DEAR EDITOR,

Re: Paul Scott, Some recent discoveries in elementary geometry, *Math. Gaz.* **81** (Nov 1997), pp. 391-397 and I. Ward, The tritet rule, *Math. Gaz.* **79** (July 1995), pp. 380-382.

Readers may like to know of some earlier references which discuss the generalisation of Pythagoras' Theorem to 3-space. The first, originally published in 1962 is George Pólya, *Mathematical discovery*, Wiley (1981), p. 34. The others were collected as Note 62.23 in the *Gazette*: (1) Lewis Hull, (2) Hazel Perfect, (3) I. Heading, Pythagoras in higher dimensions: three approaches, *Math. Gaz.* **62** (October 1978) pp. 206-211.

Yours sincerely,

A. K. JOBBINGS

*Bradford Grammar School, Keighley Road, Bradford BD9 4JP*

DEAR EDITOR,

In Note 82.53 a proof is given for a test of divisibilty by 19. I offer a shorter proof.

Let the number to be tested be $N = 10a + b$ where $b$ is the units digit. The reduced test number is given by $P = a + 2b$, so that $2N - P = 19a$. Therefore, $19|N$ if and only if $19|P$.

Yours sincerely,

E. J. PEET

*3 Queensway, Newby, Scarborough YO12 6SJ*

DEAR EDITOR,

In [1] Murray Humphreys and Nicholas Macharia show that the $(n + 1)$-digit number

$$k = \overline{a_n a_{n-1} \ldots a_0} = 10^n a_n + 10^{n-1} a_{n-1} + \ldots + a_0 \qquad (1)$$

is divisible by 19 if and only if

$$m = 10a_n + a_{n-1} + 2a_{n-2} + 4a_{n-3} + \ldots + 2^{n-2} a_1 + a_0 \qquad (2)$$

is divisible by 19. This is essentially a special case of the method of James Voss in [2] for determining divisibility by any integer $s$ relatively prime to 10. The method hinges on using the multiplicative inverse of 10 (mod $s$). When $s = 19$, the multiplicative inverse is 2 because

$$2 \times 10 = 20 \equiv 1 \pmod{19}. \qquad (3)$$

If we multiply (1) by $2^{n-1}$ we get

$$\begin{aligned} 2^{n-1}k &= 2^{n-1}\left(10^n a_n + 10^{n-1} a_{n-1} + 10^{n-2} a_{n-2} + 10^{n-3} a_{n-3} + \ldots + 10a_1 + a_0\right) \\ &= 2^{n-1}10^{n-1}10a_n + 2^{n-1}10^{n-1}a_{n-1} + 2^{n-2}10^{n-2}2a_{n-2} + 2^{n-3}10^{n-3}4a_{n-3} \\ &\quad + \ldots + 2 \times 10 \times 2^{n-2} a_1 + 2^{n-1} a_0 \end{aligned}$$