

PROJECTIVE GEOMETRY.

To the Editor of the *Mathematical Gazette*.

DEAR SIR,—I think attention should be drawn to a logical slip in the article *Generalised Metrical Theorems* in the *Gazette*, Vol. XXX, No. 290, p. 122. By taking a real circle as his starting-point, the author necessarily confines himself to a *non-degenerate* conic, and so his final theorem (that the diagonals of a square bisect each other), though indisputable, is not proved by this method. There is, as far as I know, no way of passing from the real circle of theorem A, through the non-degenerate conic ϕ (the proof would not work if it were degenerate) of theorem B, to the line-pair AB, AC of the last result.

This step is akin to the "proof" of the theorem of Pappus which every examiner meets at intervals: the two given lines are a special case of a general conic; project that conic into a circle and chase angles; hence the result.

While I am writing, may I raise the whole question of projection into the circular points at infinity? (The author of the above article very carefully avoids projection; his wording could be taken as a model for the method which he describes.) I should very much like to know whether teachers believe that pupils really understand what they talk so glibly about. The whole idea seems to me full of difficulties, and I usually feel that the phrase "project into . . ." is used as a kind of charm, but that its user could seldom say just how he would do the projection. Are we, in fact, on ground where the school-boy should not stand? The views of teachers should be very interesting and valuable.

Yours etc., E. A. MAXWELL.

TERMINOLOGY IN DYNAMICS.

To the Editor of the *Mathematical Gazette*.

SIR,—Although the word *inextensible* (of strings, etc.) is of frequent occurrence (see any examination paper in mechanics), its meaning is not always clear. For instance, when Mr. Lightfoot refers in a recent article in the *Gazette* * to an "inextensible string of length a " he appears to mean a string of unalterable length; but when he states, a few lines further on, that "no real string fulfils the condition of being inextensible", he seems to refer to another property, *viz.* that when the motion of a particle is checked by a string the particle does not rebound and the string afterwards remains taut. May I use your columns, Sir, to urge the use of a terminology that would avoid this confusion?

Two distinct physical properties are involved. First there is the deformability of a body. One says of a solid that it is rigid or non-rigid, or of a fluid that it is compressible or incompressible. Similarly, I suggest, one should say of a string that it is *extensible* or *inextensible* according as it can or cannot be stretched. All these adjectives express purely geometrical conditions.

Secondly, there is the property that distinguishes a lump of rubber when it is deformed from a lump of putty, or a collision between steel balls from a collision between lead balls. The common idea behind these phenomena is that, when a deformation or impact occurs, internal forces or stresses come into play, and the property that distinguishes the rubber and the steel from the putty and the lead is that the work done by these internal forces during a cyclic deformation or during the impact is zero. To describe this property I suggest the word *elastic*. Thus a solid is elastic if the deformation-forces are conservative. A collision is elastic if the forces of interaction between the

* *Gazette*, XXX, No. 290, p. 129 (July, 1946).

colliding particles do no work, so that the total kinetic energy is conserved (*i.e.* the coefficient of restitution is unity if there are two particles). It may be noticed that this terminology is customary in the theory of atomic collisions. A collision that involves loss of kinetic energy is *inelastic*; when the loss of energy is as great as possible (coefficient of restitution zero) the collision may be called *completely inelastic*.

I illustrate the use of these terms by a few examples. Steel can often be treated as rigid and elastic, lead as rigid and completely inelastic. Putty is deformable and completely inelastic. A spring obeying Hooke's law is extensible and elastic. Mr. Lightfoot requires of his string that it should be inextensible and completely inelastic, and his comment might read "although real strings are often practically inextensible, no real string fulfils the condition that it is *completely inelastic*".

I am, Yours, etc., F. C. POWELL.

COORDINATE NOTATION.

To the Editor of the *Mathematical Gazette*.

SIR,—A problem that crops up in many practical connections is that of the labelling, scheduling, and classifying of points, areas, and lines. I refer to such cases as the numbering and listing of the buildings of a camp or village, the preparation of a county valuation roll in such a way that the properties can readily be identified on the Ordnance Survey plan of the area, on the numbering of a system of roads and their identification on a road map.

A method of labelling that has been gaining in favour in recent years is that based on coordinates; and it is difficult to find a better one. But why is it not universally adopted? I am convinced that the answer is that the accepted method of representing the coordinates is at fault: each is shown separately; this allows of classification according to one or other of the coordinates: not according to both.

Consider, for instance, the point P with coordinates x 284, y 407. On Army grid maps this would be written 284407. Now this number gives no basis for classifying a system of points, but, if rewritten as

$$24, 80, 47,$$

a simple basis of classification is at once available: write down the points in the numerical order of their coordinates in the new forms. The point P falls in the square to the north-east of the point 24, 00, 00. And all points in that square will be scheduled together.

The next obvious reform will be to introduce negative digits as well as positive. P now takes the form

$$34, \bar{2}1, 4\bar{3}$$

(x 3 $\bar{2}$ 4, y 41 $\bar{3}$). All points commencing with the number 34 will be scheduled together in the square whose *centre* is the point 34, 00, 00, a more symmetrical arrangement. If P represents a building it might be labelled and identified locally by the last two digits only, 4 $\bar{3}$.

This notation could be extended to vectors and possibly to tensors of even higher order. But an objection would probably be raised because of the labour involved in separating the components when required for calculations involving them. Would it be possible to carry out these calculations in the new symbols as they stand? I have investigated this possibility and find that not only would it be possible, but in most cases of distinct advantage to do so. The following three examples will illustrate this; the notation is duodecimal—