ON THE OBSERVED COMPLEXITY OF CHAOTIC STELLAR PULSATION

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1. Introduction

The existence of some variable stars producing very complicated light curves is well known. Theoretical calculations suggest that the irregular behaviour of the pulsation models in the RV Tauri and W Virginis regime is low dimensional is the result of period-doubling or tangent bifurcations (see e.g. Buchler and Kovács 1987, Kovács and Buchler 1988, Tanaka and Takeuti 1988).

The light variation of the RV Tauri star R Scuti covering 150 years was analyzed by Kolláth (1990). A striking similarity was found between the reconstructed attractor of the Rössler model and that of the light variation of R Scuti. This result confirms the theoretical prediction of the existence of chaos, but a discrepancy still exists between the theory and observation. We could not find evidence for low dimension ($D = 2 - 3$). The analyses of other stars also show rather erratic behaviour (e.g. Cannizzo and Goodings 1988, Cannizzo et al. 1990). A possible answer for this discrepancy is the treatment of stochastic perturbations by convection (Perdang 1991).

It was shown (see e.g. Mitschke et al. 1988) that a relatively simple system (e.g. low pass filter) can increase the correlation and information dimension of a signal. The inner pulsation - light variation transfer mechanism can be represented by a smooth function for many kind of variable stars, but it is more complicated for red giant and supergiant stars. In the latter case the shock waves play an important role and the light variation can be modeled only with the combined dynamics of the pulsation and the transfer mechanism. We discuss the effect of a simple model to the complexity of the observable light curves: even a linear transfer mechanism can increase the dimension of the attractor.

2. Characterization of the Complexity of the Light Variation

In order to get information about the complexity of a time series one can calculate the Fourier transform of the data. Multiperiodic signals can be easily detected by this method, but the spectra cannot distinguish between stochastic and low dimensional chaotic processes.

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One can usually derive qualitative information from the observational data describing the view of the reconstructed phase trajectories. For extracting qualitative dynamics from the one-dimensional observable. i.e. the light curve, the Takens-theorem (method of delays) was applied (Takens 1980). The vectors of the reconstructed phase space are defined as $(v_k, v_{k+L}, v_{k+2L}, ..., v_{k+(N-1)L})$, where v_k means the original signal and N is the embedding dimension.

The simplest method to get quantitative information is the calculation of the correlation dimension (see e.g. Grassberger and Procaccia 1983). If the phase space has more complex structure, its dimension is usually higher. The correlation integral $(C(r))$ counts the number of point pairs whose distance is less than r. If the $\log(r) - \log C(r)$ curve has a linear component, the correlation dimension is well estimated by its slope.

While the correlation dimension misses the information of the temporal running the nonlinear prediction (see e.g. Casdagli 1989) is based on the dynamics. The future of a deterministic system can be predicted to some degree from the surrounding phase trajectories representing the history of the system in the past. A very important measure of the prediction is the run of the prediction error (σ) versus the number of predictee (M). It has the scaling law: $\log\sigma \sim \log M/D$, where D is the information dimension of the attractor (see e.g. Casdagli 1989).

A similar measure on the observed complexity is the Lyapunov spectrum. The determination of the largest Lyapunov-exponent from a time series is relatively easy (see e.g. Wolf et al. 1985).

3. Effect of Inner Pulsation - Light Variation Transfer

For yellow and red supergiant stars the shock waves play an important role in the visible light variation. High resolution spectroscopic and spectrovelocimetric observations of R Sct and AC Her were reported by Gillet et al. (1990). These observations clearly describe the shock phenomena in the photosphere of RV Tau stars. The acceleration-curves calculated from the observed radial velocities have sharp peaks indicating the driving of shocks. The kinematics of the shocks is well estimated by ballistic motion except for the shock producing phase.

We used this fact to construct a simple transfer mechanism. The input is an arbitrary time series and the output is exactly determined by the variation of the input function at its maxima. Shocks were accelerated at the maxima of this time series and the model light variation of the stars was calculated from the temporary place and the intensity of the shocks. We used the most simple approximation, we did not want to describe the real light-curves only to predict the possible effect of the transfer mechanism.

This mechanism can neither create nor destroy information, but it redis-

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tributes the information on the time axis. Since the additional Lyapunov exponent due to the transfer mechanism is negative, it is expected, that the largest Lyapunov exponent remains unchanged. The dimension of the combined system, however, may be higher than that of the original dynamics (e.g. Mitschke et al. 1988). Even an added linear part can increase the dimension of the attractor if the added Lyapunov exponent is larger than the largest negative Lyapunov exponent.

As an input, we used the simplest chaotic models, i.e. the solution of the Lorenz (Lorenz 1963) and Rössler equations (Rössler 1976) with parameters according to the well developed chaos. The reconstructed trajectories were calculate from the input and output signals for both models. The simple view of the attractors indicates the differences in the complexity. The correlation integrals and their derivatives were also investigated. Although we can find well defined scaling regions in the case of the input function, this behaviour disappears for the output, and the correlation dimension is increased.

The increased complexity is also confirmed by the predictability of the data. The prediction errors were increased and the slope of the number of predictee $-\sigma$ curves were decreased indicating the higher informational dimension. We found, that the observed light curve may be more complicated than the underlying physical oscillations. As it is expected, the largest Lyapunov exponent was unchanged.

It is very important to consider the effect of a more realistic inner pulsation - light variation transfer and to develop methods for an inverse approach: how can we describe the complexity of the underlying dynamics from the observable light variation.

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