 1987).

Most of these studies model the non homogeneous nature of the problem in slab geometry by assuming that the magnetic field and/or the density depend on a single coordinate. They also adopt asymptotically flat profiles for the above quantities and assume that resistivity can be neglected altogether in the asymptotic regions. The solutions then describe the so-called resonant absorption phenomenon, consisting of an enhanced resistive dissipation localized in a narrow region around the point where the propagating frequency matches the local Alfvén frequency. These models however, do not exhaust the possible solutions of the fourth-order resistive differential equation. Another class of solutions is investigated in the present paper and their relevance to the heating problem discussed.

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2. BASIC EQUATIONS

We shall work in a resistive MHD perturbative scheme in plane geometry and assume that the equilibrium quantitites depend only on the x coordinate. The equilibrium field is supposed to have a constant direction that defines the z-axis.

We adopt as basic scales: a, the typical scalelength of the equilibrium field, $\rho(\infty)$ and $B(\infty)$, the asymptotic values of the density and magnetic field. All other scales are derived from them.

The absence of structural variations with y and z in the equilibrium allows to write the perturbed quantities as:

$$f(\mathbf{r}, t) = f(x) \exp [i (k_y y + k_z z - \omega t)]$$

The linearized resistive MHD equations can be combined into a single fourth-order differential equation for the x-component of the velocity, w, that reads (Mok and Einaudi 1985):

$$(\varepsilon w')' - k^2 \varepsilon w = - i \omega \frac{(\rho_o c^2_A)}{S} \left\{ \frac{1}{(\rho_o c^2_A)} [(\rho_o w')' - k^2 \rho_o w] \right\}'' \qquad (1)$$

where

$$\varepsilon = \rho_o (k^2_z c^2_a - \omega^2 - i \omega k^2/S)$$

$$k = (k^2_y + k^2_z)^{1/2} \;;$$

$$S = \tau_r/\tau_a = (4 \pi \sigma/c^2) \; a \; c_a(\infty) \quad ,$$

σ being the electrical conductivity and τ_r the resistive timescale. As usual, a subscript "o" indicates equilibrium quantities and a prime the x-derivative.

All the quantities entering the equations must be understood as non-dimensional, each having been normalized with respect to the appropriate scale.

Since B_o and ρ_o are asymptotically constant, in the asymptotic region the coefficients or equation (1) are constant and four independent solutions can be easily found. They read,

$$w_{1,2} \sim e^{\pm k_z x} \;; \quad w_{3,4} \sim e^{\pm \lambda x}$$

where

$$\lambda = \left(\frac{i S \varepsilon}{\rho_o \omega} \right)^{1/2} . \qquad (2)$$

We thus see that besides the well known "ideal" solutions ($w_{1,2}$), we have a couple of "resistive" solutions. One solution in each pair is well - behaved at $x \to + \infty$, the other at $x \to - \infty$.

The solutions of the asymptotic fourth order resistive equation thus separate naturally in two distinct classes. The asymptotically ideal solutions and their extension to the whole x-axis have been extensively studied in the past years. They give rise to the well-known "resonant"solutions, where resistivity plays a role only in the vicinity of the locations where $\omega = k_z c_a$.

On the other hand, the existence of "resistive" asymptotic solutions implies the possibility of finding complete solutions of equation (1) for which the resistive terms are important everywhere. The physical distinction between the two possible choices of asymptotic solutions becomes more apparent when we move into the non-homogeneous region. In the "ideal" case we essentially insist in keeping the system non-dissipative as long as possible. As a result the dissipation is concentrated in a thin boundary layer. By choosing the "resistive" asymptotic solutions we investigate the possibility of a more widespread influence of the resistivity in the non-uniform region.

Equation (2) shows that the resistive solutions vary asymptotically on a spatial scale proportional to $S^{-1/2}$. We notice that, if the complete solution (i.e. valid for every x) also varies on the same scale, the dominant resistive term of equation (2.1) (~ to w^{IV}) is always comparable with the dominant ideal term (~ to w''), since each derivative introduces a factor proportional to $(S^{-1/2})$.

Retaining only the dominant terms of equation (1), we finally arrive at the equation that governs the resistive solutions:

$$(k_z^2 c_a^2 - \omega^2)\, w'' = -\frac{i\omega}{S} w^{IV} \quad . \tag{3}$$

3. THE MODEL

We assume for $c^2_a(x)$ the only quantity entering equation (3), the following form:

$$c^2_a(x) = 1 + \Delta \operatorname{sech}^2(x) \quad , \tag{4}$$

When $\Delta > 0$, such a profile may represent a configuration where the magnetic field stays essentially constant and the density shows a central depression (as in solar coronal holes) or the cut through a magnetic flux tube where the field intensity is considerably larger than outside. Introducing equation (4) into equation (3) we get:

$$(w'')'' + i S \left[\left(\omega - \frac{k_z \Delta}{\omega} \right) - \frac{k^2_z \Delta}{\omega} \operatorname{sech}^2(x) \right] w'' = 0 \tag{5}$$

Defining:

$$\xi = \operatorname{tgh}(x) \quad , \quad \phi(\xi) = w''(x) \quad ,$$

$$\lambda^2 = -i S\left(\omega - \frac{k^2_z}{\omega}\right) \quad , \quad \nu(\nu+1) = -\frac{i S k^2_z \Delta}{\omega} \quad . \tag{6}$$

we obtain, from equation (5),

$$(1-\xi^2) \frac{d}{d\xi}\left[(1-\xi^2) \frac{d\phi}{d\xi} \right] + \left[\nu(\nu+1) - \frac{\lambda^2}{1-\xi^2} \right] \phi = 0 \tag{7}$$

It is possible to show that in order to satisfy the boundary conditions: $\phi \rightarrow 0$ when $|\xi| \rightarrow 1$, ν and λ cannot be arbitrary but must satisfy:

$$\mathrm{Re}\,\lambda > 0 \quad \text{and} \quad \nu - \lambda = n \;, \quad n = 0, 1, 2 \ldots \tag{8}$$

The solution of equation (7) can then be conveniently written as:

$$\phi(\xi) = (1-\xi^2)^{\lambda/2} C_n^{\lambda+1/2}(\xi) \;, \tag{9}$$

where $C_n^{\lambda+1/2}(\xi)$ is the n-th Gegenbauer polynomial (Magnus et al. 1966).

Recalling the definitions (6) we see that the conditions (8) completely determine the (complex) eigenvalues ω as functions of k_z, Δ and S. General solutions of the dispersion relation, eq. (8), must be found numerically However, since S>>1, approximate solutions can be found by expanding ω in powers of $(k_zS)^{-1/2}$. Retaining only the leading terms we get:

$$\omega = k_z \{(1+\Delta)^{1/2} - (1+i)(n+1/2)(\Delta/2)^{1/2}(1+\Delta)^{-1/4}(k_zS)^{-1/2}\} \tag{10}$$

It can further be shown that $\mathrm{Re}\,\lambda > 0$ implies the existence of a maximum value of n , n_{max}, that scales as $n_{max} \sim \Delta^{1/2}(k_z S)^{1/2}$. For a given set of parameters therefore only a finite number of solutions exist.

The results of the numerical evaluation of the dispersion relation are shown in Fig. 1 as solid lines. The open circles give the values predicted by the approximate formula, eq. (10). The agreement turns out to be remarkably good for the physically interesting range $k_zS > 10^6$.

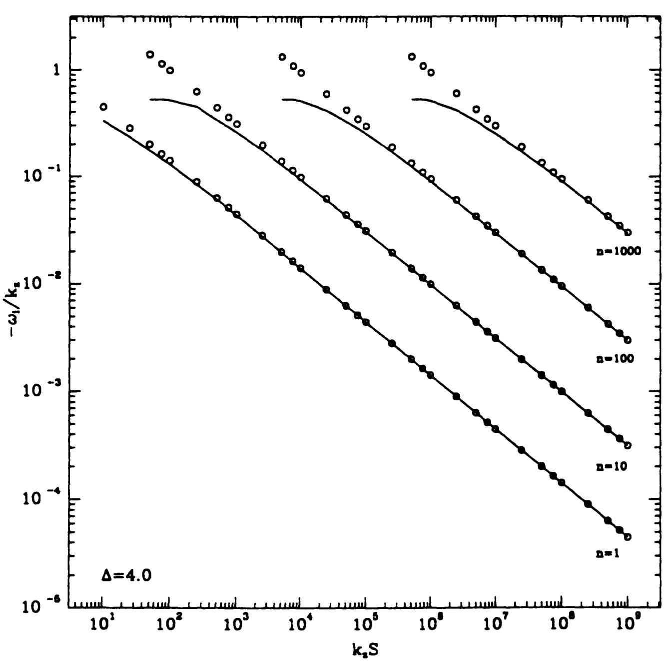

Fig. 1.

The damping rate, ω_i/k_z, as a function of k_zS for different values of n. Solid line: numerical evaluation of eq. (8). Open circles: approximate expression, eq. (10).

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Eq. (9) allows us to figure out the spatial structure of the eigenfunctions. The term $(1-\xi^2)^{1/2} \equiv [\text{sech}(x)]^{\lambda}$ is a rapidly oscillating one, that quickly damps out for large values of x. The spatial scale of both the oscillation and the damping are proportional to $\lambda^{-1} \sim S^{-1/2}$, as expected. The polynomial term is a modulating factor that, for large n's becomes increasingly small in an increasingly wide region around x = 0. The combined effects of the two terms for large n's is to confine the oscillations in two symmetrical regions that move away from the origin as n increases. This behaviour is clearly born out by Fig. 2.

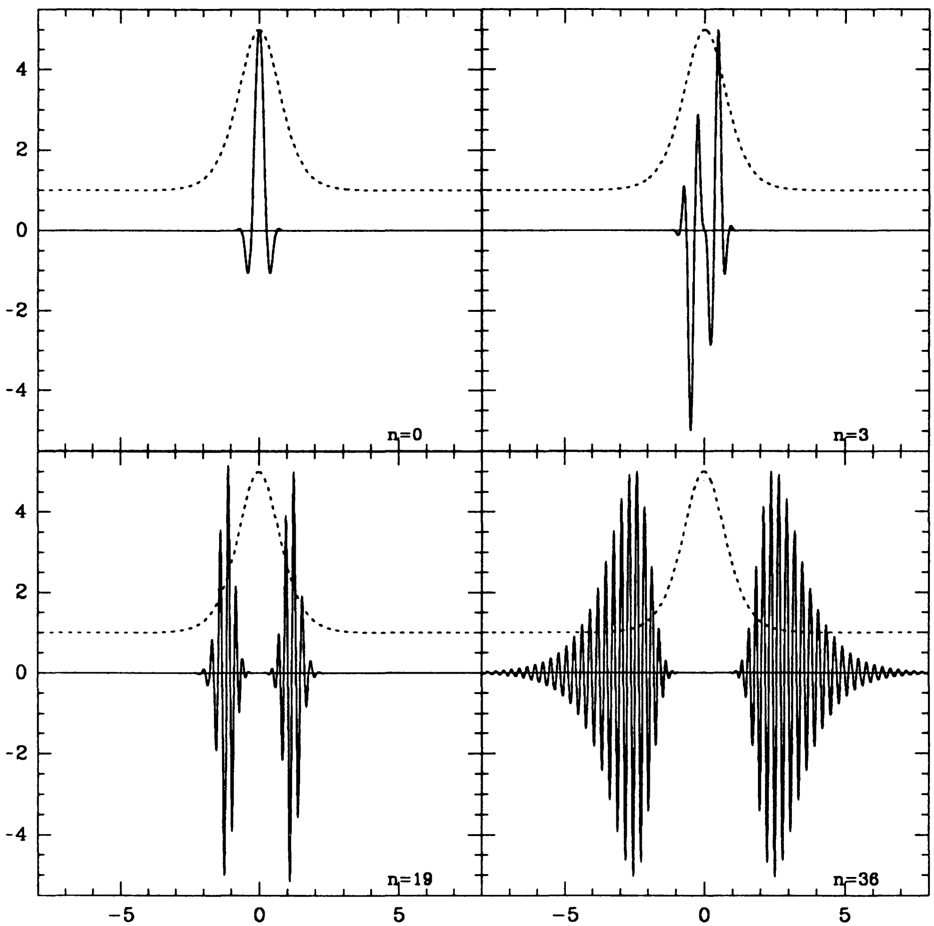

Fig. 2. - The real part of the eigenfunction, eq. (9), for $S = 10^4$, $k_z = 0.1$, $\Delta = 4$. For this case $n_{max} = 38$. The broken line is the profile of c_a^2, eq. (4).

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4. CONCLUSIONS

We have shown that consideration of non-ideal asymptotic boundary conditions gives rise to a new type of solutions for the resistively damped Alfvén waves. These solutions are characterized by the appearance of very small scales in the x-direction. Their relevance to the heating problem can be appreciated by noticing that for each k_z there are now n_{max} solutions, whose superposition spans the entire non-uniform region. The spatial structure of the normal modes, previously discussed, makes likely an efficient resistive dissipation. Since the normal modes can be thought as the asymptotic state of the temporal evolution of an initial perturbation of arbitrary scale, we may interpret the appearance of the small scales as the result of the bilinear interaction of the initial perturbation and the non-uniform equilibrium structure. If this interpretation is correct, we expect that energy will cascade towards the small scales in the x-direction already in the linear phase. The timescale of this cascade can only be determined by an initial value approach.

The preceding interpretation has been confirmed by a preliminary series of numerical experiments (Carbone and Malara 1989) that show the development of an energy cascade towards the small scales in the x-direction on times of the order of $\sim\tau_a$, the Alfvén time of the large scales and the subsequent damping of arbitrary initial perturbations. The transient time, needed to establish the normal modes, turns out to be also of the order of few τ_a.

During the transient the spectrum of modes in the z-direction remains essentially unchanged, thus showing that the formation of the small scales across the field takes place well before the possible developpment of the non-linear mode-mode coupling along the field.

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DISCUSSION

UBEROI: The fourth-order differential equation of MHD resistive effects has been previously considered by many workes who found an absorption rate varying as $S^{1/2}$, similar to your calculations. What is new in your work?

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CHIUDERI: There are essentially two new features. Firstly, the choice of "resistive" rather than ideal asymptotic boundary conditions makes the dominant resistive term balance the dominant ideal term everywhere and not simply at the resonant layer. Secondly, we have worked out an analytical solution, showing that, even with a single k_z, instead of one (or few) "resonant" region we have an extended region where large gradients of the physical quantities are present.

BUTI: I believe that your model is restricted to classical resistivity. Turbulent resistivity could be more important. Also one should consider a space-dependent resistivity consistent with the inhomogeneous magnetic field. Hasan has done this in connection with fusion plasmas and we are doing this exercise for the solar corona. Also, Alfvén waves, in the presence of a driver, can become chaotic and lead to very efficient heating.

CHIUDERI: We wanted to prove a point of principle. Therefore we used the simplest possible model. One could add other effects, but at the cost of not being able to find analytical solutions. Anyway, if dissipation works with classical resistivity it will work even better with an anomalous one.

RYUTOVA: Did you estimate the energy density of the power, released in the region of n_{max} where the scale in the x-direction is a minimum? Due to your cascade the heating power must have a very characteristic shape with a pronounced maximum. Calculations similar to yours show that the heating power grows with increasing n and at n_{max} reaches the maximum, and then rapidly decreases. This effect can lead to strong heating and brightening of the region where $n = n_{max}$.

CHIUDERI: So far we wanted simply to invesigate the possible existence of solutions different from the resonant one. A more complete study of the energetics is in progress. I was aware of your results and I am very pleased at the similarity you pointed out.

RUDERMAN: How does the damping rate of Alvén waves depend on the ratio of the magnetic field scale to wave length?

CHIUDERI: There is no "wave length" along x, since the problem is non-homogeneous in that direction. As far as the z-direction is concerned the dependences are those shown in the paper.

DAVILA: If I have understood your presentation correctly, the results you have presented demonstrate the existence of two new non-singular modes which exhibit resistive effects, and these are in addition to the better known singular modes of the ninth order dissipative wave equation.

CHIUDERI: Yes, completely correct.