b Illustrations of Euc. I., 47.

c Discussion of figures of the same shape (their areas, &c.).

The principles of mapping and drawing to scale.

How to measure the breadth of a river without crossing it.

d Use of squared paper to find areas. A Δ described and its area found (1) by counting the whole squares included and estimating the broken ones; (2) by the ordinary method. Results compared. Area of circle found in this way.

§ 9. To the foregoing I am permitted now (March 1888) to add a few remarks.

The course above described is neither complete nor systematic. If an idea came in the regular course, good and well; if a digression was necessary we digressed. We had no text-book to follow, and no examination to prepare for. As it happened, the pupils were examined after all, and it may encourage others to know that the work was characterised as "an excellent special course."

This session the course, still subject to modification, is being repeated to four classes similar to last year's three. A problem discussed this year with great interest was the finding of the distance of an inaccessible object by actual work in the open air. The pupils got out in batches of ten (the rest of the class working at another problem). A chain was used to measure the base line, and the base angles were taken by the eye applied to a ruler laid along the paper. Precautions were taken to have the books in the proper position. Further work of this kind is to be done.

Eighth Meeting, June 10th, 1887.

George Thom, Esq., LL.D., President, in the Chair.

Note on Milner's Lamp.

By Professor Tait.

This curious device is figured at p. 149 of De Morgan's *Budget of Paradoxes,* where it is described as a "hollow semi-cylinder, *but not with a circular curve,*" revolving on pivots. The form of the cylinder is

https://doi.org/10.1017/S0013091500001498 Published online by Cambridge University Press

such that, whatever quantity of oil it may contain, it turns itself till the oil is flush with the wick, which is placed at the edge.

Refer the "curve" to polar coördinates, r and θ; the pole being on the edge, and the initial line, of length a, being drawn to the axis. Then if θ_0 correspond to the horizontal radius vector, β to any definite radius vector, it is clear that the couple due to the weight of the corresponding portion of the oil is proportional to

$$\int_{\theta_0}^{\beta} r^2 d\theta\left(a\cos\theta_0 - \frac{2}{3}r\cos(\theta-\theta_0)\right).$$

This must be balanced by the couple due to the weight of the lamp. and of the oil beyond β; and this, in turn, may be taken as proportional to

$$\cos(\alpha+\theta_0).$$

Thus the equation is

$$a\cos\theta_0\int_{\theta_0}^{\beta} r^2 d\theta - \frac{2}{3}\left(\cos\theta_0\int_{\theta_0}^{\beta} r^3\cos\theta d\theta + \sin\theta_0\int_{\theta_0}^{\beta} r^3\sin\theta d\theta\right)$$
$$= b^3\cos(\alpha+\theta_0).$$

Differentiating twice with respect to θ_0, and adding the result to the equation, we have (with θ now put for θ_0)

$$2ar^2\sin\theta - 2ar\frac{dr}{d\theta}\cos\theta + 2r^2\frac{dr}{d\theta} = 0.$$

Rejecting the factor r, and integrating, we have

$$r^2 = 2ar\cos\theta + C.$$

This denotes a *circular* cylinder, in direct contradiction to De Morgan's statement!

As it was clear that this result, involving only one arbitrary constant, could not be made to satisfy the given differential equation for all values of b, α, and β, I fancied that it could not be the complete integral. I therefore applied to Prof. Cayley, who favoured me with the following highly interesting paper. It commences with the question I asked, and finishes with an unexpectedly simple solution of Milner's problem.

It appears clear that De Morgan did not know the solution, for the curve he has sketched is obviously one of continued curvature—and he makes the guarded statement that a friend "vouched for Milner's Lamp."
