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BOOK REVIEWS—COMPTES RENDUS CRITIQUES

Introduction to Spectral Theory in Hilbert Space, by G. M. Helmberg. iii+ 346 pages. Series in Applied Mathematics and Mechanics, North-Holland, Amsterdam, **6,** 1969. U.S. $19.50.

The author states in his preface that "the aim of this book is to make the reader familiar with everything needed in order to understand, believe, and apply the spectral theorem for selfadjoint operators (not necessarily bounded) in Hilbert space". The reviewer feels that he achieves this handsomely.

No previous algebraic knowledge is assumed, but a familiarity with Weierstrass' approximation theorem and Riemann-Stieltjes integration is required. Since the excellent collection of worked examples is largely concerned with the spaces $L_2(\alpha, \beta)$ where $-\infty \leq \alpha < \beta \leq \infty$, the reader without a training in Lebesgue integration, whilst still being able to understand most of the text, would miss much of the flavour of the book. To help him, there is an appendix on Lebesgue integration at the end of the book.

In Ch. I & II the concept of a Hilbert space is introduced, and the usual elementary results are proved. Following the Gram-Schmidt orthogonalization process is an excellent section in which the classical orthonormal bases are produced, viz the Legendre polynomials, the Hermite functions, and the Laguerre functions.

Ch. III discusses bounded linear operators and their adjoints. It ends with a discussion of the Fourier-Plancherel operator on $L_2(-\infty, \infty)$.

Ch. IV, entitled "General theory of linear operators", has sections on adjoint operators, closed linear operators, invariant subspaces, eigenvalues, and the spectrum of a linear operator. The differentiation operator in $L_2(\alpha, \beta)$ is extensively discussed. In particular, the differentiation operator in $L_2(-\infty, \infty)$ is shown to be selfadjoint and equivalent, via the Fourier-Plancherel operator, to the multiplication operator $f(x) \to xf(x)$ in $L_2(-\infty, \infty)$.

Compact operators are discussed in Ch. V, the spectral decomposition of a compact selfadjoint operator is obtained, and there is a section on Fredholm integral equations.

Ch. VI contains the representation of a bounded self-adjoint operator by the uniformly convergent Riemann-Stieltjes integral $\int_{-\infty}^{\infty} \lambda \, dP(\lambda)$ for a suitable spectral family of projections $P(\lambda)$. In a similar way the representation

$$U = \int_0^{2\pi} e^{i\lambda} \, dP(\lambda)$$

for a unitary operator is obtained. Finally, the representation of a bounded normal operator A by $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\xi+i\eta)\, dP(\xi)\, dQ(\eta)$ is shown, where $\{P(\xi)\}$ and $\{Q(\eta)\}$ are the spectral families of the real and imaginary parts of A.

In Ch. VII the spectral representation of an unbounded selfadjoint operator is obtained using the Cayley transform and the preceding decomposition of a unitary operator.

The text is very clear and contains some excellent worked examples which are pursued throughout the book. This compensates for the small number of exercises.

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Subgroups of Finite Groups, by S. A. Chunikhin. 142 pages. (English translation by Elizabeth Rowlinson.) Noordhoff, Groningen, 1969. U.S. \$6.25.

This book is essentially a unified exposition of research of the author and his school on certain generalizations of the Sylow theorems. For example, if Π is a set of primes, G a group, and m the largest divisor of the order of G all of whose prime factors lie in Π, then a subgroup of order m in G is called a Π-Sylow subgroup of G. One can now ask under what conditions the Sylow theorems will generalize to Π-Sylow subgroups. A typical result is this: Let G be a group with a composition series in which each index has at most one distinct prime divisor in Π (this is called Π-separability and generalizes the notion of solvability). Then for any $\Pi_1 \subseteq \Pi$, Π_1-Sylow subgroups of G exist and are all solvable and conjugate.

The chapter headings are (1) Sylow Π-properties of finite groups, (2) Factorizations of finite groups based on the indices of chief and composition series, (3) A method for finding subgroups by means of indexials, and (4) Complexes of nonnilpotent subgroups.

The book is on a very specialized topic and is probably not of wide interest. Nevertheless, any graduate student should find it accessible.

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Topics in the Theory of Lifting, by A. Ionescu Tulcea and C. Ionescu Tulcea. x+190 pages. Ergebnisse der Mathematik und ihrer Grenzgebeite Band 48. Springer-Verlag, New York, 1969. U.S. \$9.90.

As indicated by the title, this book is concerned with the subject of lifting for spaces of bounded measurable functions. The problem was first formulated by A. Haar and solved by von Neumann in 1931 (for the real line and the Lebesgue measure). The general case has remained unsolved until 1958 when D. Maharam proved the existence of a lifting for a sigma-finite measure space.