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CERTAIN RESULTS OF REAL HYPERSURFACES IN A COMPLEX SPACE FORM

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Abstract. First, we classify a real hypersurface of a non-flat complex space form with (i) semi-parallel $T(= \pounds_\xi g)$, and (ii) recurrent T. Next, we characterise a real hypersurface admitting the generalised η-*Ricci soliton* in a non-flat complex space form.

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1. Introduction. A complex n-dimensional Kaehler manifold of constant holomorphic sectional curvature c is called a complex space form and is denoted by $\overline{M}^n(c)$. A complete and simply connected complex space form is a complex Euclidean space C^n, if $c = 0$, a complex projective space $P_n(C)$, if $c > 0$ or a complex hyperbolic space $H_n(C)$, if $c < 0$. Takagi [**17**, **18**] first characterised all homogeneous real hypersurfaces in $P_n(C)$ into six model spaces A_1, A_2, B, C, D and E. Thereafter, Cecil and Ryan [**2**] (see also [**9**]) studied extensively that when the structure vector field ξ is principal and showed that they are realised as the tubes over certain submanifolds in $P_n(C)$ by using its focal map. On the other hand, Berndt [**1**] classified all homogeneous real hypersurfaces in $H_n(C)$ with ξ as principal vector and divided into four model space A_0, A_1, A_2 and B. Let M be a real hypersurface of a non-flat complex space form. Then M has an almost contact metric structure (φ, ξ, η, g) induced from the complex structure J. Many differential geometers studied real hypersurfaces of a complex space form under various conditions on the Ricci tensor, the shape operator A (in the direction of the unit normal of M), curvature tensor etc. For a real hypersurface of a complex space form, we now define the tensor T by

$$g(TX, Y) = (\pounds_\xi g)(X, Y) = g((\varphi A - A\varphi)X, Y), \tag{1}$$

for all vector fields X, Y tangent to M. A typical characterisation for a real hypersurface M of type A in a complex space form $\overline{M}^n(c)$ was given under the condition $g(TX, Y) = 0$, for any tangent vector fields X and Y on M. Under this condition Okumura [**15**], for $c > 0$, and Montiel-Romero [**13**], for $c < 0$ proved the following:

THEOREM A. *Let M^{2n-1} be a real hypersurface in a non-flat complex space form. If M satisfies $A\varphi = \varphi A$, then M is locally congruent to real hypersurface of type A.*

Let M be a real hypersurface of type A in $\overline{M}^n(c)$. Then it follows from Theorem A that M naturally satisfies $\nabla_X T = 0$. Thus, as a generalisation of Okumura's condition $g(TX, Y) = 0$, for any tangent vector fields X and Y on M, here we consider the real hypersurfaces M of a non-flat complex space form $\overline{M}^n(c)$ with semi-parallel tensor T (i.e. $R.T = 0$, where R is the curvature tensor of M) and prove that such hypersurface is the Hopf hypersurface and also locally congruent to one of type A in $P_n(C)$ or $H_n(C)$. We also consider a real hypersurface of a non-flat complex space form with recurrent T and prove that such hypersurface is locally congruent to one of type A in $P_n(C)$ or $H_n(C)$. We discuss these issues in Section 3.

It is well known [**5**] that there are no real hypersurfaces with parallel Ricci tensor in a non-flat complex space form $\overline{M}^n(c)$ when $n \geq 3$. This is also true for $n = 2$ as was pointed out by Kim [**7**]. Since the Einstein manifold has parallel Ricci tensor, it is easy to observe that there do not exist the Einstein real hypersurfaces in a non-flat complex space form. For this Kon [**10**], studied and classified the *pseudo-Einstein* (that is there exist constants λ, μ such that the Ricci tensor S satisfies $S = \lambda I + \mu\eta \otimes \eta$) real hypersurfaces of a complex space form $\overline{M}^n(c)$ when $n \geq 3$. Recently, Kim–Ryan [**8**] proved that every pseudo-Einstein hypersurface in $P_2(C)$ or $H_2(C)$ is the Hopf hypersurface. Now we recall some classification theorems of the pseudo-Einstein type real hypersurfaces in $P_n(C)$ (see [**2**, **10**]) or $H_n(C)$ (see [**12**]).

THEOREM B. *Let M^{2n-1} ($n \geq 3$) be a real hypersurface of $P_n(C)$ with Fubini-study metric of constant holomorphic sectional curvature* 4. *Then M is pseudo-Einstein if and only if M is locally congruent to one of the following:*
(A_1) *A geodesic hypersphere of radius r, where* $0 < r < \frac{\pi}{2}$.
(A_2) *A tube of radius r over a totally geodesic $P_k(C)$ $(1 \leq k \leq n-2)$, where* $0 < r < \frac{\pi}{2}$ *and* $\cot^2 r = \frac{k}{n-k-1}$.
(B) *A tube of radius r over a complex quadric Q^{n-1} and P_nR, where* $0 < r < \frac{\pi}{4}$ *and* $\cot^2 2r = n-2$.

THEOREM C. *Let M^{2n-1} ($n \geq 3$) be a real hypersurface of $H_n(C)$ with Bergman metric of constant holomorphic sectional curvature* -4. *Then M is pseudo-Einstein if and only if M is locally congruent to one of the following:*
(A_0) *A horosphere.*
(A_1) *A geodesic hypersphere or a tube over a complex hyperbolic hyperplane $H_{n-1}(C)$.*

Moreover, we remark that *a tube over a totally geodesic $H_l(C)$* $(1 \leq l \leq n-2)$ is known as a A_2-type hypersurface of $H_n(C)$, $n \geq 3$. Note that real hypersurfaces of types A_1 and A_2 (without extra restriction $\cot^2 r = \frac{k}{n-k-1}$) in $P_n(C)$ and of types A_0, A_1 and A_2 in $H_n(C)$ are simply known as a real hypersurfaces of type A.

A Ricci soliton is a generalisation of Einstein metric and is defined on a Riemannian manifold (M, g) by a vector field V and a constant λ

$$(\pounds_V g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) = 0, \tag{2}$$

where $\pounds_V$ denotes the Lie-derivative operator along V, S is the Ricci tensor of g and X, Y are arbitrary vector fields on M. It can be viewed as a fixed point of the Hamilton's Ricci flow: $\frac{\partial}{\partial t}g_{ij} = -2R_{ij}$, as a dynamical system, on the space of Riemannian metrics modulo diffeomorphisms and scalings. For details we refer to Chow–Knopf [**4**]. Recently, Cho–Kimura [**3**] considered real hypersurfaces of a non-flat complex space form that admits the Ricci soliton with $V = \xi$ and proved that such hypersurface does not exist. For this reason, Cho–Kimura [**3**] defined the so-called *η-Ricci soliton* by taking $V = \xi$ and adding an extra term $\mu\eta \otimes \eta$ in the left-hand side of (2), i.e.

$$\frac{1}{2}\pounds_{\xi}g + S + \lambda g + \mu\eta \otimes \eta = 0,$$

for constants λ and μ. Under this assumption they proved that M is *pseudo-Einstein* (or η-umbilical). Moreover, as a generalisation of η-*Ricci soliton*, one may consider real hypersurfaces M of a complex space form $\overline{M}^n(c)$ satisfying

$$\frac{1}{2}(\pounds_{\xi}g)(X, Y) + S(X, Y) + \lambda g(X, Y) = 0, \tag{3}$$

for all tangent vectors X, Y orthogonal to ξ and λ is constant. We call this a generalised η-*Ricci soliton*. Note that there exist real hypersurfaces that admit a η-*Ricci soliton* and hence generalised η-*Ricci soliton*. In fact, it is straight forward to see that any η-*umbilical* real hypersurface of a complex space form admits such a structure. Thus, as a generalisation of Cho–Kimura's result we classify real hypersurfaces M of complex space form $\overline{M}^n(c)$ satisfying equation (3). We discuss this matter in Section 4.

2. Real hypersurfaces in a complex space form. In this section we recall some basic equations and formulas that we shall use later on. For details about the real hypersurfaces of a complex space form we refer to Niebergall–Ryan [**14**]. Let M be a real hypersurface of a Kaehler manifold $(\overline{M}, J, \overline{g})$. For any vector field X tangent to M, we put

$$JX = \varphi X + \eta(X)\xi, \tag{4}$$

$$JN = -\xi, \tag{5}$$

where φ is a tensor field of type (1, 1), η is a 1-form and ξ is a unit vector field on M. We denote the induced metric of M by g. From equation (4) it is easy to see that (φ, ξ, η, g) gives an almost contact metric structure on M, that is

$$\varphi^2 X = -X + \eta(X)\xi, \eta(\xi) = 1, \tag{6}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{7}$$

for all vector fields X, Y on M. From these equations it is easy to see that $\varphi\xi = 0$ and $\eta \circ \varphi = 0$. The Gauss and Weingarten formulas for M are given by

$$\overline{\nabla}_X Y = \nabla_X Y + g(AX, Y), \overline{\nabla}_X N = -AX,$$

where $\overline{\nabla}$ and ∇ are the Levi-Civita connection of $\overline{M}$ and M, respectively. Making use of these formulas, equations (4) and (5) and $\overline{\nabla}J = 0$ (as $\overline{M}$ is Kaehler) it follows that

$$(\nabla_X \varphi)Y = \eta(Y)AX - g(AX, Y), \tag{8}$$

$$\nabla_X \xi = \varphi AX, \tag{9}$$

where A is the second fundamental tensor of M. Now we suppose that the Kaehler manifold $\overline{M} = \overline{M}(c)$ is a complex space form. Then we have the following Gauss and

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Codazzi equations:

$$R(X, Y)Z = \frac{c}{4}\{g(Y, Z)X - g(X, Z)Y + g(\varphi Y, Z)\varphi X - g(\varphi X, Z)\varphi Y \\ -2g(\varphi X, Y)\varphi Z\} + g(AY, Z)AX - g(AX, Z)AY, \tag{10}$$

for any tangent vector fields X, Y, Z on M. From equation (10), we get

$$SX = \frac{c}{4}\{(2n+1)X - 3\eta(X)\xi\} + hAX - A^2X, \tag{11}$$

where h is the trace of A. If the vector field ξ is a principal curvature vector in a non-flat complex space form, i.e. $A\xi = \alpha\xi$, then M is called the Hopf hypersurface of $\overline{M}(c)$. Such hypersurfaces have some remarkable properties. Note that for $c \neq 0$, α is constant (see [**6**, **10**, **11**, **14**]).

3. Real hypersurfaces with semi parallel T.

THEOREM 1. *Let M be real hypersurface of a non-flat complex space form. If the tensor T is semi-parallel, then M is locally congruent to a type A hypersurface.*

Proof. By hypothesis, we have

$$R(X, Y)T - TR(X, Y) = 0,$$

from which we get

$$g(R(X, Y)TZ, W) - g(R(X, Y)Z, TW) = 0. \tag{12}$$

Setting $Z = W = \xi$ the foregoing equation yields

$$g(R(X, Y)T\xi, \xi) = 0. \tag{13}$$

Now, from equation (1), $T\xi = \varphi A\xi$ and hence equation (13) reduces to $g(R(X, Y)\varphi A\xi, \xi) = 0$. Thus, in view of this we obtain from equation (10)

$$\frac{c}{4}\{g(Y, \varphi A\xi)\eta(X) - g(X, \varphi A\xi)\eta(Y)\} + g(AY, \varphi A\xi)g(AX, \xi) \\ - g(AX, \varphi A\xi)g(AY, \xi) = 0.$$

Next, putting $Y = \varphi A\xi$ and since $g(A\varphi A\xi, \xi) = 0$, the foregoing equation implies that

$$\frac{c}{4}g(\varphi A\xi, \varphi A\xi)\eta(X) + g(A\varphi A\xi, \varphi A\xi)g(AX, \xi) = 0. \tag{14}$$

Finally, taking $X = \varphi A\varphi A\xi$ in equation (14) provides $g(A\varphi A\xi, \varphi A\xi) = 0$. Making use of this in equation (14) and since M is non-flat, we see that ξ is principal, i.e. $A\xi = \alpha\xi$. Utilising this and taking $Y = Z = \xi$ in equation (12), we get $TR(X, \xi)\xi = 0$. Let X be any principal vector orthogonal to ξ corresponding to the principal curvature λ, i.e. $AX = \lambda X$. Then $R(X, \xi)T\xi = 0$ since $T\xi = 0$. Also from the Gauss equation (10) it follows that $R(X, \xi)\xi = (\alpha\lambda + \frac{c}{4})X$. Thus,

$$0 = TR(X, \xi)\xi = \left(\alpha\lambda + \frac{c}{4}\right)(A\varphi - \varphi A)X,$$

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so that unless there is a principal curvature satisfying $\alpha\lambda + \frac{c}{4} = 0$, we are finished by Theorem A. Suppose λ is such a principal curvature so that $AX = \lambda X$ and $(A\varphi - \varphi A)X \neq 0$. The well-known properties of principal curvatures of Hopf hypersurfaces (see [**14**, pp 245–246]) give a principal curvature μ such that $A\varphi X = \mu\varphi X$. Since $(A\varphi - \varphi A)X = (\mu - \lambda)X$ we have $\mu \neq \lambda$. This is a contradiction as the same argument applied to μ and φX gives $\alpha\mu + \frac{c}{4} = 0$. This completes the proof. □

REMARK 1. In [**16**], Pyo–Suh proved that a real hypersurface M of a non-flat complex space form $\overline{M}^n(c)$, $n \geq 2$, satisfying $\mathcal{L}_\xi R = 0$ is of type A. We can prove this result by applying Theorem 1. In fact, Lie differentiating the identity

$$g(R(X, Y)Z, W) + g(R(X, Y)W, Z) = 0,$$

using $\mathcal{L}_\xi R = 0$ and (1), it follows that $(R(X, Y)T)Z = 0$.

Next we prove the following.

THEOREM 2. *Let M be real hypersurface of a non-flat complex space form with recurrent T. Then M is locally congruent to one of type A in $P_n(C)$ or $H_n(C)$.*

Proof. By hypothesis T is recurrent, i.e. there exists a 1-form π such that

$$(\nabla_X T)Y = \pi(X)TY, \tag{15}$$

for all vector fields Y, Z on M. Clearly T is symmetric. Suppose T has a non-zero eigenvalue σ, for otherwise $T = 0$ and by Theorem A, M will be congruent to one of type A in $P_n(C)$ or $H_n(C)$. Let Y be a unit vector and $TY = \sigma Y$. Then by (15), we have

$$\pi(X)g(TY, Y) = g((\nabla_X T)Y, Y) = g(\nabla_X(TY), Y) - g(\nabla_X Y, TY).$$

Using $TY = \sigma Y$ the foregoing equation shows that

$$(X\sigma)g(Y, Y) + \sigma g(\nabla_X Y, Y) - \sigma g(\nabla_X Y, Y) = \sigma\pi(X)g(Y, Y),$$

which, in turn, gives $X\sigma = \sigma\pi(X)$. Writing this consequence as $d\sigma = \sigma\pi$ and operating this by d (operator of exterior differentiation) and using the Poincaré lemma, $d^2 = 0$, we obtain

$$0 = d^2\sigma = d\sigma \wedge \pi + \sigma d\pi = \sigma(\pi \wedge \pi) + \sigma d\pi,$$

i.e. $\sigma d\pi = 0$. At this point we take an open set N of all points p of M such that $\sigma(p) \neq 0$. Then on N, $d\pi = 0$, i.e.

$$(\nabla_X \pi)Z = (\nabla_Z \pi)X. \tag{16}$$

Now, for any X, Y and $Z \in T_pM$ and $p \in$ N, by differentiating (15) covariantly with respect to Z, we obtain

$$(\nabla_Z\nabla_X T)Y = \{(\nabla_Z\pi)X\}TY + \pi(Z)\pi(X)TY.$$

Interchanging Z and X we have

$$(\nabla_X\nabla_Z T)Y = \{(\nabla_X\pi)Z\}TY + \pi(X)\pi(Z)TY.$$

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Making use of these equations, together with the Ricci identity and (16) we find that

$$R(X, Z)TY - TR(X, Z)Y = 0.$$

Therefore, following the proof of Theorem 1 it is easy to see that $T = 0$ and so $\sigma = 0$ on N. Thus, we arrive at a contradiction and hence $\varphi A = A\varphi$. Using Theorem A, we complete the proof. □

4. Generalised η-Ricci soliton.

THEOREM 3. *Let M be real hypersurface of a non-flat complex space form admitting a generalised η-Ricci soliton. If the tensor $g(TX, Y)$ of M vanishes for all X, Y orthogonal to ξ, then M is pseudo-Einstein.*

Proof. In view of equation (1), the hypothesis $g(TX, Y) = 0$, for all X, Y orthogonal to ξ implies $g((A\varphi - \varphi A)X, Y) = 0$, for all X, Y orthogonal to ξ, which is equivalent to

$$\varphi A\varphi^2 X - \varphi^2 A\varphi X = 0,$$

for all X tangent to M. Operating this by φ and replacing X by φX, the foregoing equation provides

$$(A\varphi - \varphi A)X - g(A\varphi X, \xi)\xi + \eta(X)\varphi A\xi = 0. \tag{17}$$

Since M admits a generalised η-Ricci soliton, equation (3) is equivalent to

$$g(\nabla_{\varphi X}\xi, \varphi Y) + g(\nabla_{\varphi Y}\xi, \varphi X) + 2S(\varphi X, \varphi Y) + 2\lambda g(\varphi X, \varphi Y) = 0 \tag{18}$$

for all vectors X, Y tangent to M. Making use of equations (9) and (11), the foregoing equation yields

$$\varphi A\varphi^2 X - \varphi^2 A\varphi X + \varphi A^2\varphi X - h\varphi A\varphi X - \left\{2\lambda + \frac{(2n+1)c}{2}\right\}\varphi^2 X = 0$$

for all vectors X tangent to M. Therefore, use of (6) the last equation entails that

$$\begin{aligned}(A\varphi - \varphi A)X - g(A\varphi X, \xi)\xi + \eta(X)\varphi A\xi + \varphi A^2\varphi X \\ - h\varphi A\varphi X - \{2\lambda + \frac{(2n+1)c}{2}\}\varphi^2 X = 0.\end{aligned} \tag{19}$$

Feeding equation (17) into (19) provides

$$\varphi A^2\varphi X - h\varphi A\varphi X - \{2\lambda + \frac{(2n+1)c}{2}\}\varphi^2 X = 0. \tag{20}$$

Operating equation (20) by φ we get an equation and replacing X by φX in equation (20) gives another equation. Differentiating them yields

$$\begin{aligned}(\varphi A^2 - A^2\varphi)X + g(A^2\varphi X, \xi)\xi - \eta(X)\varphi A^2\xi \\ + h\{(A\varphi - \varphi A)X - g(A\varphi X, \xi)\xi + \eta(X)\varphi A\xi\} = 0.\end{aligned}$$

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Thus, in view of equation (17), the preceding equation shows that

$$(\varphi A^2 - A^2\varphi)X + g(A^2\varphi X, \xi)\xi - \eta(X)\varphi A^2\xi = 0. \tag{21}$$

In other words

$$g((\varphi A^2 - A^2\varphi)X, Y) = 0, \tag{22}$$

for all tangent vectors X, Y orthogonal to ξ. Now, operating equation (17) by A gives

$$(A^2\varphi - A\varphi A)X - g(A\varphi X, \xi)A\xi + \eta(X)A\varphi A\xi = 0. \tag{23}$$

Further, replacing X by AX, equation (17) transforms into

$$(A\varphi A - \varphi A^2)X - g(A\varphi AX, \xi)\xi + g(AX, \xi)\varphi A\xi = 0. \tag{24}$$

Adding equation (23) with (24) and taking into account equation (22) it follows that

$$g(AX, \xi)g(\varphi A\xi, Y) + g(\varphi A\xi, X)g(AY, \xi) = 0, \tag{25}$$

for all tangent vectors X, Y orthogonal to ξ. Since $\varphi\xi = 0$, the vector fields $\varphi^2 A\xi$ and $\varphi A\xi$ are orthogonal to ξ. Therefore, if we replace X by $\varphi^2 A\xi$ and Y by $\varphi A\xi$, then equation (25) shows $|\varphi A\xi|^4 = 0$, which implies $\varphi A\xi = 0$, that is $A\xi = \alpha\xi$. This, together with the hypothesis $(g(A\varphi - \varphi A)X, Y) = 0$, for all X, Y orthogonal to ξ implies that $A\varphi = \varphi A$. Moreover, using $A\xi = \alpha\xi$ in equation (11), we see that $S\xi = \beta\xi$, where $\beta = \frac{c(n-1)}{2} + h\alpha - \alpha^2$. Making use of equation (9), $g(A\varphi - \varphi A)X, Y) = 0$ in equation (18), we find that

$$S(\varphi X, \varphi Y) + \lambda g(\varphi X, \varphi Y) = 0.$$

Finally, replacing X by φX and Y by φY in the foregoing equation and since $S\xi = \beta\xi$ we see that M is pseudo-Einstein. □

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