THEORETICAL METHOD FOR TREATING FORCE-FREE BLACK HOLE MAGNETOSPHERE IN NON-STATIONARY ANI NON-AXISYMMETRIC STATE

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The force-free approximation is a very useful method in studies of the global electromagnetic structure of the magnetospheres around the relativistic astronomical bodies. However, its application has almost been restricted to the stationary and axisymmetric configuration. In this work, we remove this restriction and construct a general method for treating the force-free electromagnetic field.

The basic equations for the force-free electromagnetic field are Maxwell's equations and the force-free condition $F_{\mu\nu}J^{\nu} = 0$. Then the force-free electromagnetic field necessarily becomes a degenerate electromagnetic field, which satisfies $F_{\mu\nu} * F^{\mu\nu} = 0$. Further, physical force-free electromagnetic fields also satisfy the condition $F_{\mu\nu}F^{\mu\nu} > 0$.

We can show that a degenerate electromagnetic field is generally expressed by two Euler potentials ϕ_1 and ϕ_2 as

$$F_{\mu\nu} = \partial_\mu\phi_1\partial_\nu\phi_2 - \partial_\mu\phi_2\partial_\nu\phi_1.$$

A two-dimensional surface on which ϕ_1 and ϕ_2 are constant is a world sheet of the magnetic field line. Inversely, an intersection of this surface and the three-space is a magnetic field line. Our theory is based on this expression of the degenerate electromagnetic field.

Substituting this expression into the force-free condition, we have

$$\partial_\nu\phi_i\partial_\lambda\left\{\sqrt{-g}\left(\partial^\nu\phi_1\partial^\lambda\phi_2 - \partial^\nu\phi_2\partial^\lambda\phi_1\right)\right\} = 0, \qquad i = 1, 2,$$

where we use the independence of $\partial_\mu\phi_1$ and $\partial_\mu\phi_2$ as vectors. This is the basic equation for the force-free electromagnetic field. We can see that the force-free electromagnetic field is generally described by two equations for two

Y. Sofue (ed.), The Central Regions of the Galaxy and Galaxies, 411-412.

Euler potentials. Further, since the basic equation is written in a covariant field theory, we can apply the theory to any curved black hole spacetime.

The dynamics of these equations becomes transparent by rewriting them to the 3+1 form. In the flat spacetime, we have

$$\begin{pmatrix} \nabla\phi_2\cdot\nabla\phi_2 & -\nabla\phi_1\cdot\nabla\phi_2 \\ -\nabla\phi_1\cdot\nabla\phi_2 & \nabla\phi_1\cdot\nabla\phi_1 \end{pmatrix}\begin{pmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \end{pmatrix}+\cdots,$$

where the omitted part contains the time derivatives up to the first order and the spatial derivatives up to the second order. This implies that if the determinant of the matrix in the above equation does not vanish, the second time derivatives of the Euler potentials are calculated from the initial data. Therefore as far as the condition $(\nabla\phi_1\times\nabla\phi_2)^2\neq 0$ is satisfied all over the force-free region, we can trace the time evolution of the force-free electromagnetic field, in principle.

In many astronomical applications, we must treat a field configuration with some symmetry. The conditions for symmetry restrict forms of the Euler potentials. We can decide them from the condition for symmetry.

For example, let us consider the time-dependent axisymmetric configuration. In this case, the electromagnetic field satisfies $\mathcal{L}_{\mathbf{m}}F_{\mu\nu}=0$ where $\mathbf{m}$ is the axisymmetric Killing vector and $\mathcal{L}_{\mathbf{m}}$ is the Lie derivative with respect to $\mathbf{m}$. This yields differential equations for the Euler potentials as

$$\mathcal{L}_{\mathbf{m}}\phi_1=\partial_\varphi\phi_1=0,\qquad \mathcal{L}_{\mathbf{m}}\phi_2=\partial_\varphi\phi_2=1.$$

Solving these equations, we find that the Euler potentials are restricted to the form as

$$\phi_1=\psi_1(t,r,\theta),\qquad \phi_2=\varphi+\psi_2(t,r,\theta).$$

Note that one of the Euler potentials depends on φ even in the axisymmetric configuration. This arises from the motion of the magnetic field lines in the φ direction. The above equations are also considered as a transformation of the variables from the Euler potentials to two functions invariant under the action of the axisymmetric Killing vector. Substituting these Euler potentials to the basic equation, we can treat the time evolution of the axisymmetric force-free magnetosphere.

Similarly, we can treat the stationary and axisymmetric configuration. In this case, one of the basic equations becomes integrable. After integration, we arrive at the well-known results of the stationary and axisymmetric force-free electromagnetic field.

We have presented an outline of the general theory of the force-free electromagnetic field. A detailed exposition has been given in our two recent papers (see Uchida, T., 1997, Phys. Rev. E **56**, p.2181 and p.2196).