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The Lidov-Kozai resonance at different scales

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Abstract. The Lidov-Kozai (LK) resonance is one of the most widely discussed topics since the discovery of exoplanets in eccentric orbits. It constitutes a secular protection mechanism for systems with high mutual inclinations, although large variations in eccentricity and inclination are observed. This review aims to illustrate how the LK resonance influences the dynamics of the three-body problem at different scales, namely i) for two-planet extrasolar systems where the orbital variations occur in a coherent way such that the system remains stable, ii) for inclined planets in protoplanetary discs where the LK cycles are produced by the gravitational force exerted by the disc on the planet, iii) for migrating planets in binary star systems, whose dynamical evolution is strongly affected by the LK resonance even without experiencing a resonance capture, and iv) for triple-star systems for which the migration through LK cycles combined with tidal friction is a possible explanation for the short-period pile-up observed in the distribution of multiple stars.

Keywords. Celestial mechanics, planetary systems, planetary systems: formation, planetary systems: protoplanetary discs, planets and satellites: dynamical evolution and stability, binaries: close, stars: kinematics and dynamics, stars: formation

1. Introduction

When two bodies gravitationally interact, the problem is integrable and its solution, which consists of two fixed elliptic orbits, is known since the works of Kepler in the 17th century. However, the perturbations caused by additional bodies produce changes in the shape and orientation of the Keplerian orbits on secular timescales. In particular, the planetary perturbations induce the precession of the pericenter argument of the orbits. In the 18th century, Laplace and Lagrange built a linear approximation for the secular motion of the planets of the Solar system, showing that the semi-major axes present no secular variation while the eccentricities and inclinations of the orbits suffer from limited variations inducing no possible orbit crossing or planet collision. This is no longer true when considering planetary systems with large eccentricities and / or mutual inclinations.

Lidov (1962) and Kozai (1962) investigated the secular motion of small bodies under the effect of a pertuber on a circular orbit. Lidov (1962) focused on the orbital motion of artificial Earth satellites perturbed by the Moon, while Kozai (1962) studied the influence of Jupiter on the motion of asteroids. They showed that, for inclined orbits of the small body (considered here as a massless particle), the dynamics can be characterized by the libration of the pericenter argument ω of the small body around 90° or 270°, as well as large periodic coupled variations of its eccentricity e and inclination i. These variations are bounded by the conservation of the *Kozai constant*, $H = \sqrt{1-e^2}\cos i$ (i.e., the adimensional z-component of the small body angular momentum). As a result, the orbit of the small body will be more eccentric for smaller inclinations and less eccentric for larger inclinations. This particuliar dynamics is known as the *LK resonance* or more

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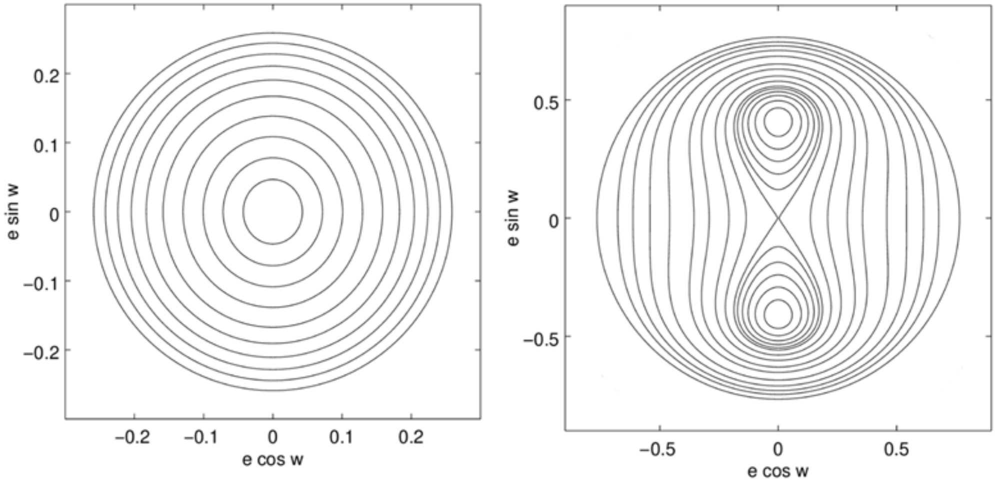

Figure 1. Level curves of the Hamiltonian $\mathcal{H}_{\text{QUAD}}$ in the $(e\cos\omega, e\sin\omega)$ phase space, for two values of the Kozai constant: $H = 0.97$ (left panel) and $H = 0.64$ (right panel).

recently *ZLK resonance*, as it turns out that the mechanism was originally discussed by von Zeipel (1910) for the motion of (long-period) comets (see e.g., Ito and Ohtsuka 2019, for a comparison between the works of von Zeipel (1910), Lidov (1962), and Kozai (1962)).

More precisely, Kozai (1962) showed that the Hamiltonian formulation of the three-body problem in the test particle approach can be reduced to two degrees of freedom after short-period averaging (i.e., averaging with respect to the mean anomalies to obtain the secular evolution of the bodies whose semi-major axis is then considered as constant) and node reduction (i.e., adoption of the invariant plane as reference plane, see e.g., Poincaré 1892). Assuming that the orbit of the perturber is circular, the problem becomes integrable. The one-degree of freedom Hamiltonian formulation at quadrupole level of the expansion in Legendre polynomials writes (e.g., Kinoshita and Nakai 1999; Naoz et al. 2013)

$$\mathcal{H}_{\text{QUAD}} = -\frac{Gm_0m_2}{16a_2(1-e_2^2)^{3/2}}\left(\frac{a}{a_2}\right)^2\left((2+3e^2)(3\cos^2 i - 1) + 15e^2\sin^2 i\cos(2\omega)\right), \quad (1.1)$$

where the index 2 refers to the perturber and with a the semi-major axis of the particle, m_0 the mass of the central star, and m_2 the mass of the perturber.

The dynamics can be represented, for a given value of the Kozai constant H (i.e., bounded values of the eccentricity and inclination), in the phase space $(e\cos\omega, e\sin\omega)$, where the level curves of the Hamiltonian correspond to the trajectories of the test particle (e.g., Thomas and Morbidelli 1996). Such a representation is shown in Fig. 1. For large values of H, or equivalently for small inclination values of the massless body (for instance, for $i = 15°$ at $e = 0$, left panel), the phase space shows a stable equilibrium at zero eccentricity of the massless body. The eccentricity (and thus the inclination) of the test particle at low inclination suffers from limited variations during the precession of the orbit. However, for higher inclinations (for instance, for $i = 50°$ at $e = 0$, right panel), the equilibrium becomes unstable and a separatrix divides the phase space in three regions: two libration islands around $\omega = 90°$ and $\omega = 270°$ and a region characterized by the circulation of ω. These two equilibria are referred to as *LK equilibria*. The bifurcation of the central equilibrium induces large eccentricity variations for the massless body initially in a circular orbit since its real motion (short periods included) will remain close

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to the separatrix of the reduced problem. From the conservation of the Kozai constant and the integral of energy, one can retrieve the well-known value of $\cos i = \pm\sqrt{3/5}$, i.e., $i = 39.23°$ and $i = 140.77°$, for the critical inclination corresponding to the bifurcation of the stable equilibrium in the hierarchical case (i.e., when the perturber is on a much wider orbit). The critical inclination decreases with increasing semi-major axis ratio, as shown by Kozai (1962).

Since the original works of von Zeipel (1910), Lidov (1962), and Kozai (1962), the LK secular resonance has been investigated in hundreds of works and for many different configurations of planetary and stellar systems. Regarding the Solar system, we may cite for instance studies on the main belt asteroids (e.g., Kozai 1985; Michel and Thomas 1996), gas giant satellites and Jovian irregular moons (e.g., Carruba et al. 2002; Nesvorný et al. 2003), and trans-neptunian objects and comets (e.g., Quinn et al. 1990; Bailey 1992; Thomas and Morbidelli 1996; Gallardo et al. 2012), all focusing on small body dynamics. As we will show in the following, the discovery, at the end of the 20th century, of extrasolar planets on eccentric and possibly inclined orbits brought a new field of applications for the LK resonance. Extensions of the previous works were achieved in order to take into account eccentric orbits for the perturber as well as different mass ratios among the bodies, in the context of planetary systems (e.g., Michtchenko et al. 2006; Libert and Henrard 2007; Migaszewski and Goździewski 2009) and multiple-star systems (e.g., Innanen et al. 1997; Naoz et al. 2013). In particular, the formation through the LK resonance of hot Jupiters with orbital periods of a few days only has been deeply investigated (e.g., Fabrycky and Tremaine 2007; Wu et al. 2007; Naoz et al. 2011).

In this work I aim to show how the LK resonance influences the dynamics at four different scales of the three-body problem: two-planet extrasolar systems in Section 2, planets perturbed by their protoplanetary disc in Section 3, circumprimary planets in binary stars in Section 4, and finally triple-star systems in Section 5. This review has not the ambition to draw an exhaustive picture of the latest results in the four different fields discussed here, but rather it aims to take a closer look at how the LK dynamics can be easily transposed among different fields and provide valuable contribution in all of them.

2. Two-planet systems

We first consider the planetary three-body problem. Many two-planet extrasolar systems were detected via the radial velocity (RV) method in the last 25 years. This detection method measures only the line of sight component of the star velocity, and thus gives no information on the orbital inclinations of the planets. The planetary masses are also unclear since they have to be scaled by the sinus of the unknown orbital inclination. It means that three-dimensional (3D) configurations for the detected planetary systems cannot be uncovered by the RV technique alone. Recent years have seen the emergence of a number of observational evidence on the existence of 3D planetary systems. A well-known example of a mutually inclined system is υ And for which the mutual inclination between the orbital planes of planets c and d was estimated to 30°, by combining different detection methods (Deitrick et al. 2015).

Because of the large eccentricities (and possibly large inclinations) of many detected exoplanets, the classical analytical theories, such as the Laplace-Lagrange linear perturbation theory, are unable to describe correctly the motion of these planets. As a result, several authors carried out analytical works on the secular evolution of the 3D planetary three-body problem (e.g., Michtchenko et al. 2006; Libert and Henrard 2007; Migaszewski and Goździewski 2009; Naoz et al. 2013). A development commonly used is the Hamiltonian expansion in Legendre polynomials to high order in the semi-major axis ratio (e.g., Kozai 1962; Ford et al. 2000; Lee and Peale 2003; Migaszewski and

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Goździewski 2009; Naoz et al. 2013), which is valid for hierarchical systems. Another well-known analytical approach consists in the generalization of the Laplace-Lagrange Hamiltonian expansion to high order in the eccentricities and inclinations, as given here (e.g., Libert and Henrard 2007)

$$\mathcal{H} = -\frac{Gm_0m_1}{2a_1} - \frac{Gm_0m_2}{2a_2} - \frac{Gm_1m_2}{a_2} \sum_{k,i_l,j_l,l\in\underline{4}} A_{i_l}^{k,j_l} \sqrt{\frac{2P_1}{L_1}}^{|j_1|+2i_1} \sqrt{\frac{2P_2}{L_2}}^{|j_2|+2i_2} \sqrt{\frac{2Q_1}{L_1}}^{|j_3|+2i_3} \sqrt{\frac{2Q_2}{L_2}}^{|j_4|+2i_4} \cos\Phi, \tag{2.2}$$

with

$$\Phi = [(k+j_1+j_3)\lambda_1 - (k+j_2+j_4)\lambda_2 + j_1p_1 - j_2p_2 + j_3q_1 - j_4q_2] \tag{2.3}$$

and where subscript 1 refers to the planet closer to the star (inner planet) and 2 to the outer one. The expansion is expressed in the classical modified Delaunay's elements:

$$\begin{aligned} &\lambda_i = \text{mean longitude of } m_i, && L_i = m_i\sqrt{Gm_0a_i} \\ &p_i = \text{- the longitude of the pericenter of } m_i, && P_i = L_i\left[1-\sqrt{1-e_i{}^2}\right] \\ &q_i = \text{- the longitude of the node of } m_i, && Q_i = L_i\sqrt{1-e_i^2}\,[1-\cos i_i]\,. \end{aligned} \tag{2.4}$$

The coefficients $A_{i_l}^{k,j_l}$ depend only on the ratio a_1/a_2 of the semi-major axes.

As we are interested in the long-term evolution of extrasolar systems, we can average the expansion over the short period terms by simply removing from the Hamiltonian the terms depending on the mean anomalies (first-order averaging), if the system is not close to a mean-motion resonance (a similar analytical development for planetary systems in mean-motion resonance can be found in Sansottera and Libert 2019). Moreover, by adopting the invariant Laplace plane which is orthogonal to the total angular momentum vector (i.e., elimination of the nodes, see Jacobi 1842), the Hamiltonian can be reduced to two degrees of freedom, namely $\mathcal{H}(e_1, e_2, \omega_1, \omega_2)$. For a fixed value of the total angular momentum

$$C = (L_1 - P_1)\cos i_1 + (L_2 - P_2)\cos i_2 \tag{2.5}$$

or equivalently for a fixed value of the angular momentum deficit (Laskar 1997)

$$AMD = L_1 + L_2 - C, \tag{2.6}$$

one can easily determine the values of the inclinations as functions of the eccentricities. It was shown that the averaged expansion limited to order 12 in the eccentricities and inclinations is accurate enough to describe precisely the secular evolution of extrasolar systems with moderate to high eccentricities and inclinations (Libert and Henrard 2007).

To visualize the dynamics, we can define, for a given value of the AMD (i.e., bounded values of the eccentricities and inclinations), a *representative plane* $(e_1 \sin\omega_1, e_2 \sin\omega_2)$ where both arguments of the pericenters are fixed to $\pm 90°$. This plane is neither a phase portrait nor a surface of section, since the problem is four dimensional. However, nearly all the orbits will cross the representative plane at several points of intersection on the same energy curve. The maximal value of the mutual inclination between the two orbital planes, $i_{\mathrm{mut}} = i_1 + i_2$, is reached at the origin of the representative plane and is given by

$$\max i_{\mathrm{mut}} = \arccos\left(\frac{C^2 - L_1^2 - L_2^2}{2L_1L_2}\right). \tag{2.7}$$

Representative planes are shown in Fig. 2 for different AMD values. Depending on the value of the AMD, one or three equilibria are visible: the central equilibrium at $e_1 = e_2 = 0$ which is stable for small mutual inclination values (for instance, for $\max i_{\mathrm{mut}} = 23°$, left panel) and, for higher mutual inclination values (for instance, for $\max i_{\mathrm{mut}} = 53°$, middle

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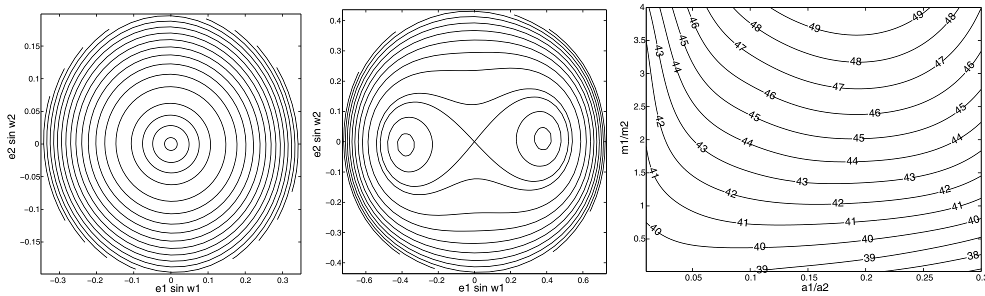

Figure 2. Level curves of the Hamiltonian $\mathcal{H}$ in the $(e_1 \sin\omega_1, e_2 \sin\omega_2)$ representative plane with both arguments of pericenters fixed to $\pm 90°$, for two *AMD* values corresponding to a maximal mutual inclination of 23° (left panel) and 53° (middle panel). Planetary parameters are fixed to $a_1/a_2 = 0.1$ and $m_1/m_2 = 1$. In the right panel, critical mutual inclinations as a function of the semi-major axis ratio and mass ratio. Adapted from Libert and Henrard (2007).

panel), the stable LK equilibria generated by the bifurcation of the central equilibrium. The critical value of the maximal mutual inclination corresponding to the change in the stability of the equilibrium at the origin depends on the planetary mass ratio and semi-major axis ratio, as shown in the right panel of Fig. 2. As expected, when the two ratios are small, namely $a_1/a_2 = 0.01$ and $m_1/m_2 = 0.01$, we retrieve the critical mutual inclination of 39.23° found by Kozai (1962).

From Fig. 2, we deduce that non-coplanar two-planet systems can be long-term stable, either at low mutual inclination between the orbital planes, whatever the planetary eccentricities (in this case the eccentricities and inclinations will stay close to their initial values), or at high mutual inclination for elliptic orbits only, since planets on quasi-circular orbits will feel some instability in the neighborhood of the unstable equilibrium. In the latter case, the LK equilibria provide stability islands for highly mutually inclined systems. In the invariant Laplace plane reference frame, LK resonant systems are characterized by large coupled variations of the eccentricity and the inclination of the inner planet as well as the libration of the argument of the pericenter of the same planet around $\pm 90°$ (e.g., Libert and Tsiganis 2009).

Volpi et al. (2019) showed that many RV-detected extrasolar systems have orbital parameters compatible with a LK resonant state when varying the (unknown) orbital inclinations with respect to the plane of the sky, denoted here I_1 and I_2, and the mutual inclination $i_{\rm mut}$. For simplicity, although the inclinations I_1 and I_2 could differ, the authors fixed them to the same value I and therefore used the same scaling factor $\sin I$ for both planetary masses. This scaling factor has an impact on the long-term evolution of the planetary system, since the dynamics of the three-body problem depends on the individual planetary masses, and not only on their mass ratio (see Eq. (2.2)). The extent of the LK resonant region is shown in the left panel of Fig. 3 for the HD11506 system considered with different I and $i_{\rm mut}$ values (dark blue region characterized by the libration of ω_1). The LK resonance is essential to ensure long-term regular evolutions for the highly mutually inclined systems, as shown in the right panel of Fig. 3, where chaotic regions are unveiled by the MEGNO chaos indicator (Cincotta et al. 2003). For moderate to high values of the eccentricities, such as the ones reported by the RV detections, significant chaos (yellow) is generally observed around the islands of the LK resonance (purple), which are the only regular regions in the case of a mutual inclination higher than 40°.

For exoplanets which are very close to the star with orbital periods of a few days only, additional effects such as tides and general relativity can greatly affect the long-term evolution of the system. Several authors found that the precession induced by the tides

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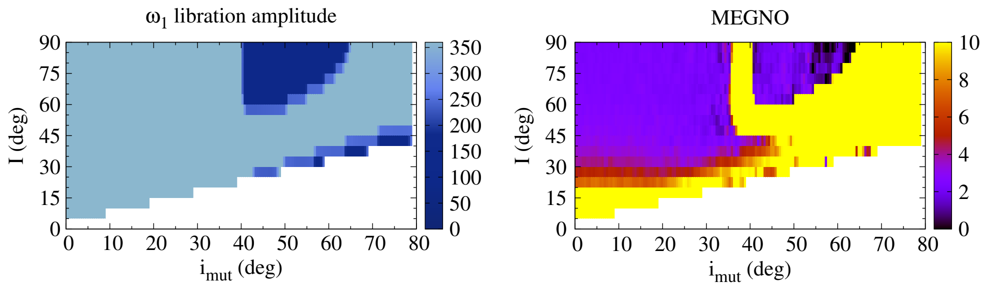

Figure 3. Long-term evolution of HD11506 (parameters from Tuomi and Kotiranta (2009)). Left panel: Libration amplitude of the argument of the pericenter ω_1. Right panel: Values of the (mean) MEGNO chaos indicator. The white region corresponds to mutual inclination values incompatible with a given value of I in the relation $\cos i_{\rm mut} = \cos^2 I + \sin^2 I \cos(\Delta\Omega)$ (i.e., $i_{\rm mut} \geq 2I$). Adapted from Volpi et al. (2019).

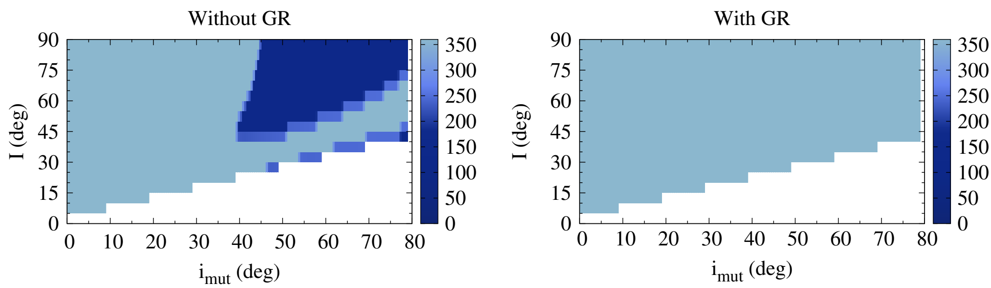

Figure 4. Libration amplitude of the pericenter argument ω_1 for GJ649 (parameters from Wittenmyer et al. (2013)), without (left) and when (right) considering relativistic corrections.

is generally negligible with respect to the one caused by the relativistic corrections (e.g., Migaszewski and Goździewski 2009; Veras and Ford 2010; Sansottera et al. 2014; Naoz 2016). The general relativity causes an advance of the pericenter which can be damaging for mutually inclined systems given that the LK resonance acts on the argument of the pericenter of the inner planet. More precisely, the secular perturbation caused by the general relativity on the pericenter argument of the inner planet is

$$\mathcal{H}_{\rm GR} = \frac{-3(Gm_0)^4 m_1^5}{c^2 L_1^3 G_1} \tag{2.8}$$

with c the speed of light (e.g., Migaszewski and Goździewski 2009). This corresponds to the following relativistic advance of the inner pericenter

$$\dot{\omega}_{1,\rm GR} = \frac{3(Gm_0)^{3/2}}{c^2 a^{5/2}(1-e_1^2)}. \tag{2.9}$$

Volpi and Libert (in preparation) investigated the possibility for RV-detected exoplanetary systems with close-in planets to be in a 3D configuration and found that, for the majority of the systems they examined, the LK resonance region disappears when the relativistic corrections are considered. An example is shown in Fig. 4 for GJ649 system.

3. Planets in protoplanetary discs

Besides the existence of non-coplanar extrasolar systems, the discovery of giant planets very close to their parent star (hot Jupiters) and the strong spin-orbit misalignment (i.e., high angle between the sky projection of the stellar spin axis and the orbit normal) of a significant fraction of them (Albrecht et al. 2012) are several characteristics that appear to

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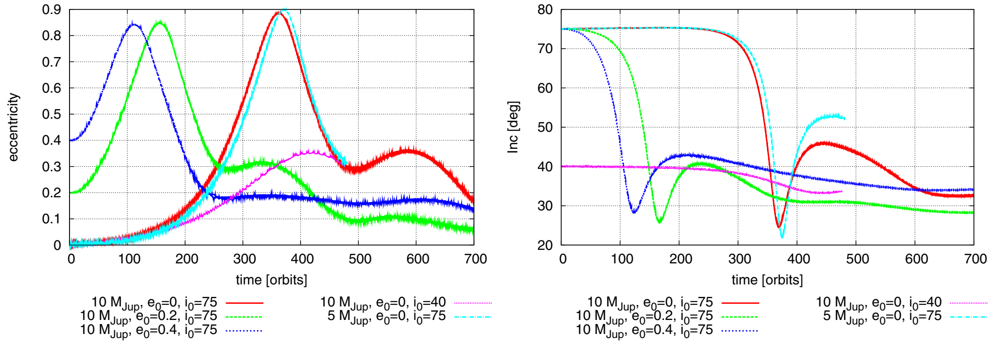

Figure 5. LK cycles of a planet evolving in the protoplanetary disc during the gas phase. The mass of the disc is $0.01 M_\odot$. The curves correspond to different planetary mass, initial eccentricity and initial inclination values. Reproduced from Bitsch et al. (2013).

be at odds with the formation of the Solar system. According to the commonly accepted core accretion scenario, the planets form in a protoplanetary disc supposedly aligned with the stellar spin. Thus, the planetary orbits should lie in the disc, and therefore in the stellar equatorial plane. Many scenarios were advanced to explain the unexpected spin-orbit misalignments, as well as the formation of highly mutually inclined systems. In this respect, the disc phase is especially important in the formation process and the effect of the gas disc on inclined (giant) planets was deeply explored in the previous decade. In particular, it was discovered that planet-disc interactions can generate LK cycles for the planet evolving in the gas disc (e.g., Terquem and Ajmia 2010; Xiang-Gruess and Papaloizou 2013; Teyssandier et al. 2013; Bitsch et al. 2013).

An example of LK evolution during the disc phase is shown in Fig. 5. It is issued from 3D hydrodynamical simulations of protoplanetary discs with embedded high-mass single planets (1 to 10 $M_{\rm Jup}$) achieved by Bitsch et al. (2013), in which they let the planet evolve freely under the influence of the disc forces. For a low initial inclination of the planet, the planetary eccentricity and inclination are both generally damped by the disk (i.e., the dynamical friction that the disc exerts on the planet when it crosses the disc), except for very massive planets (above $\sim 5 M_{\rm Jup}$) where eccentricity can increase at low inclination. For initial planetary inclinations larger than a critical value, the gravitational force exerted by the disc on the planet leads to LK cycles during which the orbital eccentricity reaches high values (~ 0.9 for an initial planetary inclination of 75°, as shown in Fig. 5), in antiphase with the inclination. Note that the critical inclination value depends on the mass ratio between the planet and the disc, and it can be as small as $\sim 20°$ in specific cases according to Teyssandier et al. (2013).

In Fig. 5, we observe that the LK cycles are damped through time by the dynamical friction exerted by the disc. If the dissipation of the disc is not rapid, the LK cycles only delay the alignment of the planetary orbit with the disc and its circularization caused by the damping forces of the disc on the planet. Otherwise, the LK cycles make it hard to predict the exact evolution of the planet and its orbital parameters at the dispersal of the disc.

4. Planets in binary star systems

Gravitational perturbations from a planetary companion and a protoplanetary disc were considered in Sections 2 and 3, respectively. We now focus on the gravitational influence of a binary companion. It is believed that half of the Sun-like stars are part of multiple-star systems (Duquennoy and Mayor 1991; Raghavan et al. 2010). More than 100

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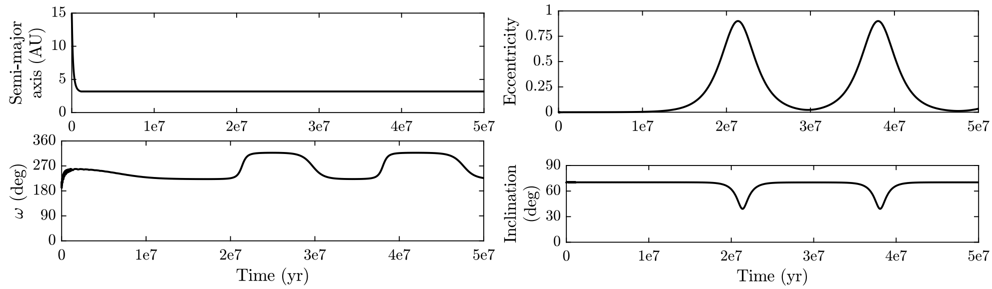

Figure 6. Evolution of a planet in LK resonance with an inclined binary companion, as shown by the libration of the argument of the pericenter in the invariant plane reference frame. The initial parameters for the binary are $m_B = 1 M_\odot$, $a_B = 500$ AU, $e_B = 0.3$, and $i_B = 70°$. The ones for the planet are $m = 2.02\ M_{\rm Jup}$, $a = 15$ AU, $e \sim 0$, $i \sim 0$, and $\omega = 16°$.

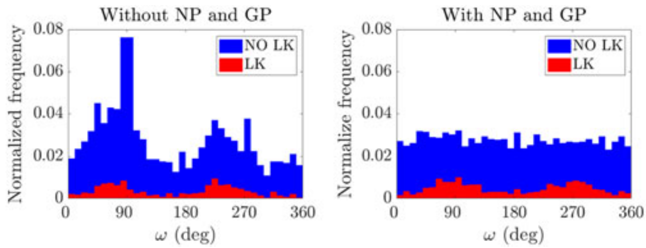

Figure 7. Normalized distribution of the pericenter argument of the planet (in the invariant Laplace plane reference frame) at the dispersal of the disc, as found in the simulations of Roisin et al. (2021), when the disc gravitational potential (GP) acting on the planet and the nodal precession (NP) induced by the binary companion on the disc are included (right panel) or not (left panel). The simulations follow the evolution of 3200 single giant planets migrating in a S-type configuration with a wide binary companion at 1000 AU and with a mass of $1 M_\odot$. The initial planetary parameters are $m \in [1, 5]\ M_{\rm Jup}$, $a = 20$ AU, $e = 10^{-3}, 0.1, 0.3, 0.5$, and $i \in [0°, 70°]$.

circumprimary planets (also called S-type planets) have been discovered in multiple-star systems (Schwarz et al. 2016), mostly in wide binaries with separation of at least 500 AU. The stellar companion can strongly impact the formation and long-term evolution of these planets (see e.g., Thébault and Haghighipour 2015, for a review).

The LK resonance was widely discussed for planets in binary star systems. In particular, it was identified as a possible origin for the hot Jupiters via the LK migration scenario (e.g., Wu and Murray 2003; Naoz et al. 2011; Petrovich 2015; Anderson et al. 2016; Naoz 2016, for a review; see also Section 5). In this section, we show how the LK resonance induced by an inclined binary companion can interfer with the planetary migration driven by the protoplanetary disc during the late-stage formation process.

An example of a planetary evolution in the LK resonance with a binary companion is shown in Fig. 6. The binary companion has an inclination of 70° with respect to the disc plane (where the planet is embedded). After the migration phase (~ 1 Myr, top left panel), the planet is rapidly captured in the LK resonance, as indicated by the libration of the pericenter argument (bottom left panel) and the high eccentricity and inclination variations (right panels).

Using a symplectic N-body integrator designed for binary star systems, in which the dissipation for the planet due to the disc is implemented following the formulas of Bitsch et al. (2013), Roisin and Libert (2021) noted that accumulations of the pericenter arguments around 90° and 270° are clearly visible at the dispersal of the disc (left panel of Fig. 7). This reflects the significant dynamical influence of a wide binary

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companion (here at 1000 AU) on the evolution of a giant planet during its migration in the protoplanetary disc. However, when considering additional effects related to the disc, such as the disc gravitational potential acting on the planet (e.g., Terquem and Ajmia 2010; Teyssandier et al. 2013) and the nodal precession induced by the binary companion on the disc (e.g., Batygin et al. 2011; Zanazzi and Lai 2018), Roisin et al. (2021) showed that these accumulations disappear (right panel of Fig. 7). The gravitational and damping forces exerted by the disc on the planet tend to maintain the latter in the midplane of the former and suppress the effect of the binary companion by preventing a LK resonance locking during the disc phase. Moreover, the color code of Fig. 7 provides information on the dynamical evolution of the planet at the end of the simulation. A capture of the planet in the LK resonance is far from being automatic, since only 30% of the systems with an inclined binary companion (inclination above 40°) are found in a LK-resonant state in the long term (red color). The non-resonant evolutions which also present high eccentricity and inclination variations are associated with circulation around the LK stability regions and the long-term stability of these systems is not assured (see Roisin and Libert 2021, for more details).

5. Hierarchical triple-star systems

For a significant proportion of triple-star systems, the inner binary has a period smaller than six days (e.g., Duquennoy and Mayor 1991; Tokovinin 2014). The LK mechanism combined with tidal friction was widely invoked to explain the observed pile-up around three days (e.g., Eggleton and Kiseleva-Eggleton 2001; Fabrycky and Tremaine 2007; Naoz and Fabrycky 2014; Liu et al. 2015; Toonen et al. 2016; Anderson et al. 2017). Due to the gravitational perturbation of an (outer) highly-inclined stellar companion, LK cycles are observed for the inner binary star. As shown in Fig. 8 (top panel), the eccentricity of the inner orbit can reach values close to unity during the LK cycles. As a result, the periastron distance can reach very small values. Tidal dissipation comes then into play and causes the inner orbit to shrink. This mechanism is often called *LK migration*. The evolution of Fig. 8 was obtained when using vectorial secular octupole-order equations for both the spin and orbital evolutions with general relativity corrections, tidal effects, stellar oblateness, and magnetic spin-down braking (Correia et al. 2016).

The evolution of the inner binary can also be displayed on the phase portrait of a simplified Hamiltonian formulation, namely the quadrupole approximation (given by Eq. (1.1)) with general relativity corrections, as done in Bataille et al. (2018). For hierarchical stellar systems as the ones considered here (i.e., an inner binary with masses m_0 and m_1 perturbed by a distant companion star with mass m_2 and mutual inclination i), the adimensional angular momentum of the inner orbit $H = \sqrt{1-e_1^2}\cos i$ can be considered as constant (i.e., the Kozai constant) and the trajectory of the system can be followed, for a given value of H, on the level curves of the one-degree Hamiltonian in the plane (ω_1, i). In the bottom panel of Fig. 8, the time evolution of the system is indicated by the color code, from its initial configuration (in blue) to the final one (in yellow). The phase portrait clearly shows the LK equilibria at $\omega_1 = 90°$ and $\omega_1 = 270°$. The system initially evolves around one of these equilibria, then, due to tidal dissipative effects, moves away from it by crossing different Hamiltonian level curves, before the final locking of the inner orbit in a quasi-circular orbit with orbital period of a few days only.

Fabrycky and Tremaine (2007) studied the secular evolution of hierarchical triple-star systems using the quadrupole approximation with general relativity and tidal effects. They observed an excess of short-period binaries in their simulations, which suggests the possible existence of distant tertiary companions for the observed population of short-period binaries, preferably with a mutual inclination of $\sim 40°$ or $\sim 140°$. In their extension of the previous study to the octupole order, Naoz and Fabrycky (2014) showed that the

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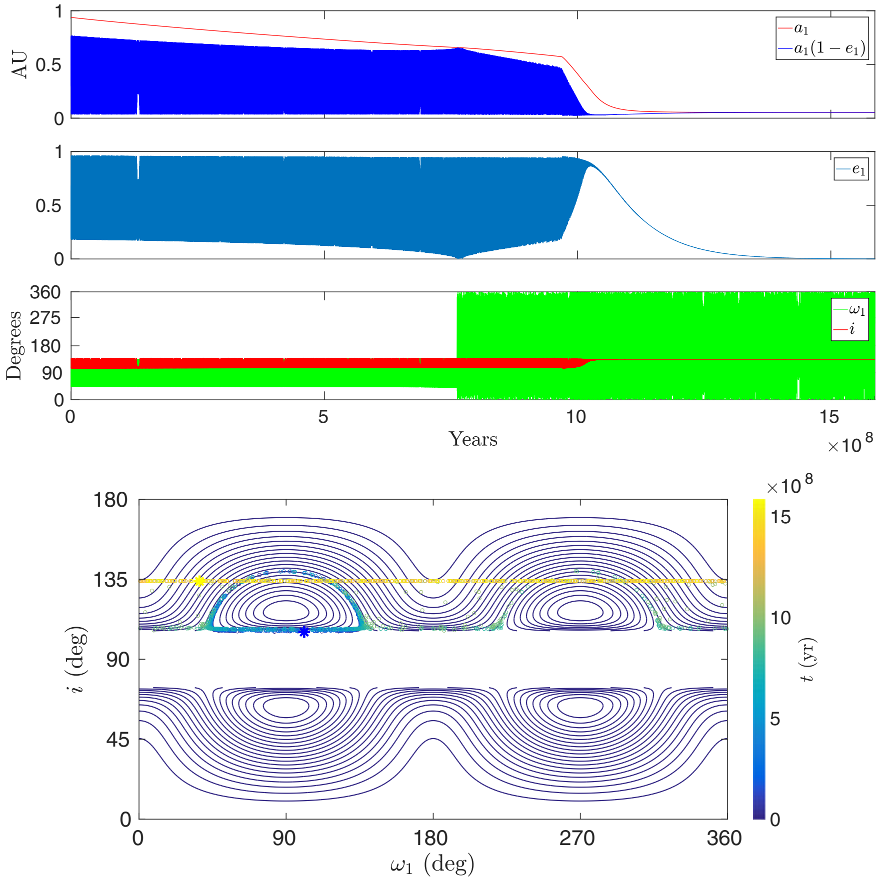

Figure 8. Triple-star system undergoing LK migration. Both the orbital and the spin evolutions are considered (octupole order approximation with general relativity corrections, tidal effects, stellar oblateness, and magnetic spin-down braking). Top: Time evolution of the orbital elements of the inner binary. Bottom: Same evolution displayed on the phase portrait of the quadrupole level Hamiltonian with general relativity corrections. Adapted from Bataille et al. (2018).

LK migration can be induced by a larger range of initial mutual inclinations. Further refinements were proposed by Anderson et al. (2017), Moe and Kratter (2018), and Bataille et al. (2018). In this last work, the authors aimed to identify the initial values of the orbital elements leading to the LK migration, by using uniform distributions of the initial elements in their simulations. They highlighted that the probability to initiate the migration, and thus the formation of the short-period pile-up around three days visible in Fig. 9 (bottom panel), is higher for the following initial orbital elements: $e_1 \sim 0.9$, $\omega_1 \sim 0°$ or $180°$, and mutual inclination $i \sim 90°$, as it can be deduced from the accumulations shown in Fig. 10.

Since very high eccentricities are needed for the migration caused by the tidal effects, either the inner binary is initially formed on very eccentric orbit, or its eccentricity has to be pumped to high values via the LK cycles (like in the evolution of Fig. 8). In the latter case, the initial mutual inclination preferably lies in the range $[40°; 140°]$. However, this condition on the mutual inclination is not sufficient to initiate the migration because when the system is very close to a LK equilibrium, the eccentricity variations are too limited for the tidal effects to operate. These findings can also be interpreted in terms of the

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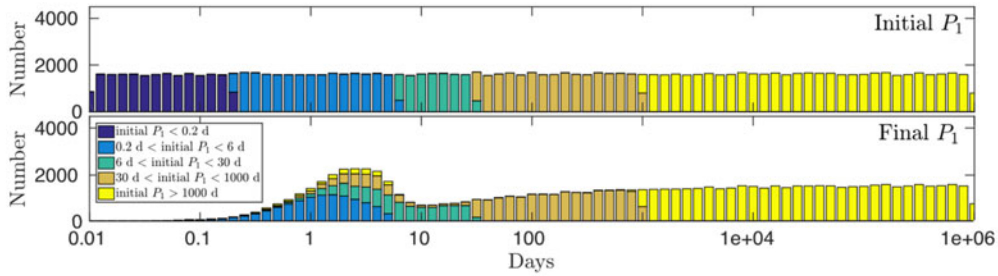

Figure 9. Histogram of the initial (top panel) and final (bottom panel) orbital periods of the inner binary, as given by the simulations of Bataille et al. (2018).

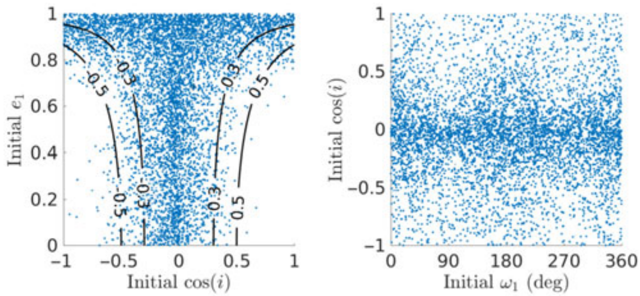

Figure 10. Initial orbital elements of the triple-star systems forming the three-day pile-up in Fig. 9. Adapted from Bataille et al. (2018).

Kozai constant: low initial H-values, namely $|H| \lesssim 0.5$, are associated with the migration into short-period binaries (see the level curves of H in the left panel of Fig. 10). As a result, the LK migration is a robust mechanism to produce the pile-up around three-day periods, but it requires demanding initial conditions for its initiation, which are maybe not encountered during the formation of the triple-star systems.

6. Conclusions

After the original works of von Zeipel (1910), Lidov (1962), and Kozai (1962), the LK resonance recently attracted renewed interest among very diverse scientific fields, from planetary and stellar systems to galaxies and black holes. This is due to the universal nature of the mechanism which clearly seems to operate at different scales. The list of the applications developed here is far from being exhaustive. The present contribution aimed to give a fairly accurate description of the LK resonance and to show how this mechanism can provide answers to current research questions in planetary and stellar systems, such as the formation of inclined planetary orbits and the existence of the hot Jupiters and short-period binaries.

The coming years will give us the opportunity to carry out deeper observations and confirm as well as quantify the occurence of the LK mechanism, a mechanism whose initial conditions in eccentricities and/or inclinations are quite demanding. This will provide valuable clues for a better understanding of the formation and long-term stability of planetary and stellar systems.

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