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NONLINEAR TIME SERIES ANALYSIS USING ORDINAL NETWORKS WITH SELECT APPLICATIONS IN BIOMEDICAL SIGNAL PROCESSING

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Over the past ten years, there have emerged numerous network-based methods for the analysis of nonlinear time series which provide novel and alternative means of extracting useful information from data measured from complex systems. The fundamental defining principle of such methods is that time series data are mapped or transformed to construct a network model, the structural properties of which can then be analysed using measures from the field of network science to infer dynamical properties of the system from which the time series has been observed. The underlying assumption is, therefore, that the network topology is consistently and measurably dependent on the time series dynamics. The subject matter of this thesis is the proposal, investigation and application of a new network-based method of nonlinear time series analysis which we term ordinal network analysis.

We define the procedure for constructing ordinal networks from univariate time series data in two stages. The data are first mapped to a sequence of discrete symbolic states called ordinal symbols by partitioning a delay embedding of the time series based on the amplitude rank of the elements within each embedded state vector. The finite symbol space is then bijectively mapped to a set of nodes in a network and directed edges are assigned between nodes based on temporal succession of states in the symbolic sequence. The ordinal network can therefore be interpreted as a Markov chain stochastic approximation of the time series dynamics.

The primary aim of this research is to investigate and establish the relationship between measures of ordinal network topology and time series dynamics for discrete-time sampled data from archetypal continuous autonomous chaotic systems. This

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aspect of the thesis is undertaken with respect to a range of simple network statistics and two newly proposed measures to characterise local transitional complexity and mixing rate respectively. Comparative numerical investigations with toy data and experimental chaotic circuit data show that the network measures reliably track the change in estimates of the largest Lyapunov exponent over a range of the bifurcation parameter, even in the presence of low levels of observational noise, demonstrating that network topology is measurably dependent on dynamics.

To test the potential of ordinal network analysis in practice, we apply our method in three separate investigations of biomedical time series. Firstly, we implement a multiscale framework for ordinal network analysis for discrimination between short time electrocardiogram recordings characterised by normal sinus rhythm, ventricular tachycardia and ventricular fibrillation respectively. Secondly, we use the same multiscale framework for the investigation of age-related effects in cardiac dynamics using interbeat interval time series from long time electrocardiograms. Thirdly, we apply the ordinal network method in a sliding window analysis of multivariate electroencephalogram time series to extract linearly separable feature vectors for epileptic seizure onset classification using binary linear support vector machines.

In addition, we investigate the ordinal network's capacity to generate new time series with similar dynamical properties to the data used to construct the model. To do this we take constrained random walks on the network to regenerate new symbolic dynamics and use a data reassignment procedure to produce surrogate time series. We then compute invariant measures and recurrence properties to compare the original time series with the regenerated surrogates. Furthermore, we test the out-of-sample predictive properties of ordinal networks for low-dimensional chaotic time series.

It has been established that the count of ordinal symbols which do not occur in a time series, called forbidden patterns, is an effective measure for the detection of determinism in noisy data. A very recent study has shown that this measure is also partially robust against the effects of irregular sampling. As a secondary investigation in this thesis, we extend said research with an emphasis on exploring the parameter space for the embedding dimension and find that the measure is more robust to under-sampling and irregular sampling than previously reported. Using numerically generated toy data from discretely sampled continuous systems, we investigate the reliability of the relative proportion of ordinal patterns in periodic and chaotic time series for various degrees of under-sampling, random depletion of data, and timing jitter.

The papers [1–5] contain results arising from the thesis. In particular, [5] gives an overview of key concepts and results.

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