ON SOME CHARACTERISTICS OF THE DISTRIBUTION OF PERIHELIA OF LONG-PERIOD COMETS

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1. Introduction

Although the problem of the origin of comets has not yet been settled, there are only two possibilities; one is to regard that comets formed within the solar system when or soon after planets formed, and the second possibility is that comets have been captured from interstellar space by planetary perturbations or by perturbations of the galactic nucleus. To decide upon the possibility, one has to take into account statistical features of cometary orbits.

In this paper, we examine if there are statistical data which support interstellar origin of comets, and in doing so, we will take into account long-period comets (comets with periods exceeding 200 years) and disregards short-period comets. There are two reasons why only long-period comets are considered. Firstly, of nearly 600 comets observed so far, some 500 are long-periodic. Secondly, short period-comets are generally regarded as long-period comets which have been perturbed into short periodic orbits by planets (mainly Jupiter and Saturn).

If it is found that statistical data of the properties of cometary orbits are such that they support interstellar origin of comets, one should proceed to investigating physical processes which lead to cometary formation in interstellar space.

2. Perihelion points and solar motion apex

A. Direction of strong concentration.-In the cometary catalogue of Marsden (1972), 503 parabolic and nearly parabolic comets are contained. We will label each of them by suffix i. Take a system of rectangular coordinates (x,y,z), such that x axis coincides with L=0°, y axis with L=90°, and z axis with B=90°, where L and B are ecliptic longitude and latitude, respectively (equinox 1950). Let (Ω,i,ω) be the geometrical orbital elements of a comet. From observed values of (Ω,i,ω), the direction cosines of perihelion point (P_i) can be readily calculated, which will be denoted by (ℓ_i,m_i,n_i). Further, let $\overline{\ell}$, $\overline{m}$, $\overline{n}$ be direction cosines of a point Q which is to be determined. The angle

R. L. Duncombe (ed.), Dynamics of the Solar System, 283–287.

(α_i) between the two points P_i and Q is given by

$$\cos \alpha_i = \ell_i \bar{\ell} + m_i \bar{m} + n_i \bar{n} \ . \tag{1}$$

We consider the sum of $\cos \alpha_i$,

$$S \equiv \sum_{i=1}^{N} \cos \alpha_i \ . \tag{2}$$

We now require that the point Q be chosen such that S takes on a minimum value. This is achieved when $\bar{\ell}$, $\bar{m}$, $\bar{n}$ are so chosen that

$$\frac{\Sigma \ell_i}{\bar{\ell}} = \frac{\Sigma m_i}{\bar{m}} = \frac{\Sigma n_i}{\bar{n}} \ ,$$

or

$$\bar{\ell} = \frac{1}{N} \Sigma \ell_i, \quad \bar{m} = \frac{1}{N} \Sigma m_i, \quad \bar{n} = \frac{1}{N} \Sigma n_i \ . \tag{3}$$

Note that if a criterion other than minimizing S were adopted, the direction $(\bar{\ell},\bar{m},\bar{n})$ would be different from the one given by equation (3). Numerical values of L and B given by equation (3) were calculated using orbital elements of 503 comets contained in Marsden's catalogue. They are given by

$$L = 259^{\circ}.7 \ , \quad B = 66^{\circ}.0 \tag{4}$$

The direction of the solar motion referred to the neighbouring stars is characterized by

$$L_s = 260^{\circ}.6 \ , \quad B_s = 43^{\circ}.2 \tag{5}$$

which differs from the direction (4) by some 20° only.

B. The closeness of the two directions (4) and (5) suggests the possibility that they are not independent but are correlated. Therefore the next problem is to examine whether the closeness is merely a chance coincidence or otherwise. We therefore assume (null hypothesis) that the perihelion points are uniformly randomly distributed in the celestial sphere and investigate if the statistical data support the hypothesis. Under the hypothesis, the average values of (ℓ_i, m_i, n_i) will all be zero, while their dispersions are all equal to 1/3. Now, the direction of the solar apex is known *a priori* so that the cosine of the angle θ_i between the apex and P_i is such that the mean value is zero, while the standard deviation is 1/3. Now, according to the central limit theorem of probability, the distribution of sum of $\cos \theta_i$ approaches, as N is increased, a normal distribution such that the mean is zero and the dispersion is N/3. Since N is large (N=503), the above theorem should hold with sufficient approximation. For a normal variate, it should lie in the interval $(-2.57\sigma, 2.57\sigma)$ with the probability of 99%, where σ is the standard deviation. On the null hypothesis, the standard deviation, σ of the normal distribution is $\sigma=(167.7)^{1/2}=12.9$, so that the above interval is (-33.15, 33.15). The actual value of $\Sigma\cos\theta_i$ is found to be 70.65, well outside the interval. Hence one may conclude that at 99% level of significance, the distribution of perihelion points are not unrelated to the solar motion apex.

3. Tamanov's law

Tamanov (1976) pointed out that if perihelia are concentrated at one point (L,B), then a relation of the form

$$\tan i = \tan B/\sin(L-\Omega) \tag{6}$$

should exist between inclination (i) and node (Ω). We insert the values $L=L_s=260^\circ\!.6$, $B=B_s=43^\circ\!.2$, and obtain the curves shown in Fig. 1.

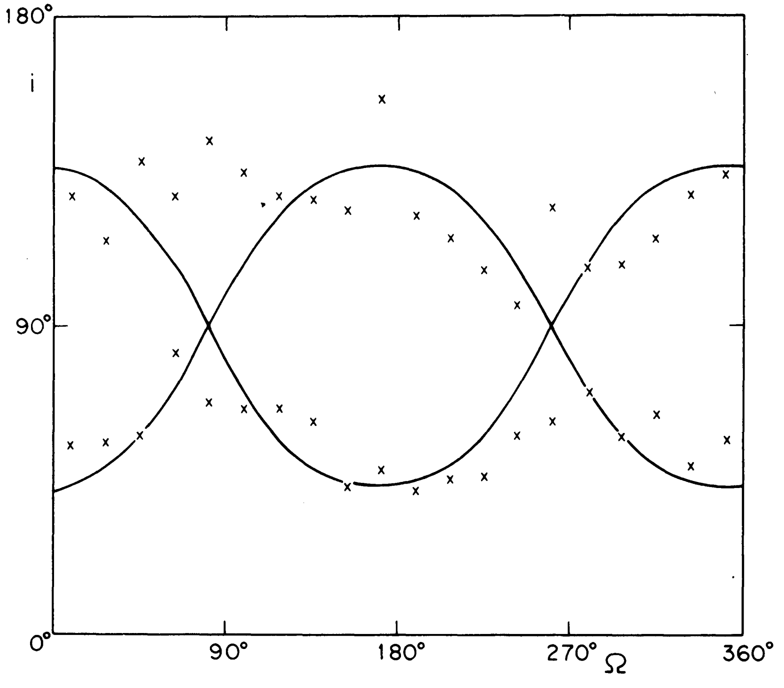

Figure 1. The curve gives the relation between i and Ω given by equation (6), while a cross denotes the average of observed i in each interval of Ω.

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In order to see whether or not the observed comets satisfy the above relation, we once again used Marsden's catalogue and computed the average of i in each 18° interval of Ω. Those with $i>90°$ and those with $i<90°$ were averaged separately. Again, only those comets whose galactic longitudes lie in (25°, 155°) and in (205°, 335°) have been taken into account. For $i<90°$, there are 176 such comets and for $i>90°$, there are 192 such comets.

From the figure, the agreement between the relation (6) and observational data appears fair, if not excellent.

4. Discussions and Conclusions

We have followed the works of Oppenheim (1922), Tyror (1957) and Hasegawa (1976) regarding perihelion distribution by making use of Marsden's catalogue, and confirmed that perihelion points are concentrated in the direction of solar motion.

As to the relation (6) which should hold true if all perihelia were concentrated at a point, the agreement with observational data appears fair, though not excellent. When all of the observed comets are included, the agreement gets worse. Therefore, it appears that the gravitational field of the galaxy perturbs cometary orbits to some extent, especially those with large values of semi-major axes.

The two results arrived at in the present work indicate that cometary orbits are strongly related to the solar motion. This might be an indication that comets have interstellar origin. It has been pointed out (Yabushita & Hasegawa 1978) that some of comets with negative original values of 1/a calculated by Marsden *et al* (1978) may represent those comets which originated in interstellar gas clouds.

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DISCUSSION

Kiang: You have exaggerated the improbability of the observed closeness of your mean perihelion point to the solar apex, on the null hypothesis. Since for any distribution of the perihelion points, you will always find a mean point, what is remarkable is then that the mean point should be within $\theta(\sim 25°)$ of some special point. On the null hypothesis, the mean point should be uniformly distributed on the sphere. Hence the probability of the observed event on the null hypothesis is $(1-\cos\theta)/2 \sim .05$.

Van Flandern: Observational selection effects have operated to greatly reduce the number of observed cometary perihelia south of the equator. When these are allowed for, the center of the observed perihelion distribution moves far (perhaps 60°) away from the solar apex. It also moves to ecliptic longitude 222°. Therefore, the proximity you have discussed must be a coincidence. [Everhart and Weissman concurred with this remark.]

Yabushita: Dr. Hasegawa told me that those comets observed in the southern hemisphere have the same tendency as those found in the northern hemisphere. See Publ.Astr.Soc.Japan 28,1976.

Marsden: We find original hyperbolic orbits only for comets of small perihelion distance. We attribute this to the fact that comets of small perihelion distance are influenced by nongravitational forces, which do not affect large-perihelion distance comets as much. How do you explain the fact that the hyperbolic orbits are restricted to the comets of small perihelion distances?

Yabushita: Your comment is very interesting but does not refer directly to what I have spoken about. Your comment refers to a paper by Hasegawa and myself to be published shortly. Hasegawa and I will closely examine whether a correlation exists between q and 1/a values in the manner you have suggested.

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