CONSTRUCTION OF RELATIVISTIC EPHEMERIDES AND APPLICATIONS

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Abstract. The present precision of VLBI observations for the Earth rotation, the accuracy of time measurement realised with modern atomic clocks, and the necessity to obtain precise definition of reference systems and links between them, imply that we compute in the near future more accurate ephemerides, based on general relativity theory (GRT), in accordance with the recent IAU resolutions (IAU, 1992). This paper is dedicated to the presentation of semi-analytical integration of the motion of bodies in the Solar system which completes the theories built at the Bureau des Longitudes by (Bretagnon and Francou, 1988), with some applications in the determination of coordinate time scales links.

1. Introduction

The construction of accurate ephemerides will be of great importance in the future to provide semi-analytical models of translation and rotation of the planets of the Solar system, which could fill the needs of astrometric observations, the precision of which has considerably increased. The recent progress in this field has been made by Brumberg (1991), Damour *et al.* (1991–1994) and Klioner (1993).

Let us recall that the present solutions VSOP (VSOP 82 and 87) were obtained on the basis of Lagrange's differential equations for the elliptic variables. Moreover, these solutions contain the whole of the perturbations up to the third order of the masses for all the planets from Mercury to Neptune. For the outer planets, the solution is completed up to the sixth order of the masses by an iterative method. The Schwarzschild relativistic contribution to the perturbations has been also taken into account. The accuracy of such solutions reaches several *mas* for the inner planets and

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less for the outer ones. In order to complete the solution VSOP87, we integrated translatory equations of the Solar system bodies based on a barycentric metric developed up to the fourth order in $1/c$, extending the well known Schwarzschild relativistic perturbation of the orbital motion in taking into account all the mutual relativistic perturbations of the planets between themselves.

This new ephemeris will be useful in determining the links between the space-time reference systems (RS) introduced in GRT, that is the barycentric RS and the planetocentric RS, both in dynamically and kinematically non-rotating sense, including the links between coordinate time scales attached to them.

2. Barycentric Metric and Equations of Motion

The space-time of GRT is the curved pseudo-Riemannian manifold described by the quasi-Galilean metric $g_{\alpha\beta}$, found by solving Einstein's field equations with complementary boundary conditions.

We consider the post-Newtonian expansion of the barycentric metric, $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$, where $\eta_{\alpha\beta}$ is the metric associated to the Minkovski space-time ($\eta_{00} = 1$, $\eta_{0i} = 0$, $\eta_{ij} = -\delta_{ij}$), and $h_{\alpha\beta}$ is expanded with respect to the linearized theory (Brumberg, 1991):

$$\left\{ \begin{array}{lcl} h_{00}(t,\mathbf{r}) & = & c^{-2}h_{00}^{(2)} + c^{-4}h_{00}^{(4)} + c^{-5}h_{00}^{(5)} + \dots \\ h_{0i}(t,\mathbf{r}) & = & c^{-3}h_{0i}^{(3)} + c^{-5}h_{0i}^{(5)} + c^{-6}h_{0i}^{(6)} + \dots \\ h_{ij}(t,\mathbf{r}) & = & c^{-2}h_{ij}^{(2)} + c^{-4}h_{ij}^{(4)} + c^{-5}h_{ij}^{(5)} + \dots \end{array} \right. \tag{1}$$

Denoting x_A^k and $v_A^k = dx_A^k/dt = \dot{x}_A^k$ the BRS rectangular coordinates and velocity components of any body A of the Solar system and writing $r_A^k = x^k - x_A^k$, $r_A = (r_A^k r_A^k)^{1/2}$, we have in harmonic coordinates :

$$\left\{ \begin{array}{lcl} h_{00}^{(2)} & = & -2\sum_A \frac{GM_A}{r_A} \\ h_{00}^{(4)} & = & -4\sum_A \frac{GM_A}{r_A}\dot{\mathbf{x}}_A^2 + 2\left(\sum_A \frac{GM_A}{r_A}\right)^2 + 2\sum_A \frac{GM_A}{r_A}\sum_{B\neq A}\frac{GM_B}{r_{AB}} \\ & & +\sum_A \frac{GM_A}{r_A}\left(\mathbf{r}_A\ddot{\mathbf{x}}_A + \frac{1}{r_A^2}\left(\mathbf{r}_A\dot{\mathbf{x}}_A\right)^2\right) \\ h_{0i}^{(3)} & = & 4\sum_A \frac{GM_A}{r_A}\left(\dot{x}_A^i\right) \\ h_{ij}^{(2)} & = & 2\sum_A \frac{GM_A}{r_A}\eta_{ij} \end{array} \right. \tag{2}$$

Associated with the metric described above, we can construct the barycentric equations of motion following Fock's method (Fock, 1955), on the form of perturbed motion :

$$\ddot{\mathbf{x}}_i = -\sum_{j\neq i}\frac{GM_j}{r_{ij}^3}\mathbf{r}_{ij} + \sum_{j\neq i}\frac{GM_j}{c^2}\left(A_{ij}\mathbf{r}_{ij} + B_{ij}\dot{\mathbf{r}}_{ij}\right) \tag{3}$$

TABLE 1. Mean mean motions.

Body	Mercury	Venus	Earth	Mars
n_k	26087.9027370761	10213.2853878520	6283.0757525711	3340.6123749029
Body	Jupiter	Saturn	Uranus	Neptune
n_k	529.6909568816	213.2990921308	74.7815974078	38.1330350465

for each body i, with :

$$\left\{\begin{array}{lll} A_{ij} & = & \frac{\dot{\mathbf{x}}_i^2}{r_{ij}^3} - 2\frac{\dot{\mathbf{r}}_{ij}^2}{r_{ij}^3} + \frac{3}{2r_{ij}^5}\left(\mathbf{r}_{ij}\dot{\mathbf{x}}_j\right)^2 + G\left[5M_i + 4M_j\right]\frac{1}{r_{ij}^4} \\ +\sum_{k\neq i,j} & GM_k & \left[\frac{4}{r_{ij}^3 r_{ik}} + \frac{1}{r_{ij}^3 r_{jk}} + \frac{4}{r_{ij} r_{jk}^3} - \frac{7}{2}\frac{1}{r_{ik} r_{jk}^3} - \frac{1}{2}\frac{1}{r_{jk}^3 r_{ij}^3}\left(\mathbf{r}_{ij}\mathbf{r}_{ik}\right)\right] \\ B_{ij} & = & \frac{1}{r_{ij}^3}\left(4\mathbf{r}_{ij}\dot{\mathbf{r}}_{ij} + \mathbf{r}_{ij}\dot{\mathbf{x}}_j\right) \end{array}\right. \tag{4}$$

3. Integration Method

We transformed (3) by introducing for a body the perturbative complex variable p and the real variable w defined by the relations:

$$\left\{\begin{array}{l} x^1 + ix^2 = a(1-p)e^{i\lambda} \\ x^3 = aw \end{array}\right. \tag{5}$$

where a is the semi-major axis, $\lambda = nt + \varepsilon$ the mean mean longitude, n being the mean motion in Table 1, x^i the rectangular coordinates of the body and $i^2 = -1$.

Then, if we denote X, Y and Z the components of the sum of the Newtonian and the relativistic perturbations, $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$, we obtain for p and w a system of second-order linear differential equations which admits the following solution:

$$\begin{array}{rl} p = & Ae^{i\lambda} - 3\bar{A}e^{-i\lambda} + inB + 3inCt - 2C \\ + & \frac{3}{4}ine^{-i\lambda}\int(3P - \bar{P})e^{i\lambda}dt + \frac{1}{4}ine^{i\lambda}\int(3\bar{P} - P)e^{-i\lambda}dt \\ - & 2in\int Pdt - \frac{3}{2}n^2\int\int(P - \bar{P})dtdt \\ w = & De^{i\lambda} + \bar{D}e^{-i\lambda} + \frac{1}{2}ine^{-i\lambda}\int We^{i\lambda}dt - \frac{1}{2}ine^{i\lambda}\int We^{-i\lambda}dt \end{array} \tag{6}$$

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where A and D are complex integration constants, B and C real ones and:

$$\left\{ \begin{array}{lcl} P & = & -\frac{3}{2}(p+\bar{p}) + (\frac{a^3}{r^3} - 1)(1-p) - \frac{1}{n^2 a}(X+iY)e^{-i\lambda} \\ W & = & -(\frac{a^3}{r^3} - 1)w + \frac{1}{n^2 a} Z \end{array} \right. \quad (7)$$

The Keplerian part of P is of second order in eccentricity and inclination and we use an iterative scheme to integrate equations (3). This means that we introduce a solution of the motion of the Solar system bodies expressed in barycentric coordinates in the right-hand side of (7) and after computing p and w with (6), we derive a new solution with the help of (5). The convergence of such a process has been checked.

In order to do that, the VSOP87 solution has been transformed so that it is expressed with TCB as time-like argument (Moisson, 1996). This new solution refers to the inertial and dynamical equinox J2000 (TCB) and we show, when determining the differences induced by this choice on the precession quantities computed in (Simon *et al.*, 1994), that this new reference frame could be completely mixed up with the commonly used J2000 reference frame.

4. Difference between Planetocentric and Barycentric Coordinate Time Scales

Let $u = TCP$ and $t = TCB$ be a planetocentric and barycentric coordinate time, respectively. The proper time of an observer being the same in both the barycentric and the planetocentric reference system, we have :

$$\begin{aligned} \frac{du}{dt} &= 1 + \frac{1}{2}c^{-2}h_{00}^{(2)} - \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{8}\frac{v^4}{c^4} + \frac{3}{4}c^{-2}h_{00}^{(2)}\frac{v^2}{c^2} \\ &\quad + \frac{1}{2}c^{-4}h_{00}^{(4)} - \frac{1}{8}(h_{00}^{(2)})^2 + c^{-3}h_{0i}^{(3)}\frac{v^i}{c} \end{aligned} \quad (8)$$

where v^i are the components of the barycentric velocity of the observer.

Using the ephemeris in TCB, we compute and integrate this relation for Mercury, Venus, the Earth, Mars and the Moon. These differences can be found in (Moisson, 1996) with a low accuracy. For practical applications (planetary ranging for instance), the difference for Mars is the most interesting. Indeed, the periodic part of this difference is much larger in the case of Mars because of the eccentricity of its orbit. We give it below with all the terms whose amplitude is greater than $0.5\,\mu s$ after one hundred years. Let us write

$$TCB - TCP_{Mars} = A_1 t + A_2 t^2 + \sum_\alpha t^\alpha \sum_i A_{\alpha i} \sin(\omega_{\alpha i} t + \phi_{\alpha i}).$$

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TABLE 2. TCP–TCB difference – MARS.

Periodic Part

$A_{\alpha i}$ [μs]	$\omega_{\alpha i}$ [rd/10^3 yr]	$\phi_{\alpha i}$ [rd]	Period [years]	Argument Me	V	E	M	J	S	U	N
11419.44044	3340.6123	.3381	1.88	0	0	0	1	0	0	0	0
532.34005	6681.2247	.6762	.94	0	0	0	2	0	0	0	0
38.74814	2810.9214	2.4544	2.23	0	0	0	1	−1	0	0	0
37.21632	10021.8371	1.0143	.62	0	0	0	3	0	0	0	0
11.63899	−3.5231	1.9600	1783.38	0	0	4	−8	3	0	0	0
7.33877	3127.3132	2.1814	2.00	0	0	0	1	0	−1	0	0
5.15983	2281.2304	4.3087	2.75	0	0	0	1	−2	0	0	0
5.25571	5621.8428	4.9046	1.11	0	0	0	2	−2	0	0	0
4.07906	−398.1489	1.2076	15.78	0	0	1	−2	0	0	0	0
3.08301	13362.4494	1.3524	.47	0	0	0	4	0	0	0	0
2.78566	213.2990	5.2467	29.45	0	0	0	0	0	1	0	0
2.70029	−7.1135	2.3207	883.25	0	0	0	0	2	−5	0	0
2.47061	2942.4633	1.7962	2.13	0	0	1	−1	0	0	0	0
2.45047	529.6909	5.8361	11.86	0	0	0	0	1	0	0	0
2.26129	6151.5337	2.7671	1.02	0	0	0	2	−1	0	0	0
2.22737	2544.3143	3.2318	2.46	0	0	2	−3	0	0	0	0
1.90053	191.4482	2.4247	32.81	0	1	0	−3	0	0	0	0
1.60894	3337.0892	2.2189	1.88	0	0	4	−7	3	0	0	0
1.57701	−3344.1354	1.5407	1.87	0	0	4	−9	3	0	0	0
1.46570	−796.2979	2.8172	7.89	0	0	2	−4	0	0	0	0
.84182	5092.1518	5.1842	1.23	0	0	0	2	−3	0	0	0
.81155	1751.5395	4.4567	3.58	0	0	0	1	−3	0	0	0
.79643	2146.1653	4.6375	2.92	0	0	3	−5	0	0	0	0
.78895	2914.0141	3.5437	2.15	0	0	0	1	0	−2	0	0
.75474	3265.8307	3.8644	1.92	0	0	0	1	0	0	−1	0
.71118	3302.4793	4.0334	1.90	0	0	0	1	0	0	0	−1
.63021	1059.3819	5.2301	5.93	0	0	0	0	2	0	0	0
.53854	74.7815	2.5184	84.01	0	0	0	0	0	0	1	0
.50886	−3340.5951	5.3898	1.88	0	2	0	−8	8	−6	0	0
.50885	3340.6296	6.0656	1.88	0	2	0	−6	8	−6	0	0
.50528	8962.4552	5.2672	.70	0	0	0	3	−2	0	0	0

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$A_{\alpha i}$ [μs]	$\omega_{\alpha i}$ [rd/10^3 yr]	$\phi_{\alpha i}$ [rd]	Period [years]	Argument Me	V	E	M	J	S	U	N
891.58759	3340.6123	5.1746	1.88	0	0	0	1	0	0	0	0
83.12462	6681.2247	5.5126	.94	0	0	0	2	0	0	0	0
8.71706	10021.8371	5.8508	.62	0	0	0	3	0	0	0	0

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Then, $A_1 = 0.971938157 \times 10^{-8}$ and $A_2 = 0.6 \times 10^{-17}$. The coefficients $A_{\alpha i}$ are reported in Table 2. In this table, $\omega_{\alpha i}$ are given both numerically in column two and under the form of linear combinations of the mean mean motions given in Table 1.

5. Conclusion

We expect the above described integration process to provide solutions ten to hundred times better than the previous semi-analytical VSOP ones. At this level of accuracy, the choice of the time as argument used in the ephemerides is relevant and we decided to follow the IAU resolutions in adopting TCB. The validity of physical approximations used to develop the barycentric metric (point masses for the bodies, no rotational model, etc.) will be checked. This new ephemeris will be useful in deriving the links between astronomical reference systems introduced in (Brumberg, 1995), (Klioner, 1993), and (Kopeikin, 1991) and the rotation parameters of the Earth (Bretagnon, 1996).

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