as those of probability space, random variable, independence, characteristic function; theorems such as Boole's Inequality, Chebyshev's Inequality, the uniqueness theorem and continuity theorem for characteristic functions. However it is somewhat surprising to discover the Lévy-Khinchine canonical representation of infinitely divisible distributions as one of the prerequisites for reading a book with this title. Here is the first indication of the influence of the author's interests on the content.

Chapter 2 is concerned with convergence concepts and relations among them. Again much is as to be expected—the notions of almost certain convergence, convergence in probability, in quadratic mean, in law (though the author emphasizes that the last mentioned is only of marginal interest in the book). Less familiar modes of convergence are also introduced here, such as almost uniform convergence, complete convergence, Λ-convergence and information convergence.

The third chapter again exemplifies the licence which the author has permitted himself. It departs from the main thread of the theory of stochastic convergence and is concerned with the question of the existence of metrics on spaces of random variables which are compatible (in the obvious sense) with different modes of convergence. This chapter might well be of more interest to functional analysts than to statisticians.

Chapter 4 returns to territory more familiar to statisticians, and here we find the celebrated convergence theorems: zero-one laws, Kolmogorov's Inequality, the three-series theorem, laws of large numbers, the law of the iterated logarithm, the Glivenko-Cantelli theorem, etc.

Each of the remaining three chapters is short (about ten pages)—one on the definition of stochastic integrals and derivatives and the final two on somewhat esoteric characterizations of the normal distribution and the Wiener process respectively.

The difficulty facing a reviewer of giving a brief description of the content of this monograph will now be apparent. The style is easier to describe: a typical concise, pure mathematical style of definition followed by lemma, followed by theorem, all in logical order but somewhat lacking in motivation to those with a more applied outlook.

The reader who hopes for a useful intermediate textbook on stochastic convergence will be disappointed. However the book will satisfy those who wish a reference book containing clear definitions of concepts and lucid proofs of theorems. Some of these concepts and theorems are standard. Others, as indicated in the description of the content above, provide an introduction to less familiar theory.

A. D. SILVEY

HANSEN, E. (Editor), *Topics in Interval Analysis* (Oxford University Press, 1969), vi+130 pp., £2·50 or 50s.

In January 1968, a symposium on interval analysis was organized by L. Fox of the University of Oxford and held at the Culham Laboratory. This book contains the notes, amplified in some cases, of the lectures which were presented at it, together with a research paper by the editor.

Interval analysis, a recent development in numerical analysis, is an attempt to provide realistic error bounds on the result of a sequence of arithmetic computations. The subject springs from the recognition that although any number stored in a computer is not often known exactly close lower and upper bounds for it can be calculated easily. The proposal is therefore that a different type of arithmetic, called *interval arithmetic*, in which the arithmetic operations are performed on the intervals defined by these bounds, should be used. If this is carried out naively the results are disappointing for it is then a crude form of forward error analysis. When it is used

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in the solution of linear algebraic equations for instance the pessimistic predictions of von Neumann and Goldstine are confirmed. The province of interval analysis is that of re-writing known algorithms and developing new ones so that the use of interval arithmetic will lead to satisfactory error estimates. In the applications given here this means that a close investigation of the given mathematical problem has to be made before actually performing the calculations, and then taking it into account when organizing the program of operations.

The book is in two parts. The first is on algebraic problems and the chapter titles are as follows:

1. Introduction to algebraic problems by R. E. Moore.
2. Triplex-Algol and its applications by K. Nickel.
3. Zeros of polynomials and other topics by E. Hansen.
4. On linear algebraic equations with interval coefficients by E. Hansen.
5. On the estimation of significance by J. Meinguet.

The second part is devoted to continuous problems:

6. Introduction to continuous problems by R. E. Moore.
7. On solving two-point boundary value problems using interval arithmetic by E. Hansen.
8. Ordinary differential equations by F. Krückerberg.
9. Partial differential equations by F. Krückerberg.
10. On the centred form by E. Hansen.
11. Distributions in intervals and linear programming by M. Dempster.

The contributions are of varying quality and length. For example the chapter on partial differential equations is three pages long and is a sketch of possible ways of dealing with two types of equation. There is clearly a great deal more work to be done in this direction. But chapter 4 contains a fairly detailed account of the problem of its title. The first two chapters are thoroughly practical and can be recommended as a useful exposition of some of the aims and techniques of interval analysis. However the final chapter, although of theoretical interest, has little to say to the practical numerical analysist.

It is clear to the reviewer from the examples discussed that the marriage of interval analysis with mathematical analysis produces worthwhile results. It remains to be seen if the union can survive all the problems, including those of cost effectiveness, which it will be called upon to solve. In the meantime the book can be recommended as a useful collection of essays on this expanding field of research. D. KERSHAW

NEILL, H. AND MOAKES, A. J., *Vectors, Matrices and Linear Equations* (Oliver and Boyd, 1967), 216 pp.

The teaching of these topics at sixth-form level in schools is now well-established, and these two experienced authors cover them very broadly. I find congenial their introduction of general ideas such as algebraic structure, mapping, equivalence relation, and the definition of an algebra as a set with a binary relation defined on it. These topics are well introduced with many examples which will be familiar and illuminating. It is also good to see linear dependence and the fundamental idea of a vector space being thoroughly explained at this stage in simple terms.