SOME INSIGHTS INTO STELLAR STRUCTURE FROM NONLINEAR PULSATIONS

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1. Introduction

Efficient tools of investigation of stellar pulsation are the integral relations which link oscillation frequencies to the static structure of stellar models, as provided by the linear theory of pulsation (for a review, see Saio, this conference).

Similarly, oscillation amplitudes and phases, which arise from nonlinear processes, can be related to the stellar structure by means of amplitude equation formalisms (for a review, see Buchler, this conference).

For the simple case of a monoperiodic oscillation, involving only one unstable marginal mode, such a formalism shows that the (limit cycle) radius variations, at time t and mass level m, can be approximated, up to second order of approximation, (Buchler and Goupil, 1984; Buchler and Kovàcs, 1986) by:

$$\frac{\delta r}{r}(m,t) = 2A|\xi_r(m)|\cos\Omega_{nl}t + 2A^2|C_1(m)|\cos(2\Omega_{nl}t+\phi) + A^2C_0(m) \quad (1a)$$

$$A^2 = -\kappa/Q_r; \;\; \Omega_{nl} = \Omega(1+\Delta\Omega/\Omega); \;\; \Delta\Omega/\Omega = A^2Q_t \quad (1b)$$

where $A, R, \Omega, \kappa, \xi_r(m)$ respectively are the amplitude, stellar radius, linear nonadiabatic frequency, growth rate, radius eigenfunction. Second order nonlinearities generated first harmonic oscillations and change in equilibrium radius about which the star oscillates, as represented by the last two terms in (1a) respectively. Analogous expressions are obtained for velocity and light variations, that can be compared with observations.

The nonlinear, nonadiabatic coefficients, C_1, C_0, ϕ, Q_r, Q_t, are integrals over mass of kernels which depend on eigenfrequencies, eigenfunctions, on second and third order Taylor quantities from the equations modelling the star. They can either be computed from static models (Klapp et al., 1985) or obtained by numerical fits of hydrodynamical results (Kovàcs and Buchler, 1989).

2. Nonlinearities as Probes of Stellar Structure

When local quasiadiabaticity is assumed, approximated expressions for the eigenfunctions and their adjoints in terms of adiabatic ones can be used to

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simplify the nonlinear coefficients entering (1).

The amplitude and $\Delta\Omega/\Omega$ are then found to be composed of two types of competiting contributions. The first one comes from nonlinearities of the restoring forces and energy transfer. The radius variations, initially sinusoidal (small amplitude), thereby become nonlinear i.e. are contaminated by harmonics (as in (1a)). This nonlinear contamination changes the magnitude of the restoring forces and energy transfer. This, in turn, modifies the radius variations. This feedback gives rise to the second contribution.

Further simplification (taking the adiabatic exponent as $\Gamma_1 \sim 5/3$ and neglecting spatial derivatives of $\xi(m)$ in linear variations of densities) leads to

$$\frac{\Delta\Omega}{\Omega} = A^2 \int dm\ 18\xi_r^3(3\Gamma_1 - 4)(\frac{5}{4}F_1 - \xi_r(m)) \tag{2}$$

For a fundamental mode, $\xi_r \geq 0$. Then, each stellar region contributes negatively or positively to (2), according to its position with respect to the critical value $5/4F_1$, which depends on the whole stellar structure (consequence of the aforementioned feedback). To F_1's value, however, mainly contributes the exterior where $\xi_r^3(m)dm$ peaks. Roughly, $F_1 \sim 1$ for a fundamental mode, therefore $\Delta\Omega/\Omega > 0$. The nonlinear period is longer than the linear one, in agreement with results from numerical models. The discussion must be slightly changed when stable modes are taken into account.

Contributions to the amplitude, $A^2 = (\mathcal{G}_2 + \mathcal{G}_3 + \mathcal{G}_4 + \mathcal{G}_{st})^{-1}$, come from eigenvectors nonadiabaticity, nonlinear dependence of pressure on entropy, nonlinear energy gains and losses and indirect influence of stable modes. Saturation effects that contribute to the existence of a limit cycle ($A^2 > 0$) can be locally discussed as above. In particular, energy nonlinearities play an important role in modifying the impact on the pulsation of the nonlinear variation of pressure with entropy.

To lowest quasiadiabatic order, the nonlinear change in equilibrium radius is $A^2C_0 = 2A^2F_1\xi(m)$. It increases towards the exterior with ξ_r as it is observed in numerical models (Fadeyev, this conference). In quasiadiabatic regions, a phase lag of π exists between the main and first harmonic oscillations and their amplitude ratio, $(1/3)F_1A$, is independent of mass level. Existence of stable modes introduces a slight dependence with mass level.

In conclusion, the above work can be extended to more realistic cases involving mode interactions. Though the necessary assumptions (local quasiadiabaticity, simplified boundary conditions) limit the discussion to qualitative behaviors, they provide simplified expressions of nonlinear coefficients which enable to investigate which processes, in what stellar regions, affect finite amplitude pulsations.

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References

Buchler, J. R. and Goupil, M. J.: 1984, *Astrophysical Journal*, **279**, 394.
Buchler, J. R. and Kovàcs, G.: 1986, *Astrophysical Journal*, **303**, 749.
Klapp, J., Goupil, M. J., Buchler, J. R.: 1985, *Astrophysical Journal*, **296**, 514.
Kovàcs, G. and Buchler, J. R.: 1989, *Astrophysical Journal*, **346**, 898.