OBITUARY

RICHARD RADO

Richard Rado was born in Berlin; he was the second son of Leopold Rado, from Budapest. At one stage of his education he had to decide whether to become a concert pianist or a mathematician. He chose the latter in the belief that he could continue with music as a hobby, but that he could never treat mathematics in that way. He studied at the University of Berlin, but also spent some time in Göttingen. He took a DPh at Berlin with his thesis 'Studien zur Kombinatorik' [**3**] under Issai Schur in 1933. During this period he was also influenced by Erhard Schmidt.

On 16 March 1933, he married Luise Zadek, the elder daughter of Hermann Zadek, whom he had earlier come to know when he needed a partner to play piano duets. It was indeed a remarkable partnership.

As Hitler came to power in 1933, the Rados, being Jewish, made their way to England, Richard having obtained a scholarship of £300 p.a. from Sir Robert Mond through the recommendation of Professor Lindemann (later Lord Cherwell), who had interviewed him in Berlin, to enable him to study at Cambridge.

Richard entered Fitzwilliam House (now College) in 1933, and studied for a PhD under G. H. Hardy. He was awarded his degree in 1935 for his thesis on 'Linear transformations of sequences' [**13**]. He stayed on at Cambridge with a temporary lectureship until 1936. During this period, 1933–36, the Hardy–Littlewood seminar was the main meeting place for mathematicians at Cambridge, there being neither department nor institute there at that time. Among the many resident mathematicians, those who influenced Rado most seem to have been G. H. Hardy, J. E. Littlewood, P. Hall and A. S. Besicovitch. B. H. Neumann, like Richard one of the many refugees from Nazi Germany, also joined Fitzwilliam House in 1933, and became one of Richard's lifelong friends. For a time he lived in the same lodgings as Hans Heilbronn. He also saw H. Davenport regularly. Paul Erdős, who had previously written to Richard, first met him on 1 October 1934 ⟨**2**⟩; some of their joint work will be described later.

In 1936, Richard obtained a post as Assistant Lecturer at Sheffield, and later became a Lecturer there. When Leon Mirsky arrived at Sheffield in 1942, the Rados befriended him and formed lasting friendships; see ⟨**1**⟩.

In 1947 Richard was appointed Reader in Mathematics at King's College, London. In 1954 he became Professor of Mathematics in the University of Reading, remaining there until and after his retirement in 1971. He spent the academic year 1971–72, immediately after his retirement, as Visiting Professor in the Department of Combinatorics and Optimization at the University of Waterloo, Ontario.

Richard Rado had extremely wide mathematical interests. Many themes run through his work. Erdős ⟨**2**⟩, writing about their joint work, says 'I was good at discovering perhaps difficult and interesting special cases and Richard was good at generalizing them and putting them in their proper perspective'. Richard himself,

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when replying to a presentation that was made on his retirement, said: 'There are almost as many types of mathematician as there are types of human being. Among them are technicians, there are artists, there are poets, there are dreamers, men of affairs and many more. I well remember rising from my chair after having just solved what seemed to me an interesting and difficult problem, and saying aloud to myself: "This is beautiful music!" And only after I had said this did it strike me that I had strayed into the wrong category.' Richard was fascinated by mathematical beauty and sought after it. He always tried to formulate his results at their natural level of generality, so that their full power was exhibited without their content being obscured by over-elaboration.

Richard was very methodical. He made verbatim shorthand notes of the lectures and seminars (and even Senate meetings) that he attended. He also used his shorthand in his mathematical workings and in the 64 diaries that he wrote. The National Cataloguing Unit for the Archives of Contemporary Scientists at the University of Bath is cataloguing Rado's manuscripts for the Reading University Library.

Richard and Luise had a double partnership. She went with him to conferences and meetings all over the world, and kept contact with all his mathematical friends. He was an accomplished pianist, she was a singer of professional standard. They gave many recitals, both public and private, often having musical evenings in their home in Reading. A road accident in 1983 affected Richard's health, and made it impossible for Luise to walk more than a few steps, and then only on the level. This sadly diminished their lives. Luise survived Richard by only a few months, leaving their son Peter with his wife and two children alone.

He was the kindest and gentlest of men.

Mathematical work

Although two main themes dominate Rado's mathematical work, it has many minor themes and many apparently isolated notes. We first discuss some of the minor themes in an attempt to show the breadth of his work, then turn to the major themes in an attempt to examine its depth.

Convergence of sequences and series. See [**13**, **17**, **18**, **19**, **59**]. If (κ_i, λ_i), $i = 1, 2, \ldots,$ is an ordering of the pairs of positive integers, we have a formal product

$$\left(\sum_{\kappa=1}^{\infty} x_\kappa\right)\left(\sum_{\lambda=1}^{\infty} y_\lambda\right) = \sum_{i=1}^{\infty} x_{\kappa_i} y_{\lambda_i}$$

for two series. In one paper [**19**] Rado determines all the sequences (κ_i, λ_i), $i = 1, 2, \ldots,$ for which the series on the right converges to the product of the two series on the left, whenever these two series converge. Surprisingly, his proof depends on an application of Ramsey's theorem; see below.

In a further paper [**59**] Rado proves that if f is a function from a real Banach space X to a real Banach space Y, and $\Sigma f(x_n)$ has bounded partial sums in Y whenever Σx_n converges in X, then f is continuous and linear near the origin.

Inequalities. See [**11**, **12**, **14**, **27**, **36**, **39**, **47**, **50**, **80**, **82**]. Here we quote a typically atypical result. Whereas most of the papers on this theme are concerned with inequalities of classical types, in one paper [**47**] Rado investigates the minimal sum that can be obtained by a suitable rearrangement of a transfinite sequence of ordinal numbers. In particular, he determines the least sum that can be obtained by rearranging the sum of all ordinals less than any given ordinal.

Geometry and measure theory. See [**29**, **34**, **38**, **40**, **42**, **43**, **55**, **72**, **73**, **74**]. Let $\mathscr{K}$ be a family of convex bodies in $\mathbb{R}^n$. Write

$$\sigma^*(\mathscr{K}) = \inf_{\Omega} \sup_{\Theta} \left\{ \left| \bigcup_{\theta\in\Theta} K_\theta \right| \Big/ \left| \bigcup_{\omega\in\Omega} K_\omega \right| \right\},$$

the infimum being over all bounded non-empty families $\{K_\omega : \omega \in \Omega\}$ of sets from $\mathscr{K}$, and the supremum being over all disjoint subfamilies $\{K_\theta : \theta \in \Theta\}$ with $\Theta \subset \Omega$. Rado introduces this definition in [**34**] and studies $\sigma^*(\mathscr{K})$ for various families $\mathscr{K}$ of convex bodies. In particular, he shows that

$$(3^n - 7^{-n})^{-1} < \sigma^*(\mathscr{C}_n) \leqslant 2^{-n},$$

when $\mathscr{C}_n$ is the family of all cubes in $\mathbb{R}^n$ with their faces parallel to the coordinate planes. He conjectures that $\sigma^*(\mathscr{C}_n) = 2^{-n}$; this conjecture remains open.

Besicovitch and Rado constructed a plane set of measure zero that contains circles of each positive radius [**74**]. This particular result was obtained independently by J. R. Kinney $\langle \mathbf{6} \rangle$ at about the same time. The method of Besicovitch and Rado seems to be more general; they claim rather casually: 'It will be clear from the method we use that there are other families of curves which can be treated in the same way, such as confocal conics or, more generally, any one-parameter family of algebraic curves, and many more.' It seems to me that the phrase 'such as confocal conics' must be taken to imply that the one-parameter family of algebraic curves depends in a very smooth way on the parameter. These papers rekindled interest in this type of problem (see J. R. Marstrand $\langle \mathbf{9} \rangle$). The problem of whether a set of measure zero can contain translates of all plane algebraic curves remains open.

Graphs. See [**56**, **60**, **64**, **78**, **86**, **88**, **91**, **97**, **106**, **109**], and other papers [**22**, **24**, **32**] which we prefer to regard as part of the first main theme. Most of Rado's work on graph theory is concerned with properties of hypergraphs and of infinite graphs of various types.

Number theory. See [**3**, **7**–**10**, **20**, **26**, **30**, **41**, **89**, **98**]. Apart from one paper [**3**], which we discuss in some detail below, most of these papers are expository.

Miscellaneous articles. See [**1**, **2**, **25**, **28**, **37**, **47**, **49**, **51**, **54**, **57**, **61**, **63**, **72**, **85**, **87**, **94**, **95**, **104**, **105**, **108**, **110**, **112**–**117**]. Here and elsewhere, the titles usually indicate the subject of the papers. We draw special attention to a paper [**61**], written with Chao Ko and Erdős in 1938 but published only in 1961. It is shown that if n and k are positive integers with $k \leqslant \frac{1}{2}n$, and $A_1, A_2, \ldots, A_t$ are subsets of a set S with cardinal $|S| = n$ and

$$A_i \cap A_j \neq \varnothing, \quad \text{for } 1 \leqslant i \leqslant j \leqslant t,$$
$$A_i \not\subset A_j, \quad \text{for } 1 \leqslant i \leqslant t,\ 1 \leqslant j \leqslant t,$$
$$|A_i| \leqslant k, \quad \text{for } 1 \leqslant i \leqslant t,$$

then necessarily

$$t \leqslant \frac{n-1}{k-1}.$$

This result and the problems raised in the paper have given rise to a substantial body of combinatorial theory with many interesting results and conjectures (see Erdős ⟨**2**⟩).

Some of the other articles are concerned with the popularization of mathematics, most are of genuine interest, some are important, and others may well prove to be important. Some of Rado's work has borne its best fruit many years after it was written.

Hall's theorem and abstract independence. See [**6**, **16**, **21**, **22**, **24**, **31**, **32**, **33**, **53**, **62**, **67**, **69**, **70**, **81**, **90**, **96**, **99**, **110**, **111**, **119**]. The theorem of P. Hall ⟨**5**⟩ to which we refer is simple to state. Let $T_1, T_2, \ldots, T_m$ be a finite system of subsets of a finite set S. In order that it be possible to find $a_1, a_2, \ldots, a_m$ with

$$a_i \neq a_j, \quad \text{for } 1 \leqslant i < j \leqslant m,$$

and

$$a_i \in T_i, \quad \text{for } 1 \leqslant i \leqslant m,$$

it is necessary and sufficient that for each k, $1 \leqslant k \leqslant m$, *each selection of* k *sets from* $T_1, T_2, \ldots, T_m$ shall contain between them at least k distinct elements of S.

As Hall remarks, this generalizes a result of D. König ⟨**7**⟩ and also Rado's [**6**] generalization of König's result. This beautiful result clearly captured Rado's imagination; he obtained various parallel results and generalizations. In particular, to obtain a common generalization of a result on vectors and on polynomials [**27**], he introduced an abstract notion of the independence of subsets of a given set, a notion that he later realized had been introduced earlier by H. Whitney ⟨**16**⟩. Rado proved the following result.

Let a relation of independence, satisfying appropriate axioms, be defined on the subsets of a given set S. Let $A_1, A_2, \ldots, A_n$ be subsets of S. There will be a set of independent elements $a_1, a_2, \ldots, a_n$ in S with

$$a_i \in A_i, \quad 1 \leqslant i \leqslant n,$$

if, and only if, for each k, $1 \leqslant k \leqslant n$, and each set $v_1, v_2, \ldots, v_k$ with

$$1 \leqslant v_1 < v_2 < \ldots < v_k \leqslant n,$$

the union $\bigcup\{A_{v_i} : 1 \leqslant i \leqslant k\}$ contains some set of k independent elements of S.

This theorem subsequently proved to be of great importance in transversal theory (and in the equivalent theory of matroids).

Some articles [**16**, **22**, **24**] obtain Hall-type conditions on two-measure functions f and g, of a very general nature, defined on the vertices of a directed graph, and are necessary and sufficient to ensure that f can be transformed into g by a sequence of moves that transfer positive elements of measure along the directed edges of the graph. These give far-reaching versions of Hall's theorem.

Marshall Hall ⟨**4**⟩ extended Philip Hall's theorem to the case when $\{T_v : v \in N\}$ is an arbitrary family of *finite* subsets of an arbitrary set S.

A year later, Rado [**33**] extended Whitney's theory of abstract independence for finite sets to infinite sets. To do this, he introduced the concept of an independent base for an infinite set, and showed that all independent bases for a given set have the same cardinal. This cardinal becomes the cardinal rank of the infinite set. A key to this theory is the following selection lemma.

Let A and N be sets and let A_ν be a finite subset of A for each $\nu \in N$. Suppose that for each finite set L contained in N, we are given a choice function $f_L : L \to A$ such that

$$f_L(\nu) \in A_\nu \quad \text{for } \nu \in L,$$

and

$$f_L(\nu) \neq f_L(\mu) \quad \text{when } \nu, \mu \in L \text{ with } \nu \neq \mu.$$

Then there is a choice function $f^* : N \to A$ such that

$$f^*(\nu) \neq f^*(\mu) \quad \text{when } \nu, \mu \in N \text{ with } \nu \neq \mu,$$

and for any finite subset L of N there is a second finite subset M of N with $L \subset M$ and

$$f^*(\nu) = f_M(\nu) \quad \text{for all } \nu \text{ in } L.$$

The proof is rather complicated; Rado gave a much simpler proof [**81**]. Again, this selection lemma has proved to be of great importance in transversal theory.

He obtained several results concerning the possibilities of representing independence structures by the linear independence of suitably chosen vectors in a vector space over a field or a division ring [**53**].

Any attempt to describe the major significance of Rado's work for transversal theory would take us too far from our aim of describing Rado's direct mathematical contributions. So we refer the reader to the books by L. Mirsky ⟨**10**⟩ and D. J. A. Welsh ⟨**15**⟩.

Ramsey's theorem and partition relations. See [**3**, **4**, **5**, **15**, **19**, **23**, **35**, **43–46**, **48**, **49**, **52**, **56**, **58**, **65**, **66**, **68**, **71**, **75**, **76**, **77**, **79**, **80**, **83**, **84**, **92**, **93**, **100–103**, **107**, **118**, **120**]. The main starting points for Rado's first substantial paper [**3**] were theorems of B. L. van der Waerden and I. Schur. Van der Waerden (see ⟨**14**⟩ for an interesting account of the discovery of this result) had proved that: if k and l are positive integers, then there is a number $f(k, l)$ such that, if N is a positive integer with $N \geqslant f(k, l)$ and $\{1, 2, \ldots, N\}$ is divided into k sets, then at least one set contains an arithmetic progression of length $l+1$.

Rado, with good reason, describes this result as extraordinarily interesting. When he first heard about it, he disbelieved it and tried hard to disprove it. As often happens, his attempts to disprove the result led to a deep understanding of its nature. The result of I. Schur was of a similar nature concerning solutions of the Diophantine equation $x+y=z$ in one at least of the subsets of $\{1, 2, \ldots, N\}$.

Rado obtains far-reaching generalizations of these results. In particular, he considers a system of equations

$$\sum_{\nu=1}^{n} \alpha_{\mu\nu} x_\nu = 0, \quad 1 \leqslant \mu \leqslant m,$$

with integral coefficients, having solutions $(x_1, x_2, \ldots, x_n)$ in positive integers. He gives fairly complicated conditions on the coefficients $\alpha_{\mu\nu}$ (that can be easily checked in special cases) that are necessary and sufficient to ensure that when the sequence of positive integers is divided into a finite number of sequences, then at least one of the sequences contains a solution of the system of equations. The results of van der Waerden and Schur follow as immediate consequences. These investigations, which include many other results, are taken further in [**4**, **23**].

F. P. Ramsey ⟨**12**⟩ had shown that if n is a positive integer and Δ is an arbitrary distribution of all sets of n positive integers into a finite number of classes, then there exists an infinite set M of positive integers which has the property that all sets of n numbers from M belong to the same class of Δ. Early in 1934, Erdős wrote to Rado and asked: 'Is it true that when S is a set of infinite cardinal, and the countable subsets of S are split into two classes, then there is always an infinite subset S^* of S all of whose countable subsets are in the same class?' Rado replied with a counterexample almost immediately. A little later, Rado used Ramsey's theorem in a quite unexpected way in his paper [**19**] on products of infinite series; see above.

In a further study [**35**] Erdős and Rado introduce canonical distributions. For each positive integer n, they introduce $2n$ 'canonical' distributions $\Delta_i^{(n)}$, $1 \leqslant i \leqslant 2^n$, of the n-element subsets of the positive integers into a number (usually infinite) of classes. There are two extreme canonical distributions, $\Delta_i^{(n)}$ say, where all n-element sets belong to a single class, and $\Delta_{2^n}^{(n)}$ say, where each n-element set is assigned to its own class. They prove the following surprising and far-reaching generalization of Ramsey's theorem.

Let Δ be an arbitrary distribution of the n-element sets of positive integers into classes. Then there is an infinite subset N^* of the positive integers and an i with $1 \leqslant i \leqslant 2n$ such that the distribution Δ, when restricted to the n-element subsets of N^*, coincides with the distribution $\Delta_i^{(n)}$. Later, this paper had a major influence on the development of Ramsey theory (see R. L. Graham *et al.* ⟨**3**⟩).

It is now time to introduce the notation of the partition calculus invented by Rado. In one of its simpler forms, this uses the symbol

$$a \longrightarrow (b_h)^r_{h\in H}$$

if H is an arbitrary set, or more simply

$$a \longrightarrow (b_1, b_2, \ldots, b_k)^r$$

if $H = \{1, 2, \ldots, k\}$, where a, r and b_h for $h \in H$ are either cardinals or order types, as an abbreviation of the following statement. If an arbitrary set A of type a (that is, of cardinal a or order type a) is given and the system $[A]^r$ of subsets of A of type r is partitioned in the form $\{I_h : h \in H\}$, then there is an h in H and a set B_h of type b_h such that the system of subsets $[B_h]^r$ of B_h of type r all belong to I_h. The symbol

$$a \longrightarrow\!\!\!\!\!\!\not\quad (b_h)^r_{h\in H}$$

is used to abbreviate the negation of the above statement. One advantage of this shorthand notation is that the statement

$$a \longrightarrow (b_h)^r_{h\in H}$$

implies

$$a' \longrightarrow (b'_h)^{r'}_{h\in H}$$

whenever

$$a', r, b_h, \quad h \in H,$$

include

$$a, r', b'_h, \quad h \in H.$$

Thus the statement is stable if the variables on the left are reduced and those on the right are increased. If all the b_h, $h \in H$, have a common value, b say, we can write

$$a \longrightarrow (b)^r_{h \in H}$$

without risk of confusion.

In this notation, Ramsey's theorem takes the form

$$\aleph_0 \longrightarrow (\aleph_0)^r_{1 \leqslant h \leqslant n}$$

with r and n any positive integers. Rado's negative response to Erdős' 1934 question becomes

$$\aleph_0 \not\longrightarrow (\aleph_0, \aleph_0)^{\aleph_0}.$$

In [**65**] Rado and Milner investigate the partition relation

$$\alpha \longrightarrow (\alpha_0, \ldots, \alpha_\kappa, \ldots)_{\kappa < k}$$

where k, α and α_κ, $\kappa < k$, are all ordinal numbers. Note that the exponent r takes the value 1 (by implication), and so the relation is concerned with elements from α rather than with subsets of α. They describe a procedure that leads, in a finite number of steps, from any choice of a finite sequence of ordinals $\alpha_0, \alpha_1, \ldots, \alpha_{k-1}$ to a calculation of the least ordinal α for which the relation

$$\alpha \longrightarrow (\alpha_0, \alpha_1, \ldots, \alpha_{k-1})$$

holds. Once the reader is familiar with the notation, and provided note is taken of the context within which it is used, it becomes a very convenient and concise way of expressing partition relations and their negations.

Rado himself, and Rado with Erdős and with other collaborators, continued to study partition relations for many years, investigating canonical partition relations and partition relations for cardinals, for ordinals, for order types and for matrices. Erdős ⟨**2**⟩ gives an excellent account of his joint work with Rado. Most of the work is described in detail in the book [**118**] by Erdős, Hajnal, Máté and Rado. This book goes some way towards an exposition of the importance of the theory for the theory of cardinal numbers and for mathematical logic. This lies, in part, in the fact that the truth of various partition relations turns out to be independent of the Zermelo–Fraenkel axioms, and so we are provided with whole scales of potentially important new axioms for use in set theory.

Mathematical work in general

This inadequate account of Rado's mathematical work can best be supplemented by reading his original papers and the book he wrote with Erdős *et al.* [**118**]. The paper by H. Lenz ⟨**8**⟩ is also worth study. The account by Erdős ⟨**2**⟩ of his joint work

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with Rado should certainly be read. An account edited by C. Richards ⟨**13**⟩ of the presentation to Rado on his 65th birthday contains interesting speeches by Mirsky and by Rado, and also a good photograph of Richard and Luise.

Honours and appointments

Lecturer, University of Sheffield, 1936–47. Reader, King's College, University of London, 1947–54. Professor of Pure Mathematics, University of Reading, 1954–71, and Emeritus Professor from 1971. Canadian Commonwealth Fellow, University of Waterloo, Ontario, 1971–72. Visiting Professor, University of Calgary, 1973–74.

London Mathematical Society: Council, 1948–57; Hon. Sec., 1953–54; Vice-President, 1954–56; Senior Berwick Prize, 1972.

Fellow of the Royal Society, 1978.

Chairman of the British Combinatorial Committee, 1977–83; Richard Rado Lecture instituted at the British Combinatorial Conference, 1985.

Dr rer. nat. hc., Freie University, Berlin, 1981. Hon. DMath, University of Waterloo, Canada, 1986. Hon. Fellow, Fitzwilliam College, Cambridge, 1987.

Foundation editor of *Mathematika*, 1954. Member of the editorial boards of *Aequationes Mathematicae*, *Discrete Mathematics*, *Journal of Combinatorial Theory*, *Combinatorica*, *Asian Journal of Graphs and Combinatorics*.

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⟨**9**⟩. J. M. MARSTRAND, 'Packing smooth curves in $\mathbf{R}^q$', *Mathematika* 26 (1979) 1–12.

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⟨**11**⟩. L. MIRSKY (ed.), *Studies in pure mathematics: Papers in combinatorial theory, analysis, geometry, algebra and the theory of numbers, presented to Richard Rado on the occasion of his sixty-fifth birthday* (Academic Press, London, 1971).

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⟨**14**⟩. B. L. VAN DER WAERDEN, 'How the proof of Baudet's conjecture was found', in ⟨**11**⟩ 251–260.

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This list of the papers of Richard Rado and his collaborators is based on his own list. The numbers seem to give the order in which the papers were submitted for publication. Number 103a has been included for completeness; it was omitted from Rado's list probably because similar results had been recently published by L. Lovász.

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2. 'Zur Boltzmannschen Theorie des zweiten Hauptsatzes', *Erkenntnis* 3 (1931) 101–102.
3. 'Studien zur Kombinatorik', *Math. Z.* 36 (1933) 424–480.
4. 'Verallgemeinerung eines Satzes von van der Waerden mit Anwendung auf ein Problem der Zahlentheorie', *Sitzungsber. Preuss. Akad. Wiss. Phys.-Math. Kl.* (1933) 589–596.
5. 'Fragen der Kombinatorik in der Theorie der diophantischen Gleichungen', *Jahresber. Deutsch. Math.-Verein.* 42 (1933) 121–124.
6. 'Bemerkungen zur Kombinatorik im Anschluss an Untersuchungen von Herrn D. König', *Sitzungsber. Berlin. Math. Ges.* 42 (1933) 60–75.
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16. 'A theorem on general measure functions', *Proc. London Math. Soc.* 44 (1938) 61–91.
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26. 'A theorem on the geometry of numbers', *J. London Math. Soc.* 21 (1946) 34–47.
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