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QUASI-SUPRABARRELLED SPACES

J. C. FERRANDO and M. LÓPEZ-PELLICER

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Abstract

In this paper a proper class of barrelled spaces which strictly contains the suprabarrelled spaces is considered. A closed graph theorem and some permanence properties are given. This allows us to prove the necessity of a condition of a theorem of S. A. Saxon and P. P. Narayanaswami by constructing an example of a non-suprabarrelled Baire-like space which is a dense subspace of a Fréchet space and is not an (LF)-space under any strong locally convex topology.

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In [9], S. A. Saxon and P. P. Narayanaswami prove that if E is a Fréchet space and F is a barrelled dense subspace, then F is not suprabarrelled if and only if there exists a subspace G of E such that G contains F and G with a topology stronger than the relative one is an (LF)-space. In this paper we give an example which allows us to prove that G cannot be replaced by F in the former theorem. We introduce the quasi-suprabarrelled spaces which satisfy the inclusion relationships indicated by

$$\text{suprabarrelled} \Rightarrow \text{quasi-suprabarrelled} \Rightarrow \text{barrelled}.$$

A closed graph theorem together with some permanence properties are given. We use some of those results later in order to construct the quoted example.

The linear spaces we use are defined over the field of the reals or complex numbers. By "space" we mean "Hausdorff locally convex space". If A is a

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subset of a linear space we denote by $\langle A\rangle$ its linear span. If B is a bounded closed absolutely convex subset of a space E, E_B denotes the normed space over the linear hull of B. If $\{E_i : i \in I\}$ is a family of spaces, $E := \prod\{E_i : i \in I\}$ and J is a part of I, we write $E(J)$ to denote the subspace of E of those elements which have zero in any coordinate position indexed by $I-J$. A space E is suprabarrelled or (db), [12], [6] and [9], if given an increasing sequence of subspaces of E covering E there is one of them which is barrelled and dense in E. A space E is Baire-like, [7], if given an increasing sequence of closed absolutely convex subsets of E covering E there is one of them which is a neighbourhood of the origin. A space E is quasi-Baire, [7], if it is barrelled and given an increasing sequence of subspaces of E covering E there is one of them which is dense.

1. Quasi-suprabarrelled spaces and the closed graph theorem

We say that a space E is *quasi-suprabarrelled* if given an increasing sequence of subspaces of E covering E there is one of them which is barrelled.

We have that suprabarrelled implies quasi-suprabarrelled and this last class of spaces is contained in the class of the barrelled ones. The following examples distinguish these classes.

EXAMPLE 1. A non-complete (LF)-space is not quasi-suprabarrelled. In fact, if $E(\tau)$ is quasi-suprabarrelled, from the sequence $\{E_n(\tau_n): n = 1, 2, \dots\}$ of Fréchet spaces which defines $E(\tau)$ we can extract a sequence $\{E_{n(p)}(\tau/E_{n(p)}): p = 1, 2, \dots\}$ of barrelled spaces which are of Fréchet by Pták's homomorphism theorem, since the identity $i: E_{n(p)}(\tau_{n(p)}) \to E_{n(p)}(\tau/E_{n(p)})$ is continuous. Now $E(\tau)$ is the strict inductive limit of the $E_{n(p)}(\tau/E_{n(p)})$ since $E(\tau)$ is barrelled, [7, Lemma 2.17]. We must conclude then that $E(\tau)$ is complete, [3, p. 225], a contradiction. We note that if besides $E(\tau)$ is metrizable, then $E(\tau)$ is Baire-like.

The L. Schwartz space $D(\Omega)$ is not quasi-suprabarrelled as we are going to show. We first note that every separated quotient of an (LF)-space is an (LF)-space or a Fréchet space ([2, p. 147, lines 5.6]; also, see [4, Theorem 9]). Actually, let $E(\tau) = \varinjlim_n E_n(\tau_n)$ be an (LF)-space and let H be a closed subspace of E. Every canonical mapping from $E_n(\tau_n)/H \cap E_n$ into $E(\tau)/H$ is continous. Therefore the identity i from $\varinjlim_n E_n(\tau)/H \cap E_n$ onto $E(\tau)/H$ is continuous. The conclusion follows now from Grothendieck's closed graph theorem. As $D(\Omega)$ has a noncomplete metrizable quotient F, [14], and every quotient of a quasi-suprabarrelled space is quasi-suprabarrelled as we shall see later, by the above F is not quasi-suprabarrelled and thus neither is $D(\Omega)$.

EXAMPLE 2. The topological direct sum $\phi\omega = \omega \oplus \omega \oplus \cdots$ is complete and barrelled. Now $\phi\omega$ is covered by the subspaces $R_n = \omega^n \times \phi_\omega$, where ϕ_ω denotes ϕ with the topology induced by ω. No F_n is barrelled, since ϕ_ω is not-barrelled, and so $\phi\omega$ is not quasi-suprabarrelled.

EXAMPLE 3. The space ϕ is quasi-suprabarrelled since every subspace is of countable codimension, ([11] and [8]). Now ϕ is covered by an increasing sequence of finite-dimensional subspaces, therefore ϕ is not quasi-Baire and so ϕ is not suprabarrelled. In fact, if d is a cardinal number and ϕ_d denotes a linear space of dimension d with the strongest locally convex topology, then every subspace of ϕ_d has the strongest locally convex topology and, therefore, is closed and barrelled. It follows that ϕ_d is quasi-suprabarrelled. If H is a countable codimensional subspace of ϕ_d, then $\phi_d = H \oplus \phi$. As ϕ is not suprabarrelled we conclude that ϕ_d is not suprabarrelled.

We give now a characterization of quasi-suprabarrelled spaces which are not suprabarrelled.

PROPOSITION 1. *Let E be a quasi-suprabarrelled space. E is not suprabarrelled if and only if there exists an increasing sequence of barrelled subspaces covering E having each of them a copy of ϕ with a topological supplement.*

PROOF. Obviously, E is suprabarrelled if and only if E is quasi-Baire. Thus Theorem 1 of [4] applies.

It is obvious that a space E is suprabarrelled if and only if it is quasi-suprabarrelled and quasi-Baire. It follows from [4] that if E does not contain a complemented copy of ϕ (in particular, if its completion is a Baire space (see [15]), then the notions of being quasi-suprabarrelled and suprabarrelled are equivalent for E.

Theorem 4 of [12] suggests the following result, where we change suprabarrelled by quasi-suprabarrelled, losing the localizatoin property given there. We suppose E is the convex hull of a family of quasi-suprabarrelled spaces, that $\{F_n(\tau_n): n = 1, 2, \ldots\}$ is an increasing sequence of Γ_r-spaces, [13], and $F := \bigcup\{F_n: n = 1, 2, \ldots\}$ has a locally convex topology coarser than the final topology defined by the $F_n(\tau_n), n = 1, 2, \ldots$.

THEOREM 1. *If f is a linear mapping from E into F with closed graph, then f is continuous.*

PROOF. We can suppose that E is quasi-suprabarrelled. Working with a subsequence if necessary we can also suppose that every space $f^{-1}(F_n)$ is barrelled. From f we consider the restrictions $f_n: f^{-1}(F_n) \to F_n(\tau_n)$ which clearly have closed graph in $f^{-1}(F_n) \times F_n(\tau_n)$. By the closed graph theorem given in [13] we have that the f_n are continuous and from the continuity of $f_n: f^{-1}(F_n) \to F$ it follows that f is continuous, since E is the inductive limit of the subspaces $f^{-1}(F_n)$, $n = 1, 2, \dots$.

2. Permanence properties of quasi-suprabarrelled spaces

The quotients and the countable-codimensional subspaces of quasi-suprabarrelled spaces are quasi-suprabarrelled. This follows immediately from the analogous permanence properties of barrelled spaces.

PROPOSITION 2. *Let E_1 and E_2 be two quasi-suprabarrelled spaces. If E_1 is metrizable, then $E_1 \times E_2$ is quasi-suprabarrelled.*

PROOF. If $E = E_1 \times E_2$ is not quasi-suprabarrelled there exists an increasing sequence $\{F_n : n = 1, 2, \dots\}$ of proper subspaces of E covering E, none of them barrelled. Let T_n be a barrel in F_n which is not a neighbourhood of the origin in F_n. If B_p denotes the closure of T_p and $G_p := \bigcap\{\langle B_n\rangle : n \geq p\}$, then $\{G_p : p = 1, 2, \dots\}$ is an increasing sequence of subspaces of E covering E, since F_p is contained in G_p for $p = 1, 2, \dots$. No G_p is barrelled, since $B_p \cap G_p$ is a barrel in G_p which is not a neighbourhood of the origin in G_p.

Since E_1 and E_2 are quasi-suprabarrelled there is a strictly increasing sequence $(p(r))_r$ of positive integers such that $G_{p(r)} \cap E$, and $G_{p(r)} \cap E_2$ are barrelled for $r = 1, 2, \dots$. By metrizability, E_1 is suprabarrelled and so there is a positive integer q such that $G_{p(q)} \cap E_1$ is dense in E_1.

If $y \in E_1$ there is a sequence $(y_n)_n$ is the barrelled space $G_{p(q)} \cap E_1$ which converges to y. This sequence is absorbed by the barrels $B_n \cap G_{p(q)} \cap E_1$ for each $n \geq p(q)$. Hence $y \in \langle B_n\rangle$ for each $n \geq p(q)$. Thus $y \in G_{p(q)}$ and so we have proved that E_1 is contained in $G_{p(q)}$. Now, from this inclusion it follows that $G_{p(q)} = E_1 \times (G_{p(q)} \cap E_2)$ is barrelled, a contradiction.

LEMMA 1. *Let $\{E_n : n = 1, 2, \dots\}$ be a sequence of closed absolutely convex sets in E. If $G_p := \bigcap\{\langle B_n\rangle : n \geq p\}$, $p = 1, 2, \dots$, and $(p(r))_r$ is a strinctly increasing sequence of positive integers with $\bigcup\{G_{p(r)} : r = 1, 2, \dots\} = E$, there is a positive integer q such that $G_{p(q)}$ contains $E(q, q+1, q+2, \dots)$.*

PROOF. If the property is not ture we find a sequence $(x_r)_r$ in E with $x_r \in E(r, r+1, \dots) - G_{p(r)}$. This sequence converges to zero and is contained in a complete subspace of E since its projection on E_n is contained in a finite-dimensional subspace of E_n, therefore by Tychonoff's Theorem, $A = \overline{\Gamma\{x_r : r = 1, 2, \dots\}}$ is a compact set and E_A is a Banach space which is covered by the $G_{p(r)}$, $r = 1, 2, \dots$.

Thus, there exists a $G_{p(q)} \cap E_A$ which is of second category in E_A. Therefore $B_n \cap E_A$ is a neighbourhood of the origin in E_A for each $n \geq p(q)$. So we have that A is contained in $\langle B_n \rangle$ for each $n \geq p(q)$. This implies that $A \subset G_{p(q)}$, contradicting the fact that $x_q \notin G_{p(q)}$.

PROPOSITION 3. *Let $\{E_n : n = 1, 2, \dots\}$ be a sequence of spaces such that $E(1, 2, \dots, n)$ is quasi-suprabarrelled for $n = 1, 2, \dots$. Then $E = \prod\{E_n : n = 1, 2, \dots\}$ is quasi-suprabarrelled.*

PROOF. If E is not quasi-suprabarrelled there exists an increasing sequence $\{F_n : n = 1, 2, \dots)$ of proper subspaces of E covering E such that no F_n is barrelled. If T_n is a barrel in F_n which is not a neighbourhood of the origin in F_n, B_p is the closure of T_p in E and $G_p := \bigcap\{\langle B_n \rangle : n \geq p\}$, we saw in the proof of Proposition 2 that $\{G_p : p = 1, 2, \dots\}$ is an increasing sequence of non-barrelled subspaces of E which covers E.

We determine a subsequence $\{G_{p(r)}, r = 1, 2, \dots\}$ with $p(r) < p(r+1)$ for each r such that $G_{p(r)} \cap E(1, 2, \dots, r)$ is barrelled. In fact, since $E_1 = E(1)$ is quasi-suprabarrelled, there exists a $p(1)$ such that $G_{p(1)} \cap E(1)$ is barrelled. With $p(1), p(2), \dots, p(s-1)$ determined, we have that $\{G_n, n = 1, 2, \dots\}$ is an increasing sequence of spaces covering the quasi-suprabarrelled space $E(1, 2, \dots, s)$, therefore there is a $G_{p(s)}$, $p(s) > p(s-1)$, such that $G_{p(s)} \cap E(1, 2, \dots, s)$ is barrelled.

Since $\bigcup\{G_{p(r)}, r = 1, 1, \dots\} = E$, by Lemma 1 there is a positive integer q such that $G_{p(q)} \supset E(q, q+1, \dots)$. Thus $G_{p(q)}$ is the topological direct sum of the subspaces $G_{p(q)} \cap E(1, 2, \dots, q)$ and $E(q+1, q+2, \dots)$. Therefore $G_{p(q)}$, is barrelled, a contradiction.

EXAMPLE 4. We give now an example of a non-complete quasi-suprabarrelled space which is not suprabarrelled. In fact, there exists a linear form u defined on ϕ^N which is not continuous. Setting $H := u^{-1}(0)$, H is a dense hyperplane of ϕ^N. As ϕ^n is isomorphic to ϕ for every $n \in N$, then it follows from the last result that ϕ^N is quasi-suprabarrelled. This implies that H is quasi-suprabarrelled. If H is suprabarrelled then its completion ϕ^N is suprabarrelled, [12], but ϕ^N fails to be suprabarrelled since it contains ϕ as a factor.

We need the following result given in [15, p. 20] by M. Valdivia.

LEMMA 2. *Let $\{E_i : i \in I\}$ be a family of spaces and let $E = \prod\{E_i : i \in I)$. If $\{B_n : n = 1, 2, \dots\}$ is a sequence of closed convex sets of E covering E such that $0 \in B_n$ for every positive integer n, there exists a finite subset J of I and a positive integer s such that $B_s \supset E(I - J)$.*

PROPOSITION 4. *Let $\{E_i : i \in I\}$ be a family of spaces such that $E(H)$ is quasi-suprabarrelled for every countable set $H \subset I$. Then $E := \prod\{E_i : i \in I\}$ is quasi-suprabarrelled.*

PROOF. If E is not quasi-suprabarrelled there exists an increasing sequence $\{F_n : n = 1, 2, \dots\}$ of proper non-barrelled subspaces covering E. Let T_n be a barrel in F_n which is not a neighbourhood of the origin in F_n and let B_n be the closure of T_n in E. Since $\{mB_n : m, n = 1, 2, \dots\}$ covers E it follows from Lemma 2 that there is a pair $(m(1), n(1))$ of positive integers and a finite subset H_1 of I such that $m(1)B_{n(1)} \supset E(I - H_1)$. Thus $\langle B_{n(1)} \rangle \supset E(I - H_1)$. Now $\{mB_n : n \rangle n(1), m = 1, 2, \dots\}$ covers E and therefore there is a $n(2) > n(1)$ such that $\langle B_{n(2)} \rangle \supset E(I - H_2)$. By recurrence we obtain a sequence $n(1) < n(2) < \cdots < n(p) < \cdots$ of positive integers and a sequence $\{H_p : p = 1, 2, \dots\}$ of finite subsets of I such that $\langle B_{n(p)} \rangle \supset E(I - H_p)$ for $p = 1, 2, \dots$.

Let $H := \bigcup\{H_p : p = 1, 2, \dots\}$ and $G_p := \bigcap\{\langle B_{n(r)} : r \geq p\}$. Then $\{G_p : p = 1, 2, \dots\}$ is an increasing sequence of subspaces of E covering E with $G_p \supset E(I - H)$ for $p = 1, 2, \dots$. Since H is countable we have that $E(H)$ is quasi-suprabarrelled, therefore there is a positive integer q such that $G_q \cap E(H)$ is barrelled. Now G_q is the topological direct sum of $G_q \cap E(H)$ and $E(I - H)$, so G_q is barrelled, a contradiction.

THEOREM 2. *Let $\{E_i : i \in I\}$ be a family of spaces such that for every finite subset H of $IE(H)$ is quasi-suprabarrelled. Then $E = \prod\{E_i : i \in I\}$ is quasi-suprabarrelled.*

PROOF. It is a direct consequence of Propositions 3 and 4.

EXAMPLE 5. From Example 3 and from the last theorem it follows that $\prod\{\phi_d : d \in \Omega\}$, where Ω is an arbitrary family of cardinals numbers, is a quasi-suprabarrelled space which is not suprabarrelled. In particular, if I is an index set, ϕ^I is quasi-suprabarrelled and it is not suprabarrelled.

REMARK 1 A space E satisfies condition (G), [5], if given a sequence of subspaces of E covering E there is one of them which is barrelled. A space E is said to be totally barrelled, [16], if given a sequence of subspaces of E covering E there is one of them which is Baire-like (see Proposition 1 of [5]). In [16] it is proved that $l_0^\infty \otimes_\pi l^2$ is a suprabarrelled space which is not totally barrelled. Now, it is known that if E satisfies condition (G) and E does not contain a copy of ϕ then E is totally barrelled. As $l_0^\infty \otimes_\pi l^2$ is metrizable, this space does not contain a copy of ϕ. We must conclude then that $l_0^\infty \otimes_\pi l^2$ is a quasi-suprabarrelled space which does not satisfy condition (G).

3. Proof of the necessity of a condition in a theorem of S. A. Saxon and P. P. Narayanaswami

We consider in the product space ω^N the sequence $\{E_n, n = 1, 2, \dots\}$ of non-barrelled subspaces of the form $E_n = \omega \times \cdot\cdot^{(n)}\cdot \times \omega \times \phi_\omega \times \phi_\omega \times \cdots$, where ϕ_ω denotes ϕ with the topology induced by ω, and the subspaces $E := \bigcup\{E_n : n = 1, 2, \dots\}$ and $F := \phi_\omega^N$. Clearly the space E is dense in ω^N and it is not quasi-suprabarrelled since none of the spaces E_n, $n = 1, 2, \dots$ is barrelled.

Let $E(\delta)$ be the barrelled space which is the inductive limit of the quasi-suprabarrelled spaces $E_n(\delta_n) = \omega \times \cdot\cdot^{(n)}\cdot \times \omega \times \phi \times \phi \times \cdots$. From the next result it follows that E is barrelled.

PROPOSITION 5. *E and $E(\delta)$ are isomorphic.*

PROOF. If $u \in E(\delta)'$ we have that $u/E_n \in E_n(\delta_n)' = \phi \oplus \cdot\cdot^{(n)}\cdot \oplus \phi \oplus \omega \oplus \omega \oplus \cdots$. Analogously, $u/E_{n+1} \in \phi \oplus \cdot\cdot^{(n+1)}\cdot \oplus \phi \oplus \omega \oplus \omega \oplus \cdots$. Since $(u/E_{n+1})/E_n = u/E_n$, it follows that $u/E_{n+1} \in \phi \oplus \cdot\cdot^{(n)}\cdot \oplus \phi \oplus \omega \oplus \omega \oplus \cdots$. Proceeding by recurrence we obtain that u/E_n is an element of $\phi \oplus \phi \oplus \phi \oplus \cdots$ independent of n. This element defines a linear form on ω^N and, therefore, on E. Hence, $u \in E'$. As $E' \subset E(\delta)'$, we conclude that E and $E(\delta)$ are isomorphic since they are two Mackey spaces with the same dual.

PROPOSITION 6. *If τ is a locally convex topology strictly stronger than that of E, then $E(\tau)$ is not an (LF)-space.*

PROOF. If $E(\tau)$ is an (LF)-space, then the identity $i: E \to E(\tau)$ is continuous as a consequence of the Theorem 1. Thus τ must coincide with the topology of E.

The next result can be viewed either as a special case of Lemma 2, due to M. Valdiva, [15, page 20] or as a special case of A. Todd and S. Saxon's Lemma 4.5 of [10]. We provide a concise proof.

LEMMA 3. *If the product $L = \prod\{L_n : n = 1, 2, \dots\}$ is covered by an increasing sequence $\{F_n : r = 1, 2, \dots\}$ of closed subspaces, then there is a positive integer m such that $F_m \supset \{0\} \times \cdots \times \{0\} \times L_{m+1} \times L_{m+2} \times \cdots$.*

PROOF. If the property is not true, there is some $x_n \in \{0\} \times \cdots \times \{0\} \times L_{n+1} \times L_{n+2} \times \cdots - F_n$ for every $n \in N$. Now the compact $A := \overline{\Gamma\{x_n : n = 1, 2, \dots\}}$ generates a Banach space E_A which is covered by the sequence $\{F_n \cap E_A : n = 1, 2, \dots\}$. One of these subspaces, $F_P \cap E_A$, is of second category in E_A and, therefore, coincides with E_A since it is closed in E_A. It follows that $A \subset F_P$. But $x_P \notin F_P$. Contradiction.

PROPOSITION 7. *E is not an (LF)-space.*

PROOF. Suppose that E is the inductive limit of the increasing sequence $\{F_n(\rho_n): n = 1, 2, \dots\}$ of Fréchet spaces covering E.

Working with a subsequence if necessary we can suppose, since $F(\delta) = \phi^N$ is quasi-suprabarrelled (see Example 4), that $F \cap F_n$ is a barrelled subspace of $F(\delta)$, $n = 1, 2, \dots$, which we are going to show is closed. Now the injections i_1 and i_2 from $F \cap F_n$ and $F_n(\rho_n)$ into ω^N respectively, are continuous. So the injection i from $F \cap F_n$ into $F_n(\rho_n)$ has closed graph, hence it is continuous. If $\{z_i: i \in I, \geq\}$ is a net in $F \cap F_n$ which converges to z in $F(\delta)$ we have from the continuity of i that $\{z_i: i \in I, \geq\}$ converges to v in $F_n(\rho_n)$ and from the continuity of i_1 and i_2 that $z = v$. Therefore $\{F \cap F_n: n = 1, 2, \dots\}$ is an increasing sequence of closed subspaces covering ϕ^N. The previous lemma implies that there is a positive integer m such that $F \cap F_m \supset \{0\} \times \cdot \overset{m}{\cdot} \cdot \times \{0\} \times \phi \times \phi \times \cdots$.

The Fréchet space $F_m(\rho_m)$ is covered by $\{E_n \cap F_m: n \geq m\}$, hence there exists an $s \geq m$ such that $E_s \cap F_m$ is a Baire subspace dense in $F_m(\rho_m)$. The injections j_1 and j_2 from $E_s \cap F_m$ and $E_s(\delta_s)$ into ω^N respectively, are continuous. So the injection j from $E_s \cap F_m$ into $E_s(\delta_s)$ is continuous since j has a closed graph, $E_s \cap F_m$ is a Baire space and $E_s(\delta_s)$ is a webbed space. We prove just as before that $E_s \cap F_m$ is a closed subspace of $F_m(\rho_m)$. And from density we conclude that $E_s \supset F_m$.

Setting $H_s := \{0\} \times \cdot \overset{(s)}{\cdot} \cdot \times \{0\} \times \phi \times \phi \times \cdots$, we have that $H_s \subset F_m \subset E_s$. Now, endowing H_s with the topology induced by $E_s(\delta_s)$ it follows that the injections k and l from H_s into $F_m(\rho_m)$ and from $F_m(\rho_m)$ into $E_s(\delta_s)$ respectively, are continuous since they have closed graph, H_s is barrelled, $F_m(\rho_m)$ is a Fréchet space and $E_s(\delta_s)$ is a webbed space. We must conclude that the topology of the non-metrizable space H_s coincides with the topology induced by the Fréchet space $F_m(\rho_m)$. Contradiction.

In brief, the space E has the following properties: (1) E is dense in ω. (2) E is Baire-like. (3) E is not quasi-suprabarrelled. (4) $E(\tau)$ is not an (LF)-space under any locally convex topology τ stronger that the topology induced by ω. (5) E is an inductive limit of a sequence of quasi-suprabarrelled spaces. (6) E is a webbed space.

REMARK 2. If s is the subspace of ω of the rapidly decreasing sequences, s is a Fréchet space under the topology defined by the sequence of seminorms of the form $\|x\|_P = \sum\{n^P|x_n|: n = 1, 2, \dots\}$, $p = 0, 1, 2, \dots$. In [17] it is proved that if $G_n := \omega \times \cdot \overset{(n)}{\cdot} \cdot \times \omega \times s \times s \times \cdots$ and $G := \{G_n: n = 1, 2, \dots\}$, then G is a proper dense subspace of ω which is an (LF)-space. Our space E is a subspace of this space G. Having in mind this fact it also follows from the quoted theorem of S. A. Saxon and P. P. Narayanaswami that E is not suprabarrelled.

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Departamento de Matemáticas (ETSIA)
Universidad Politécnica
Apartado 22012
46022-Valencia
Spain