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AN APPROXIMATIVE CALCULATION OF ELECTRIC CONDUCTIVITY IN THE LOWER LAYERS OF THE SOLAR ATMOSPHERE

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ABSTRACT

Recently Nagasawa determined a method of calculation of the electric conductivity in a lowly ionized gas, which leads to more precise results than the formula given by Alfvén. The numerical calculation, however, is much more complicated for the Nagasawa method. This paper simplifies the calculation giving a relation, by means of which the Alfvén values of Nagasawa are obtained in a rather simple way.

For an approximative calculation of the electric conductivity in a lowly ionized gas we may use the relation [1]

$$\sigma = \frac{e_0^2}{m_e^{1/2}} \cdot \frac{1}{S(3kT)^{1/2}} \cdot \frac{n_e}{n_n}, \qquad (1)$$

where e_0, and m_e denote the electric charge and the mass of an electron, n_e the number of free electrons, n_n the number of neutral atoms, k the Boltzmann constant, T the temperature, and S the effective cross-section of the particles in question.

A much more precise method of calculation of the electric conductivity was recently developed by Nagasawa [2]. His method leads to a system of equations, the approximative solution of which gives the following relation for σ_0

$$\sigma_0 = \tfrac{3}{16}\, e_0^2 \frac{n_e(B_{13}+B_{23})}{n_e B_{12}(B_{13}+B_{23}) + n_n B_{13} B_{23}}, \qquad (2)$$

where

$$B_{12} = \frac{\sqrt{\pi}}{2}\left(\frac{m_e m_i}{m_e + m_i}\right)^{1/2} A e_0^4 (2kT)^{-3/2}, \qquad (3)$$

$$B_{13} = \frac{\sqrt{\pi}}{2}\left(\frac{m_e m_n}{m_e + m_n}\right)^{1/2} S(2kT)^{1/2} \qquad (4)$$

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$$B_{23}=\frac{\sqrt{\pi}}{2}\left(\frac{m_i m_n}{m_i+m_n}\right)^{1/2} S(2kT)^{1/2}, \tag{5}$$

$$A=\ln\left(\frac{4kT}{e_0^2 n_e^{1/3}}\right)^2. \tag{6}$$

In these equations m_i and m_n denote the mass of ions and neutral atoms. The given solution assumes that atoms are only singly ionized.

The electric conductivity σ_0 determined from (2) is closer to the real conditions than the value of σ, given by Eq. (1). The numerical calculation of σ_0, however, is much more complicated than the procedure connected with the calculation of σ. To simplify this calculation, we will show that there exists a simple relation between σ_0 and σ, which may be used for the numerical calculation of the electric conductivity in some cases, especially in the lower layers of the solar atmosphere.

For these layers we may put

$$\frac{m_e m_i}{m_e+m_i}=m_e, \tag{7}$$

$$\frac{m_e \cdot m_n}{m_e+m_n}=m_e, \tag{8}$$

$$\frac{m_i m_n}{m_i+m_n}=1{\cdot}5 m_H, \tag{9}$$

where m_H denotes the mass of a hydrogen atom. Then the relation (2) can be written in the form

$$\sigma_0=\frac{3}{8\sqrt{\pi}}\cdot\frac{e_0^2 n_e}{m_e^{1/2} S n_n (2kT)^{1/2}}\cdot\frac{m_e^{1/2}+1{\cdot}22 m_H^{1/2}}{\frac{n_e}{n_n}\cdot\frac{A}{S}\, e_0^4 (2kT)^{-2}\,(m_e^{1/2}+1{\cdot}22 m_H^{1/2})+1{\cdot}22 m_H^{1/2}}. \tag{10}$$

The effective cross-section, S, may be assumed according to Alfvén [1] equal to $10^{-15}\,\text{cm}^2$. The last measurements show, however, that the hydrogen effective cross-section should be taken lower, about $10^{-14}\,\text{cm}^2$. For these values of the cross-section and in the case of low ionization ($n_e/n_n < 10^{-2}$) we may put in the first approximation

$$\frac{m_e^{1/2}+1{\cdot}22 m_H^{1/2}}{\frac{n_e}{n_n}\cdot\frac{A}{S}\, e_0^4 (2kT)^{-2}\,(m_e^{1/2}+1{\cdot}22 m_H^{1/2})+1{\cdot}22 m_H^{1/2}}. \tag{11}$$

Then we get, with regard to the relation (1), that σ_0 is proportional to σ. Therefore, as far as the assumed conditions are fulfilled, we may determine more precise values of the electric conductivity σ_0 by means of the values of σ, the calculation of which is substantially easier. We denote the electric conductivity deduced in this way by $\bar{\sigma}$. We get

$$\bar{\sigma} = 0{\cdot}26\sigma. \tag{12}$$

An error which appears by using the approximation (11) is the same or less than the difference between various models of the solar photosphere. This fact is well demonstrated in Fig. 1, which contains the following

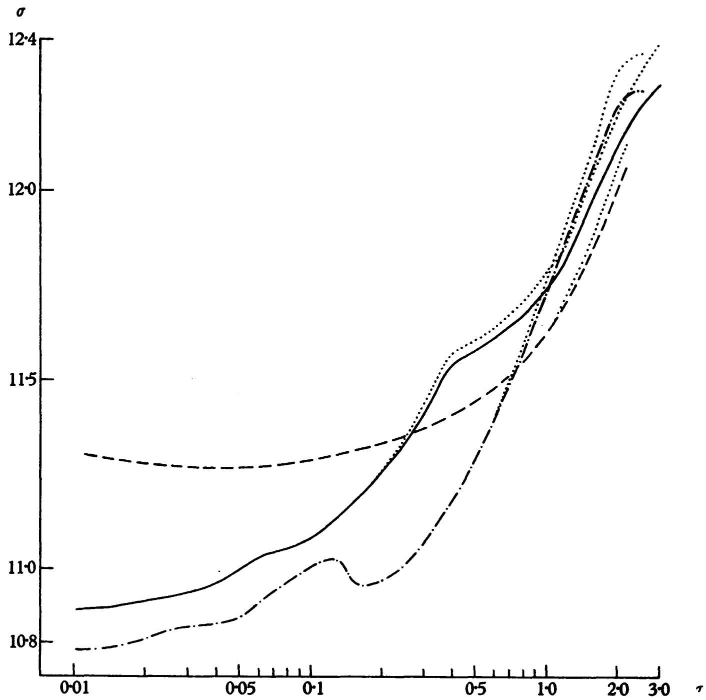

Fig. 1. The electric conductivity σ_0 calculated according to Eq. (2) for the photospheric models of Minnaert (dashed curve), de Jager (full curve), Berdičevskaja (dash and dot curve) and the deviation from these results calculated from Eq. (12) (dotted curve).

curves: the course of the electric conductivity σ_0, calculated according to (2) for the photospheric model of Minnaert [3] (dashed curve), of de Jager [4] (full curve) and Berdičevskaja [5] (dash and dot curve); and the course of the electric conductivity, calculated according to (12), as far as it differs for the various models from the course of σ_0 (dotted curve). All throughout these calculations we used the less favourable value of the effective cross-section S for the approximation (11), i.e. $S = 10^{-15}\,\mathrm{cm}^{-2}$.

Fig. 1 demonstrates that the calculation of the electric conductivity by means of the relation (12) is fully satisfactory.

A more detailed paper on this theme has been published in [6].

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