A NOTE ON SALEH'S PAPER 'ALMOST CONTINUITY IMPLIES CLOSURE CONTINUITY'†

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Recently, Saleh [3] claimed to have solved 'a long standing open question' in topology; namely, he proved that every almost continuous function is clousure continuous ($=\theta=$continuous). Unforunately, this problem was settled long time ago and even a better result is known. Consider the following implications:

$$\text{Cont.} \Rightarrow \text{Almost cont.} \Rightarrow \text{Almost } \alpha\text{-cont.} \Rightarrow \eta\text{-cont.} \Rightarrow \theta\text{-cont.} \Rightarrow \text{Weakley cont.}$$

The first two implications are trivial. In [2], the second author showed that every almost α-continuous function is η-continuous. In [1], Dickman, Porter and Rubin proved that every η-continuous is θ-continuous and hence weakly continuous.

Although θ-continuous functions behave, in general, nicely, they may cause some unexpected problems. For example, if $f: X \to Y$ is θ-continuous, then $f: X \to f(X)$ is not necessarily θ-continuous. Also, the set of all points of continuity of $f: X \to Y$ may be dense in X and f may not be θ-continuous at any of these points. We show that with the following example.

Example. Consider the classical Dirichlet function $f: (\mathbb{R}, \tau_d) \to (\mathbb{R}, \tau_d)$, where $\mathbb{R}$ is the real line with the density topology τ_d:

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q}, \\ 0, & \text{otherwise.} \end{cases}$$

It is easily observed that f is continuous at every irrational point (furthermore, $\mathbb{P}$, the set of all irrationals is dense in the density topology). On the other hand, f is not weakly θ-continuous at any irrational point.

REFERENCES

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